Impacts of resonant magnetic perturbations on stellarator plasmas

S. Nishimura[1], S. Toda[1], M. Yagi[2,3], Y. Narushima[1]

[1] *National Institute for Fusion Science, Toki, Japan*

[2] *Japan Atomic Energy Agency, Naka, Japan*

[3] *Research Institute for Applied Mechanics, Kyushu Univ., Kasuga, Japan*

Forced magnetic reconnection[1] driven by externally imposed resonant magnetic perturbations is of great interest to magnetic confinement fusion using toroidal plasma, because it might be applicable to a control method of magnetohydrodynamic (MHD) instability. Changing external perturbations or plasma equilibrium parameters slowly, sudden appearances and disappearances of imposed magnetic islands have been observed in stellarator plasmas[2]. Historically, magnetic islands and stochastic layers in stellarator plasmas have been investigated in the context of resistive MHD instability driven by averaged bad curvature[3], however mechanism of the sudden transitions is not clarified yet. In tokamak plasmas, similar transitions of externally imposed magnetic islands have been observed, and theoretical work revealed that plasma rotations can screen penetration of external perturbations[4]. In stellarator plasmas, poloidal rotations are generated by neoclassical particle diffusion associated with strongly rippled toroidal magnetic field. Therefore, poloidal rotations might play essential roles for stability of magnetic islands in stellarator plasmas.

In this study, the transition mechanism of magnetic islands in stellarator plasmas is investigated using a theoretical model of forced magnetic reconnection with neoclassical poloidal flows. The model is an extended version of that in our previous report[5]. In this paper, effects of averaged curvature and resistive flows are newly taken into account.

A stellarator plasma with toroidal magnetic field B_0, major radius R_0 and averaged minor radius a in cylindrical coordinates (r, θ, z) is considered, where r is position in the minor radial direction, θ is poloidal angle and z is toroidal position, respectively. We assume that perturbation of poloidal magnetic flux is given by $\tilde{\psi}_{m,n}(r)\exp(m\theta - nz/R_0 - \int \omega dt)$, where $\{m, n, \omega\}$ are poloidal and toroidal mode numbers and rotation frequency, respectively. Modified Rutherford equations are given by

$$\frac{dw}{dt} = \frac{r_s^2}{I_1 \tau_R}\left[\mathrm{Re}\left(\Delta'\right) + \Delta_{\mathrm{GGJ}}\right], \tag{1}$$

$$\frac{\partial \Theta}{\partial t} = \left(k_\theta v_\theta\right)\big|_{r_s} + \frac{2r_s^2}{I_1 \tau_R}\frac{1}{w}\mathrm{Im}\left(\Delta'\right), \tag{2}$$

where $I_1 = 0.82$, $k_\theta = m/r$, and τ_R is resistive diffusion time. Rational surface is located at $r = r_s$. v_θ is poloidal flow velocity. Magnetic island width is given by $w = 4\sqrt{L_s \psi_s / B_0}$, where

$\psi_s = \tilde{\psi}_{m,n}(r_s)$ and L_s is magnetic shear length. In Eq.(2), the rotation frequency is expressed as $\partial\Theta/\partial t$. $\Delta' = [\partial_r\tilde{\psi}_{m,n}|_{r_s+0} - \partial_r\tilde{\psi}_{m,n}|_{r_s-0}]/\psi_s$ characterizes an outer layer solution given by perturbed ideal MHD equilibrium with a boundary condition $\tilde{\psi}_{m,n}(a) = \psi_a$, where ψ_a indicates amplitude of externally imposed magnetic perturbations. In toroidal current-less equilibrium of helical plasmas, Δ' is given by

$$\mathrm{Re}\left(\Delta'\right) = -\Delta'_0\left(\frac{w_{\mathrm{vac}}^2}{w^2}\cos\Theta - 1\right), \tag{3}$$

$$\mathrm{Im}\left(\Delta'\right) = \Delta'_0\left(\frac{w_{\mathrm{vac}}^2}{w^2}\right)\sin\Theta, \tag{4}$$

where $w_{\mathrm{vac}} = 4\sqrt{(r_s/a)^m(L_s\psi_a/B_0)}$ and $\Delta'_0 = -2k_\theta/[1-(r_s/a)^m]$. Phase angle of magnetic islands matches that of externally imposed magnetic perturbations at $\Theta = 0$. In the derivation of Eqs.(1) and (2), a so-called constant-ψ approximation characterized by $|w\Delta'| \ll 1$ is used. Δ_{GGJ} in Eq.(1) indicates an effect of perturbed current driven by magnetic field line curvature[3]. Considering electron pressure perturbation near magnetic islands, we obtain

