Drift interchange turbulence driven magnetic islands

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In tokamaks, macro-scale MHD instabilities coexist with micro-scale turbulent fluctuations. Magnetic islands can, in particular, coexist with pressure driven instabilities such as interchange modes and/or turbulence. Several experiments and numerical studies report the coexistence of turbulence and MHD activities showing some correlated effects [1, 2, 3, 4]. We address here the multi-scale-nonlinear dynamics between macro-scale tearing instabilities and gradient pressure driven micro-instabilities (resistive interchange) by solving reduced MHD equations numerically.

We consider a two-dimensional slab plasma model that includes magnetic curvature effects and consists of cold ions and isothermal electrons. The basic evolution equations are [4],

$$\frac{\partial}{\partial t}\nabla_\perp^2\phi + \left[\phi, \nabla_\perp^2\phi\right] = \left[\psi, \nabla_\perp^2\psi\right] - \kappa_1\frac{\partial p}{\partial y} + \mu\nabla_\perp^4\phi, \tag{1}$$

$$\frac{\partial}{\partial t}p + [\phi, p] = -v_\star\left((1-\kappa_2)\frac{\partial\phi}{\partial y} + \kappa_2\frac{\partial p}{\partial y}\right) + \rho_\star^2\left[\psi, \nabla_\perp^2\psi\right] + \chi_\perp\nabla_\perp^2 p, \tag{2}$$

$$\frac{\partial}{\partial t}\psi = [\psi, \phi - p] - v_\star\frac{\partial\psi}{\partial y} + \eta\nabla_\perp^2(\psi - \psi_0), \tag{3}$$

where the dynamical field quantities are ϕ, the electrostatic potential, p the electron pressure, ψ the magnetic flux and ψ_0 the equilibrium magnetic flux. The equilibrium magnetic field is given by $\mathbf{B}_{eq} = B_{0z}\hat{\mathbf{y}} + \psi_0'(x)\hat{\mathbf{z}}$ where B_{0z} is constant. The equilibrium quantities consist of a constant pressure gradient and a magnetic field corresponding to a Harris current sheet model [5]. Equations (1-3) are normalized using the characteristic Alfvén speed v_A, the Alfvén time τ_A and a characteristic magnetic shear length scale $L_\perp$. Further, $\kappa_{i=\{1,2\}}$ include curvature and gradient pressure effects. μ is the viscosity, $\chi_\perp$ the perpendicular diffusivity, η is the plasma resistivity, $v_\star$ is the normalized electron diamagnetic drift velocity and $\rho_\star$ is the normalized Larmor radius. This model use in fact a reduced version of the four fields model derived in reference [6], neglecting parallel ion dynamics.

The impact of interchange turbulence on the formation of a magnetic island is investigated by means of linear and nonlinear simulations of equations (1-3). A semi-spectral code is used including a $2/3$ dealiasing rule in the y (poloidal) direction, a resolution of 256 grid points

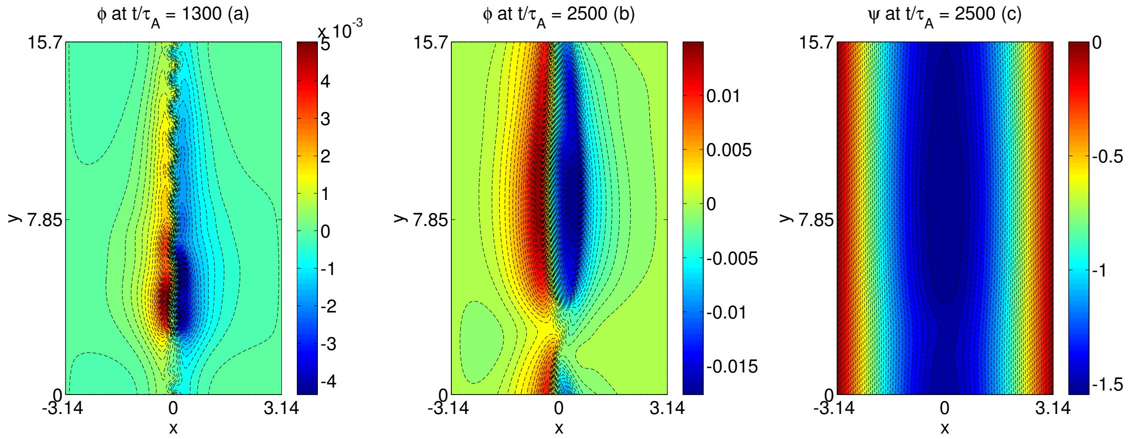

Figure 1: $\Delta' = -0.45$. Snapshots of the electrostatic potential ϕ at $t = 1300\tau_A$ (a), at $t = 2500\tau_A$ (b) and snapshots of the magnetic flux ψ at $t = 2500\tau_A$ (c).

