Non-linear modelling of curvature and diamagnetic effects on tearing modes

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Introduction

Tearing mode is a class of resistive MHD instabilities, which should be avoided in fusion devices, for causing confinement degradation and plasma disruptions. It consists of a magnetic island that can be linearly unstable or metastable [1] in the presence of stabilization mechanisms such as toroidal curvature [2] or diamagnetic rotation [3]. In the present work we investigate these issues using the non linear full MHD code XTOR-2F [4], where anisotropic heat transport, diamagnetic and geometrical effects are described, and by means of analytical theory for the linear dispersion relation which includes transport, diamagnetic and curvature effects. In spite of its simplicity, this model allows one to recover qualitatively previously known results in the limit of zero diamagnetic frequency. One of the fully non-inductive pulses on Tore Supra (TS-32299, $t = 234s$) is chosen as a test bed for our studies. Previous attempts based on one-fluid MHD simulations and using realistic transport coefficients [5] failed to explain successful discharges without detectable magnetic island at $q = 2$ surface.

Diamagnetic and curvature effects on linear stability and metastability

In order to study numerically the impact of diamagnetic rotation on the $n = 1$, $m = 2$ mode we perform a scan of the linear growth rate with diamagnetic frequency. For this purpose the number of toroidal modes in the simulations was limited to 2, i.e. only $n = 0$ and $n = 1$ were taken into account. Linear growth rate is defined as a half-logarithmic derivative of the magnetic energy stored in a mode with time, averaged over linear phase. It is important to point out that realistic Lundquist numbers (for Tore-Supra typically $S \sim 10^8 - 10^9$) are hardly achievable for present numerical capabilities due to the fact that modes develop thin layer, of width $\delta \sim S^{-2/5}$, and their growth rates are small ($\gamma \propto S^{-3/5}$). The central value $S(0) = 10^7$ is used, to be compared to its experimental value $S(0)_{exp} = 4.6 \times 10^8$. Using lower Lundquist numbers, imposes to rescale simulation parameters in order to keep important physical behaviour unchanged. In

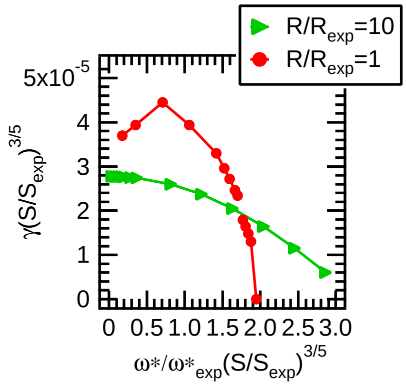

Figure 1: *Linear growth rate vs diamagnetic frequency for different curvatures.*

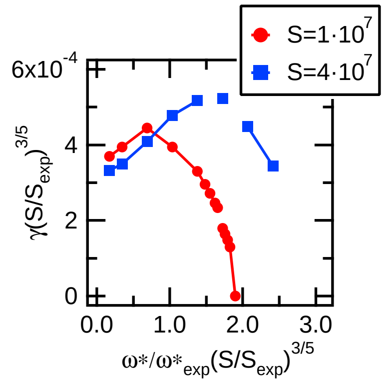

Figure 2: *Linear growth rate vs diamagnetic frequency for different values of S*

present simulations we keep experimental ratio of the energy confinement time to the resistive time, $\tau_E/\tau_R = (\tau_E/\tau_R)_{exp}$, which corresponds to the choice $S\chi_\perp = (S\chi_\perp)_{exp}$. We also use $\chi_\parallel/\chi_\perp = 10^8$, and the viscosity is chosen such that magnetic Prandtl number $Prm = \mu_0 \nu/\eta = 5$. The experimental value of Alfvén time is $\tau_A = 2.1 \times 10^{-7} s$, normalized electron diamagnetic frequency at the resonant surface ($\sqrt{\Phi}_{q=2} = 0.45$) $\tau_A \omega_e^* = 4.1 \times 10^{-3}$. The results obtained have to be rescaled in the proper way so as to be compared with the experiment. As a starting point we have chosen rescaling $(S/S_{exp})^{3/5}$ for growth rate and frequencies, implied by a linear dispersion relation $\gamma \propto \omega - \omega^*$, where S_{exp} is the experimental S-number and $\omega^* = -\vec{k}_\theta \nabla p \times \vec{B}/qnB^2$ is the diamagnetic frequency.

Simulations of linear growth regime show that diamagnetic rotation tends to destabilize the mode at low values, and to stabilize at higher values. In order to discriminate the impacts of the diamagnetic and curvature effects we reduced impact of curvature. To this end, the major radius was increased by a factor of ten ($R = 10R_{exp}$). All aspect ratio dependent quantities were rescaled accordingly. These simulations show that higher curvature makes mode stabilization by diamagnetic effect more efficient Fig. 1.