$$\Delta_{\mathrm{GGJ}} = \frac{I_2 D_{\mathrm{R}}}{\sqrt{w^2 + (I_3 w_{\mathrm{ce}})^2}}, \tag{5}$$

with $w_{\mathrm{ce}} = (\chi_\perp/\chi_\parallel)^{1/4}(L_s/k_\theta(r_s))^{1/2}$, $I_2 = 6.3$ and $I_3 = 5.4$. $\chi_\parallel$ and $\chi_\perp$ are thermal transport coefficients parallel and perpendicular to magnetic field lines, respectively. D_{R} is given by $D_{\mathrm{R}} = \kappa\beta_e L_s^2/L_{pe}$, where $\{\kappa, \beta_e, L_{pe}\}$ are averaged magnetic field line curvature in the toroidal direction, electron beta and electron pressure gradient scale length, respectively. Contribution of ion pressure perturbation is less important. An original nonlinear model of forced magnetic reconnection[1] is recovered by setting $\Theta = 0$ and $\Delta_{\mathrm{GGJ}} = 0$. When the second term on the right-hand side (RHS) of Eq.(2) is negligible, a so-called non-slip condition defined in Ref.[4] is satisfied. We give a time evolution equation of poloidal flows as

$$\frac{\partial v_\theta}{\partial t} = \sigma\frac{k_\theta v_A^2 w^3 \mathrm{Im}\left(\Delta'\right)}{256 L_s^2} + \mu\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right)\right] + f_\theta^{\mathrm{NC}}. \tag{6}$$

The first term on the RHS of Eq.(6) comes from Lorentz force among magnetic islands and external magnetic perturbations, where σ separates the inside and outside of magnetic islands as $\sigma = 1$ in $|r - r_s| \le w/2$ and $\sigma = 0$ in $|r - r_s| > w/2$. The second term on the RHS of Eq.(6) mimics viscous drag force by micro turbulence. The third term on the RHS of Eq.(6) represents neoclassical drag force among trapped particles and passing particles. f_θ^{NC} is given by

$$f_\theta^{\mathrm{NC}} = \frac{D_i^{\mathrm{neo}}}{\rho_i^2}\left(V_i^{\mathrm{neo}} - v_\theta\right) + \frac{D_e^{\mathrm{neo}}}{\rho_e^2}\left(V_e^{\mathrm{neo}} - v_\theta\right), \tag{7}$$

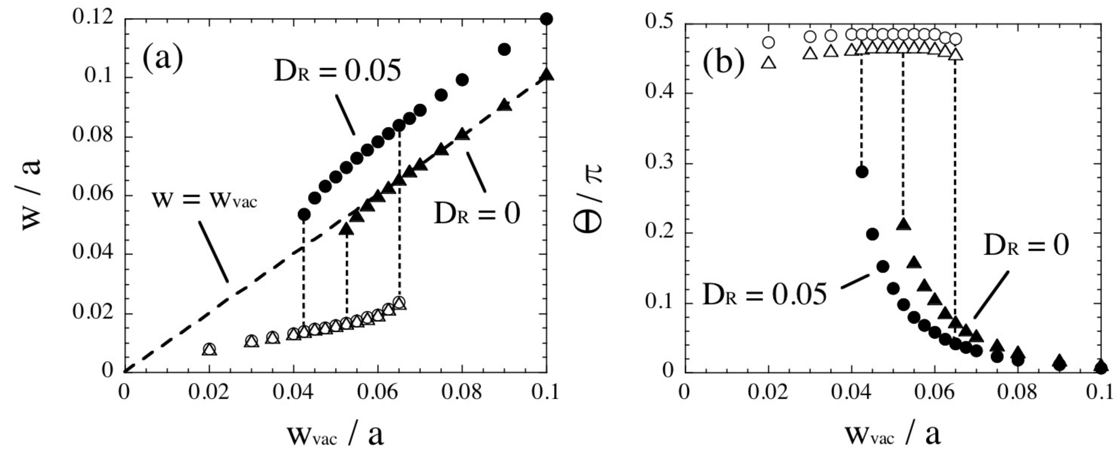

Figure 1: Vacuum island width dependence of (a) magnetic island width and (b) phase angle in stationary state.

where $\{D_{\mathrm{a}}^{\mathrm{neo}}, V_{\mathrm{a}}^{\mathrm{neo}}, \rho_{\mathrm{a}}\}$ are neoclassical diffusivity, neoclassical flow velocity and Larmor radius of $\mathrm{a} = \mathrm{i}, \mathrm{e}$ (ion, electron). When trapped particle orbit in toroidally rippled magnetic field is not strongly affected by magnetic islands, $D_{\mathrm{a}}^{\mathrm{neo}}$ might be well approximated by kinetic closure using an average over original, nested magnetic surface[6]. The modeling, (6) and (7), is basically the same as that in our previous report[5].

