

Analytical and Numerical Modelling of Transport Barrier Formation Using Bifurcation Concept

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In this work, the formation of a transport barrier is analyzed within the framework of the bifurcation model introduced by M. Malkov and P. Diamond [1]. This model aims at describing the L-H transition phenomenon in tokamak plasmas. In this model, the variation of the system state, namely pressure or density gradients, with respect to control parameters such as heat or particle fluxes, respectively, exhibits an S-curve. The present study focuses on the one-field model, where pressure and density profiles are decoupled. In this case, the pressure gradient depends non-monotonically on the heat flux (see fig.2, left): the two stable branches of the S-curve stand for the L and H modes, while the other branch, unstable, is physically irrelevant because it would correspond to a negative diffusivity. According to the model, the heat transport equation is analyzed using slab geometry in one dimension with the form:

$$\frac{\partial p}{\partial t} - \frac{\partial}{\partial x} \left[\chi_0 + \frac{\chi_1}{1 + \alpha v'_E^2} \right] \frac{\partial p}{\partial x} = H(x). \quad (1)$$

Here, p is the plasma pressure, χ_0 and χ_1 are the neoclassical and turbulent diffusivities, respectively, v'_E is the shear of the poloidal component of the electric drift, H is the heat source, and α is a positive constant. The main ingredient for the stabilization of the transport is the flow shear v'_E , which accounts for the known reduction of turbulent transport by sheared radial electric field [2].

Analytical Study

The stationary state is such that the total flux balances the injected flux:

$$-\left[\chi_0 + \frac{\chi_1}{1 + \alpha v'_E^2} \right] \frac{\partial p}{\partial x} = Q(x), \quad (2)$$

where $Q(x) = \int_0^x H(x') dx'$ is the heat flux. In the spirit of reference [3], it is further assumed that the turbulent diffusivity χ_1 scales like the local pressure gradient to the power m , with m being a positive

constant: $\chi_1 = c|p'|^m$. Also, the velocity shear is assumed to be driven by only the local pressure gradient: $v'_E = |p'|$. As a result, the steady state of heat transport equation can be written as

$$-\left[\chi_0 + \frac{c|p'|^m}{1+\alpha|p'|^\beta} \right] p' = Q. \quad (3)$$

This function can be analyzed to find conditions for the existence of a bifurcation curve, i.e. having both local maximum and minimum as turning points, which are possible only when the following conditions are satisfied:

$$\beta > m + 1, \quad (4)$$

$$\lambda = \frac{c}{\chi_0} > f \alpha^{\frac{m}{\beta}}, \quad (5)$$

$$f = f(m, \beta) = \frac{(f_1 + \beta f_2 + 2f_3)^2 (2f_3)^{\frac{m}{\beta}-1}}{(f_1 + \beta f_2)^{\frac{m}{\beta}} [(\beta - (m+1))(f_1 + \beta f_2) - 2f_3(m+1)]}, \quad (6)$$

$$f_1 = f_1(\beta, m) = \beta^2 + \beta(2m+1) - 2m(m+1), \quad (7)$$

$$f_2 = f_2(\beta, m) = \sqrt{\beta^2 + 2\beta(2m+1) - 4m(m+1) + 1}, \quad (8)$$

$$f_3 = f_3(\beta, m) = (\beta - m)(\beta - (m+1)). \quad (9)$$

In the case of $m=0$, in which the turbulent transport coefficient is constant, the threshold for *L-H* transition given by equation (5) becomes

$$\lambda = \frac{\chi_1}{\chi_0} > f(0, \beta) = \frac{4\beta}{(1-\beta)^2}. \quad (10)$$

This result agrees with Ref. [1]. Moreover, the general forms of equation (5) are verified to be correct by applying a simple searching algorithm to the plots of equation (3) in order to locate a local maximum which is the turning point for *L-H* transition.

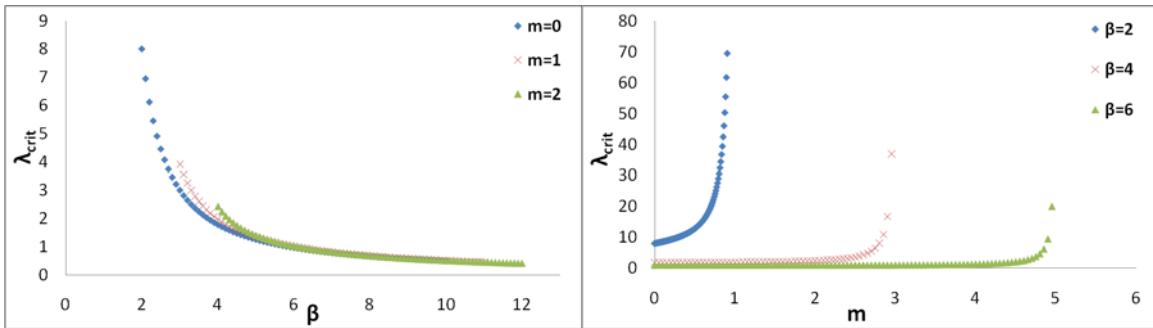


Figure 1 Plots of critical ratio λ_{crit} as a function of constants β (left) and m (right)