In order to investigate the issue of metastability, seed islands with different sizes were introduced into the plasma with values of ω^*, corresponding to linearly stable domain. The results of these simulations are summarized in Fig. 4. Depending on the seed island size and diamagnetic frequency three different regimes are observed Fig. 4. Small seeds grow in size and saturate at low amplitude (blue), middle size seeds grow exponentially and saturate at high amplitudes (in longer simulations)(green) and large island are stationary (red).

Analytical model

The dispersion relation including both diamagnetic and curvature effects has been derived using constant-ψ approximation. Assuming that pressure equilibrium is determined by thermal

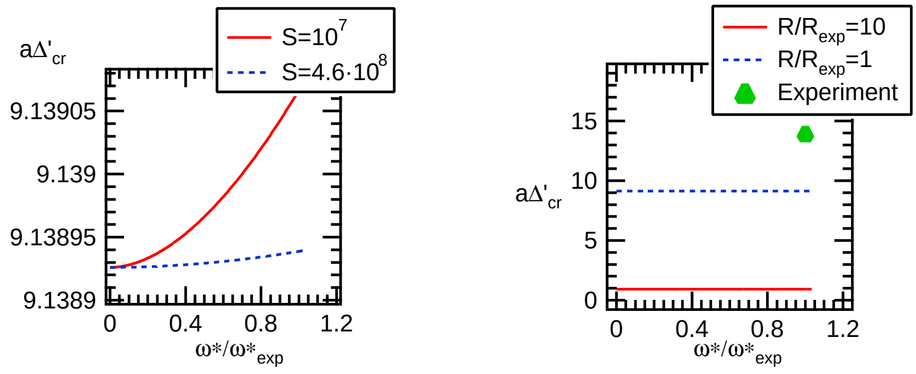

Figure 3: *Critical value of* $a\Delta'$ *vs. diamagnetic frequency for different values of major radius (R) and Lundquist number (S) in case of diffusive transport (*D_R *from equilibrium code CHEASE.)*

conductivities, i.e. $\nabla\cdot(\chi_\perp\nabla_\perp p)+\nabla\cdot(\chi_\parallel\nabla_\parallel p)=0$ then the dispersion relation takes the form:

$$a\Delta' = 2\pi\frac{\Gamma(3/4)}{\Gamma(1/4)}\frac{x_s^{1/2}}{(\varepsilon ns)^{1/2}}\frac{1}{\eta^{3/4}}(\lambda+i\omega_i^*)^{1/4}(\lambda+i\widehat{\omega}_e^*)-\frac{\pi^{3/2}}{2}\frac{D_s}{\varepsilon_\chi} \tag{1}$$

with $\varepsilon_\chi^4=\chi_\perp x_s^2/\chi_\parallel(\varepsilon ns)^2$, $\lambda=-i\omega$, $\varepsilon=a/R$, resistivity η and magnetic shear s. In toroidal geometry, the cylindrical resistive interchange parameter $D_s=-2x_s p'/s^2B^2$ is typically replaced by its toroidal version $D_R\approx(1-q^2)D_s$. This dispersion relation casts into Lütjens-like one [6] in the limit $\omega^*=0$. Taking into account the fact that D_R is negative in toroidal geometry, there is a positive threshold $a\Delta'_{cr}=-\pi^{3/2}D_R/2\varepsilon_\chi$ for tearing mode stability. As shown on Fig. 3, toroidal curvature is the dominant stabilization mechanism in this case and critical $a\Delta'$ exhibits very weak dependency on diamagnetic frequency. In the case with no diffusive transport the dispersion relation is found to be