in the x (radial) direction and 64 poloidal modes. The computational box size is $L_x = 2\pi$ and $L_y = 5\pi$. The perturbed fields are periodic in the y (poloidal direction) and are set to zero at the radial boundaries. The Fourier decomposition of the fields is typically defined as $\psi(x,y,t) = \sum_{m\in Z} \psi_m(x,t)\exp(ik_m y)$ with $k_m = 2\pi m/L_y$. The parity (odd or even symmetry in the spatial coordinate) of the eigen-functions $\psi_m(x,t)$, $\phi_m(x,t)$, $p_m(x,t)$ provides a distinct marker of identification of a given mode m and helps in pinpointing the instability mechanism generating it. The resistive interchange mode m has (odd, even, even) parities with respect to $x \in [-L_x/2, L_x/2]$, for (ψ_m, ϕ_m, p_m) respectively, and (even, odd , odd) parities for tearing modes.

Our goal is to study the non linear impact of turbulence on magnetic island. The parameters are choosen in order to to let resistive interchange instability develop at small-scales and to let marginally stable tearing mode at large-scales: we have fixed $\hat{\rho} = 0.04$, $\nu_\star = 10^{-2}$, $\kappa_2 = 0.36$ and the dissipative parameters (μ, $\chi_\perp$, η) are taken to be equal to 10^{-4}. Thus, a large range of modes $m \geq 2$ are unstable with respect to the interchange instability and, therefore, present an interchange parity. We next categorize the most interchange modes number by $m_\star$ and its growth rate by $\gamma_{m_\star}$ such that $m_\star \gg 1$ (in our study, $m_\star = 17$). Moreover, the nature (parity) of the $m = 1$ mode depends on the competition between the interchange and tearing instabilities. In our study, the stiffness of the magnetic equilibrium profile is such as the mode $m = 1$ is stable with respect to the tearing mode ($\Delta' = -0.45$). It follows that $m = 1$ mode is unstable with respect with interchange instability even if $\gamma_1 \ll \gamma_{m_\star}$. In other words, linearly, no magnetic island can develop.

To investigate how the small scales interchange modes affect the formation of a magnetic island, we carry out nonlinear simulations. Fig.(1) presents the snapshots of the electrostatic potential ϕ at the end of the quasi linear regime at $t = 1300\tau_A$ (a) and during the fully nonlinear regime at $t = 2500\tau_A$ (b), as well as the the magnetic flux ψ (c): In the quasilinear phase, unstable interchange modes grow at small scales around the resonant surface and there is no magnetic island. However, during the nonlinear phase, despiste the negative value of Δ', large scales structures dominate on small scales structures and a magnetic island grows nonlinearly .

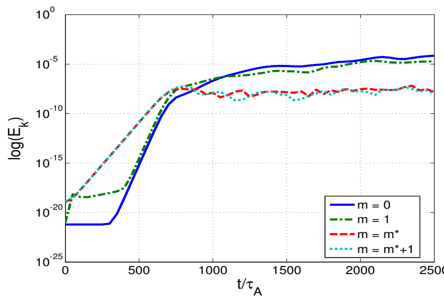

Figure 2: $\Delta' = -0.45$. Time evolution of the kinetic energy of the poloidal modes: $m = 0$, $m = 1$, $m = m\star$ and $m = m\star+1$.

To understand the mechanism leading to the nonlinear formation of the magnetic island, we present on Fig.(2) the kinetic energy time evolution for the modes of the simulation which dominate energetically either in the quasilinear phase or asymptotically . The dynamics presents different regimes. First, during the linear regime $t < 250\tau_A$, all the modes present the interchange parity and there is no island as expected. During the qualinear regime, $t \in [250, 750]\tau_A$, a beating of the interchange modes at small scales impacts on the growth of the large scale modes. More precisely, as shown by the Fig.(2), the most unstable mode $m_\star$ at small scales drives the generation of the large scale mode $m = 0$ and $m = 1$: Owing to bracket structures, $\gamma_0^{NL} = 2\gamma_{m\star}$ and $\gamma_1^{NL} = \gamma_{m\star} + \gamma_{m\star+1} \sim 2\gamma_{m\star} \gg \gamma_1^L$.