Equations (1)-(7) are solved in typical parameters of the LHD applying $(m,n) = (1,1)$ external magnetic perturbations[2], where the classical tearing mode is linearly stable, $a\Delta_0' = -7$. Our parameters are in so-called visco-resistive regime[4]. Because we focus on forced magnetic reconnection, we assume $\Delta_0' + \Delta_{\mathrm{GGJ}} < 0$.

In Fig.1, we change w_{vac} slowly, and find stationary states of magnetic island width and phase angle. Two cases are plotted: $D_{\mathrm{R}} = 0$ (open and closed triangles) and $D_{\mathrm{R}} = 0$ (open and closed circles). Two bifurcation branches are observed, i.e., fully reconnected, large state $w \sim w_{\mathrm{vac}}$ and suppressed, small state $w \ll w_{\mathrm{vac}}$. In the latter state, growth of magnetic islands is suppressed by screening effect of poloidal rotations. Note that magnetic islands do not rotate in both states. The bad curvature effect is found to enhance magnetic island width in the fully reconnected state, and changes range of the multiple state. Strictly speaking, the constant-ψ assumption is weakly violated in the suppressed state, i.e., $|w\Delta'| \sim 1$. For this reason, comparison of our results with fluid simulations is desirable.

Figure 2 shows radial profile of poloidal flow velocity, where the negative sign indicates the ion diamagnetic direction. The dashed line is an unperturbed poloidal flow. Two solid lines

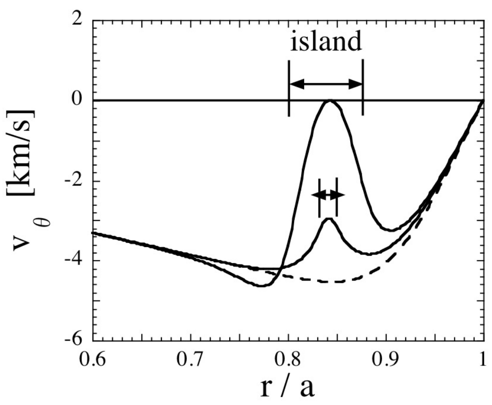

Figure 2: Radial profile of poloidal flows near magnetic islands.

indicate poloidal flows in the fully reconnected and suppressed states at $w_{\mathrm{vac}}/a = 0.06$. $(m,n) = (1,1)$ rational surface is located at $r_{\mathrm{s}}/a = 0.84$. Poloidal plasma flow is damped near magnetic islands in the fully reconnected state. Whereas, the plasma flow is not strongly damped in the suppressed state, and the plasma flow and the resistive flow cancel each other to stop magnetic island rotation.

In summary, a nonlinear model of forced magnetic reconnection in stellarator plasmas is developed. With neoclassical poloidal flows, externally imposed magnetic islands have two branches of stationary state, i.e., non-rotating large islands and non-rotating small islands. It is found that averaged bad curvature enhances width of large islands and helps to sustain this state. Detailed comparison between our results and both experimental observations and fluid simulations are left as future works.

This work was supported by Grand-in-Aid for JSPS Fellows. We also acknowledge the collaboration program of Research Institute for Applied Mechanics of Kyushu University.

References

[1] T. S. Hahm, and R. M. Kulsrud, Phys. Fluids **28**, 2412 (1985).

[2] Y. Narushima *et al.*, Nucl. Fusion **48**, 075010 (2008).

[3] M. Kotschenreuther, R. D. Hazeltine, and J. Morrison, Phys. Fluids **28**, 294 (1985).

[4] R. Fitzpatrick, Nucl. Fusion **33**, 085003 (1993).

[5] S. Nishimura *et al.*, Plasma and Fusion Research **5**, 040 (2010).

[6] K. C. Shaing, Phys. Fluids **27**, 1567 (1984).