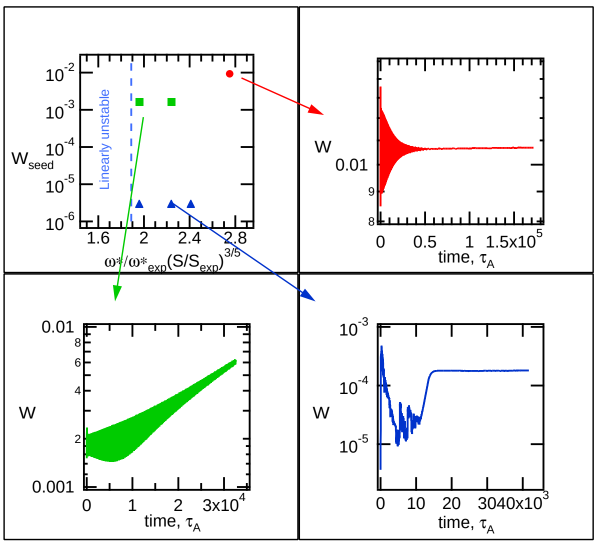

Figure 4: *Different metastable regimes*

$$\begin{aligned} a\Delta' &= 2\pi\frac{\Gamma(3/4)}{\Gamma(1/4)}\frac{x_s^{1/2}}{(\varepsilon ns)^{1/2}}\frac{1}{\eta^{3/4}}\lambda^{1/4}(\lambda+i\omega_i^*)^{1/4}(\lambda+i\omega_e^*)^{3/4}- \\ &-\frac{\pi^{3/2}}{2}\frac{(\varepsilon ns)^{1/2}}{x_s^{1/2}}\frac{1}{\eta^{1/4}}\frac{(\lambda+i\omega_e^*)^{1/4}}{\lambda^{1/4}(\lambda+i\omega_i^*)^{1/4}}D_s \end{aligned} \tag{2}$$

The linear stability threshold $a\Delta'_{cr}$ can be obtained by cancelling imaginary part $Im(Eq.\ 2)=0$.

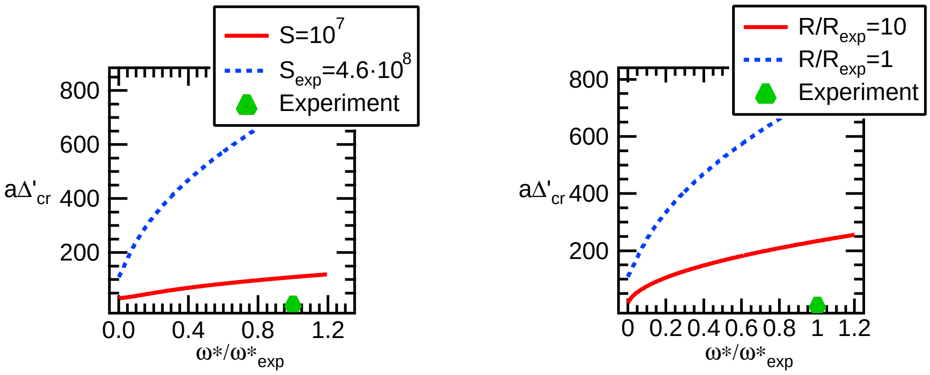

Figure 5: *Critical value of aΔ' vs. diamagnetic frequency for different values of major radius (R) and Lundquist number (S) in case of convective transport*

In this case $\gamma = 0$ and so $\lambda = -i\omega_r$. One obtains

$$a\Delta'_{cr} = \frac{\pi^{5/4}}{\eta^{1/2}} \left(\frac{\Gamma(3/4)}{\Gamma(1/4)} (-D_s) \right)^{1/2} \cdot G \cdot (\omega_r - \omega_e^*)^{1/2} \tag{3}$$

with $G \approx 2.68$. In the limit $\omega^* = 0$ expression (Eq. 3) becomes similar to GGJ result [2] with $\omega_r \propto (-D_s)^{2/3}\eta^{1/3}$ and $a\Delta'_{cr} \propto (-D_s)^{5/6}/\eta^{1/3}$. Numerical solution of (Eq. 3) is shown on Fig. 5. In this case critical $a\Delta'$ depends both on curvature and diamagnetic frequency and stabilization is more efficient for higher Lundquist number.

Conclusions

The dependence of tearing mode stability on diamagnetic rotation and toroidal curvature has been studied. Our simulations show that diamagnetic rotation can fully stabilize the (2,1) tearing mode and curvature plays an important role for this stabilization. We found that strong diamagnetic rotation leads to metastability of the mode and depending on the size of the seed island, different saturated states were observed. A reduced analytical model for two different transport regimes was developed to support the simulation results. According to the model for diffusion dominant regime, experimental point is in unstable domain for any values of ω^* while it is in stable region for any ω^* in case when convective transport is dominant. It seems that experimental situation is in between these two asymptotic regimes. The fact that full stabilization and metastability of the (2,1) mode is obtained at values of diamagnetic frequency higher than experimental one suggests that the proposed mechanism may not be sufficient to explain the results observations made on Tore Supra.

References

[1] M. Ottaviani, F. Porcelli, and D. Grasso, Physical Review Letters **93**, (2004).
[2] A. H. Glasser, J. M. Greene, and J. L. Johnson, Physics of Fluids **18**, (1975).
[3] B. Coppi, Physics of Fluids **7**, 1501 (1964).
[4] H. Lütjens and J. F. Luciani, Journal of Computational Physics **229**, (2010).
[5] P. Maget *et al.*, Plasma Physics and Controlled Fusion **51**, (2009).
[6] H. Lütjens, J.-F. Luciani, and X. Garbet, Physics of Plasmas **8**, (2001).