Three-dimensional modeling of density and Mach number profiles in the SOL of a limiter plasma

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Introduction

This work is focused on the interaction of edge plasma with wall components (limiters) in the SOL-edge region. Limiters of various geometries and configurations are placed in the SOL-edge region to absorb the heat flux coming from the core. Our purpose is to study how the 3D geometry of limiters can modify the density and Mach number profiles at the steady state in the SOL. Physics in the proximity of the plasma-limiter transition is complex, even though in a simplified model of the quasi-neutral plasma outside of the sheath, limiters can be considered like density and momentum sinks: once the plasma reaches the limiter, it is completely absorbed. This physical property of limiters will let us implement a volumetric condition within a penalization method.

Simulations modeling SOL-edge region in 1D [1] and 2D [2] have been performed using a penalization method which we have extended to a 3D version. The advantage of this method is that all information about the limiters geometry is carried by a mask function χ which allows us to study several configurations by simply changing its definition, without re-meshing the calculation domain. In addition, Bohm boundary conditions on limiter surfaces can be recovered from a volumetric condition which is easier to implement.

The model and the equations

The tokamak's domain is topologically equivalent to a periodic cylinder and so a cylindrical coordinate system $(r,\theta,\phi R_0)$ can be used, where R_0 is the major radius, $r \in [0.7,1]$, $\theta \in [0,2\pi]$ and $\phi \in [0,2\pi]$. We are interested in the steady-state profiles, in this situation the magnetic field $\vec{B}$ is considered fixed and has two components B_θ and B_ϕ , its direction is fully determined by safety factor q= rB_ϕ/R_0B_θ.

In the SOL-edge region an isothermal and quasi-neutral plasma can be modeled like a compressible fluid for which the density field N and the momentum field Γ projected along the magnetic field direction verify the system of dimensionless conservative equations:

$$\partial_t N + \nabla_\parallel \Gamma - D\nabla_r^2 N = 0, \tag{1}$$

$$\partial_t \Gamma + \nabla_{\|} \left(\frac{\Gamma^2}{N} + N \right) - D\nabla_r^2 \Gamma = 0, \tag{2}$$

where ∇_r^2 is the Laplacian in the radial direction describing a background micro-turbulence and $\nabla_{\|}$ is the gradient along the magnetic field direction describing the parallel dynamics which is the dominant in the system. From the point of view of hyperbolic systems the Mach number Mach$= \frac{\Gamma}{N}$ on the plasma-limiter interface is a sonic point (|Mach| =1).

The penalization method

This method is used to model the plasma-limiter interaction and to recover the boundary condition |Mach|=1 on the limiter sides from a volume condition. Limiters are introduced in the previous system of Equations (1) and (2) as objects absorbing density and momentum through two terms, which bring density and momentum to zero in the limiter.

$$\partial_t N + \nabla_{\|} \Gamma + \underbrace{\frac{\chi}{\eta}(N - N_\Omega)}_{\text{Density penalization}} - D\nabla_r^2 N = 0, \tag{3}$$

$$\partial_t \Gamma + \nabla_{\|} \left(\frac{\Gamma^2}{N} + N \right) + \underbrace{\frac{\chi}{\eta}(\Gamma - N M_\Omega)}_{\text{Momentum penalization}} - D\nabla_r^2 \Gamma = 0 \tag{4}$$

The characteristic function χ carries the information of limiter configuration, it takes the value 1 inside the limiters and 0 outside while inside the limiter the density is imposed to the value $N_\Omega = 10^{-7}$ and the Mach number profile is fixed via M_Ω. The penalization parameter η is taken $\eta < 10^{-7}$.

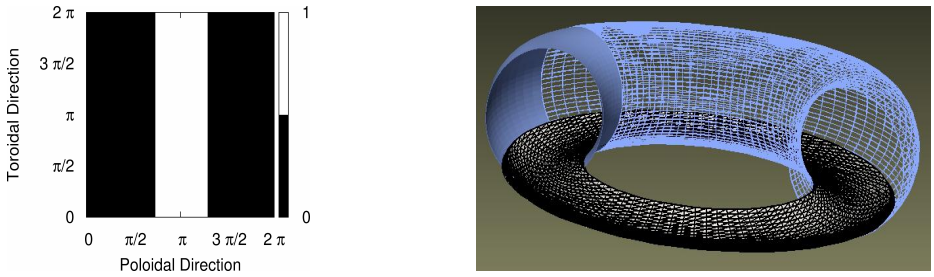

Figure 1: Left: example of a mask function χ for an axisymmetric limiter plotted in the last magnetic surface. The white region represents the limiter. Right: 3D representation of χ, the axisymmetric limiter is in black.

We apply this method to the study of two cases: • We replace a toroidally symmetric limiter by a set of non-axisymmetric limiters. • We study a configuration composed by two limiters one toroidally symmetric and a secondary limiter non-axisymmetric. We want to study the toroidal asymmetry in density and Mach number profiles induced by the secondary limiter.

Results

Set of non-axisymmetric limiters

We have replace an axisymmetric limiter by a set of three non-axisymmetric limiters representing the 60% of the original volume.