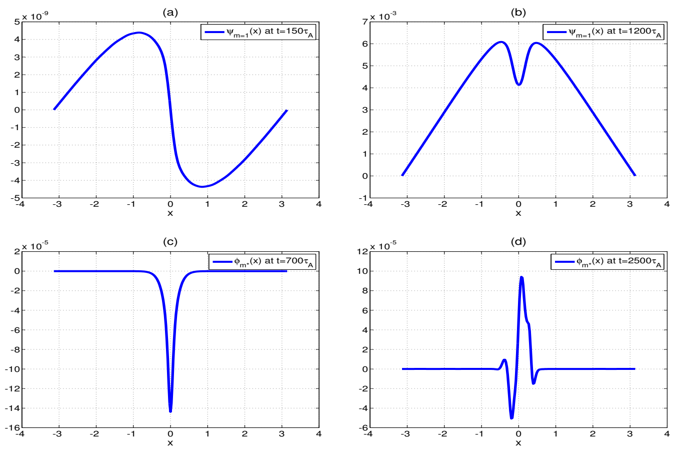

Figure 3: $\Delta' = -0.45$. Eigen functions of the magnetic flux for the mode $m = 1$, $\psi_1(x)$, during the linear regime at $t = 150\tau_A$ (a) and during the non linear regime at $t = 1200\tau_A$ (b). Eigen function $p_{m\star}(x)$, during the quasi linear regime at $t = 700\tau_A$ (c) and during the non linear regime at $t = 2500\tau_A$ (d).

Fig.(3) presents the eigen functions of the magnetic flux for the mode $m = 1$, $\psi_1(x)$, during the linear regime (a) and during the nonlinear regime (b). It shows that the beating of the interchange modes also leads to a change in the parity of the driven

$m = 1$ mode. As a consequence a magnetic island is nonlinearly generated by the pumping of the energy of small scale unstable interchange modes even when $\Delta' < 0$. Indeed, an important property of *all* the nonlinearities in eqns. (1-3) is that, if initially, the system is driven by small scale interchange modes int_{ss}, their mutual interactions can only drive tearing parity large scale fluctuations tear_{ls}: $\{\text{int}_{ss}, \text{int}_{ss}\} \rightarrow \text{tear}_{ls}$.

Then, from $t/\tau_A > 750$, island growth is slowed down and saturates asymptotically . Energetic saturation of small scales ($m \sim m_*$) is already reached while the large scale modes $m = 0$ and $m = 1$ are still feeded by it and dominate (Fig. 2). As shown on the Fig.(3) which presents the $m = m_\star$ pressure eigen functions $p_{m_\star}(x)$ during the quasilinear and the nonlinear regime, the interchange mode $m_\star$ starts to lose its parity and tends to get a tearing parity. A cascade directly from the large tearing scale to the small scales becomes dominant. Indeed, the nonlinear properties of eqns. (1-3) show that the mutual nonlinear interaction of large scale tearing modes tear_{ls} can drive only tearing parity small scale fluctuations tear_{ss} : $\{\text{tear}_{ls}, \text{tear}_{ls}\} \rightarrow \text{tear}_{ss}$. This mechanism changes the nature of the turbulence and together with the ohmic disssipation balances the pumping of the small-scales energy by the magnetic island. Let us underline the complexity of the dynamics by precising that asymptotic cascade properties are intermittent in time.

In conclusion, we have studied the effect of small-scale interchange turbulence on a marginally stable tearing mode. The presence of the interchange turbulence has a major influence on the excitation and evolution mechanisms of a magnetic island. As soon as the growth of the interchange modes is fast enough (*i.e.* $2\gamma_\star > \gamma_1$), a magnetic island can be generated at large scales thanks to a nonlinear beating of interchange modes at small scales. The presence of the island at large scales nonlinearly affects back the nature of the small scales turbulence, noteworthy caracterized by a modification of the small scale mode parities.

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