The average mean density $\langle N \rangle = \int\int \mathrm{d}\theta \mathrm{d}\phi N$ in the SOL region is plotted for two different safety factor values and is compared to the respective $\langle N \rangle^{axi}$ profile of an axisymmetric limiter.

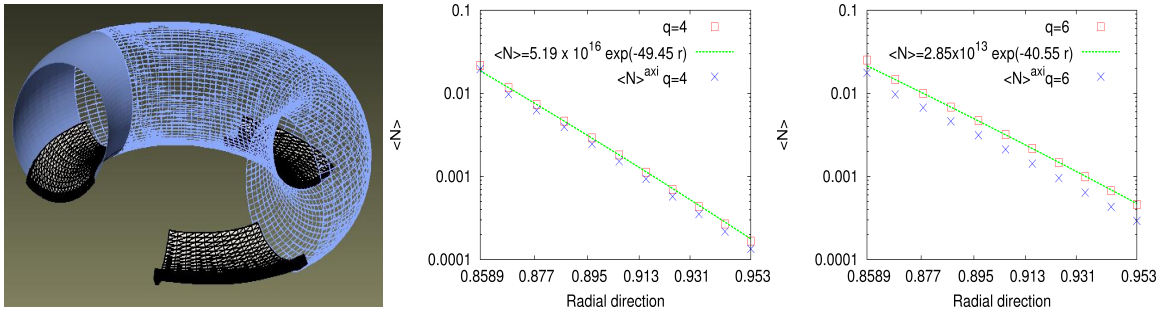

Figure 2: Left:3D representation of a set of three non-axisymmetric limiters. Right:Mean average density profile $\langle N \rangle (r)$ in the SOL (r>0.8) for two different values of "q".

In both cases $\langle N \rangle$ decays exponentially with e-folding lengths $\lambda_{q=4} = 202 \times 10^{-4}$ and $\lambda_{q=6} = 246 \times 10^{-4}$, while the corresponding e-folding lengths for the axisymmetric limiter configuration are $\lambda^{axi}_{q=4} = 204 \times 10^{-4}$ and $\lambda^{axi}_{q=6} = 246 \times 10^{-4}$. The set of non-axisymmetric limiters behaves like a axisymmetric one.

This result is not overly surprising since, in the chosen limiter configuration and with the chosen safety factors, the toroidal gap between limiters is not sufficient to let magnetic field lines perform one poloidal turn before reaching one of the limiter sides. The difference between the configuration described and an axisymmetric limiter is that a fraction of the plasma here impacts the limiter on the toroidally facing sides of the limiter (instead of only on the poloidally facing sides).

Axi-symmetric limiter and a non-axisymmetric limiter

Mach number profile in the last magnetic surface is presented. It is important to note that the Bohm condition (Mach=$\pm$1) in the limiter surfaces is still verified regardless of the limiter geometry or configuration.

Strong variations are induced in the toroidal direction, as can be seen in the toroidal density profile Figure 3 Right which crosses the non-axisymmetric limiter. At the same poloidal angle ($\theta = \frac{23\pi}{18}$) the ratio between the minimum and maximum values of the density is 5.

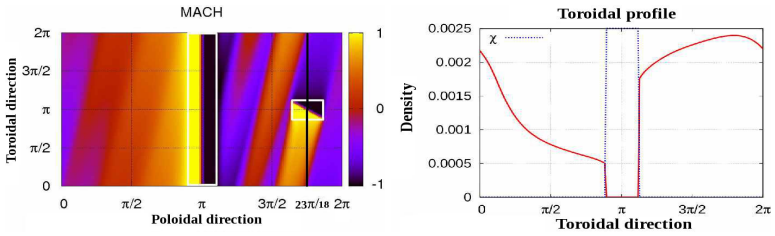

Figure 3: Left: Mach profile in the last magnetic surface. The areas within the solid line corresponds to the limiters. Right: Toroidal density profile at the poloidal coordinate $\theta = \frac{23\pi}{18}$ crossing the secondary limiter (black line on the left plot).

Concluding remarks

The penalization technique used in the 3D simulation let us recover the Bohm boundary condition |Mach| =1 on the limiters sides from a volumetric condition and independently of the limiters geometry or its configurations. We have shown two numerical experiments where an axisymmetric limiter has been replaced by a set of non-axisymmetric limiters. In both cases the mean average density $\langle N \rangle$ decays exponentially in the SOL region and the calculated e-folding lengths λ_q coincide with the respective λ_q^{axi}, so the set of non-axisymmetric limiters behave like an axisymmetric one. The non-axisymmetric geometry of a secondary limiter can give rise to a significant density variations.

References

[1] L. Isoardi *et al.* 2009 Penalization modeling of a limiter in the Tokamak edge plasma. *Journal of Computational Physics* **229** 2220-2235

[2] H. Bufferand *et al.* 2010 Applications of SOLEDGE-2D code to complex SOL configurations and analysis of Mach probe measurements. *Journal of Nuclear Materials* In Press, Corrected Proof