To analyze the effect of constants β and m , the critical ratio λ_{crit} in equation (5) are plotted with respect to the two constants as shown in Fig. 1. Constant m can be viewed as the stiffness of turbulent transport function which indicates the sensitivity of relationship between χ_1 and the pressure gradient. It can be seen that the higher m is the harder to suppress the transport, and consequently, the more difficult for plasma to make a transition. The constant β represents the suppression order. The higher β

is the easier to suppress the transport. Furthermore, the vertical asymptotic behaviour in both plots stands for the case $\beta = m+1$ in equation (4) which is the limit for bifurcation curve. In summary, this analysis shows the existence of a critical ratio of turbulent to neoclassical transport coefficients above which the L to H transition becomes a bifurcation.

Numerical Study

For simplicity, $m=0$ and $\beta=2$ are used in these numerical simulations. The numerical code solves the heat transport equation using discretization method with Crank-Nicolson and Trapezoidal leap-frog algorithms. Note that in all simulations, the hyperdiffusion term as discussed in Ref. [1] is added in order to ensure the continuity of the solution at the bifurcation point. So the total flux becomes:

$$Q = -\chi_0 p' - \frac{\chi_1}{1+\alpha p'^2} p' + \nu p''' \quad (11)$$

Here ν is a small constant. This additional term serves to regularize the discontinuity in the pressure gradient at the transition. In the following, Q_s denotes the maximum heat flux injected into the system to explore the transitional threshold at critical flux Q_{crit} . The constant parameters χ_0 , χ_1 , and α are chosen so as to be in the bifurcation regime.

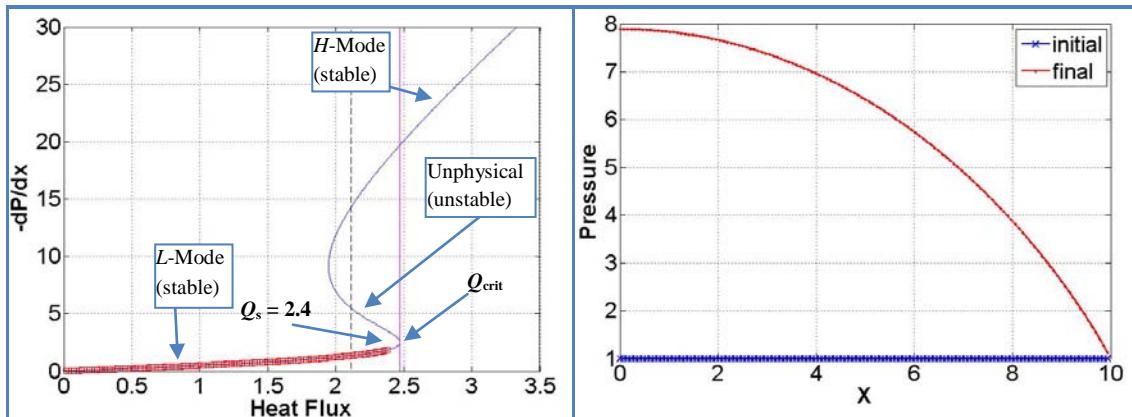


Figure 2 Bifurcation result (left) and pressure profile (right) when $Q_s < Q_{\text{crit}}$.

Case $Q_s < Q_{\text{crit}}$: Fig. 2 shows the numerical result when the heat flux Q_s is smaller than the critical value Q_{crit} necessary for plasma to make a transition to H -mode. At steady state, the system cannot reach H -mode so it remains in L -mode branch. In the left plot, the blue line represents the analytical result and the red squares represent the numerical simulation, one for each radial position. It can be observed that there is no drastic change in pressure profile implying no formation of transport barrier.

Case $Q_s > Q_{\text{crit}}$: Fig. 3 shows the result when Q_s is higher than Q_{crit} . At stationary state, the system makes a transition to H -mode so some data points reach in H -mode branch. It also shows that there exists a transient hysteresis, that is once the transport barrier has formed the required heat flux to maintain H -mode is reduced. As evidence in the right plot of the left panel, after H -mode is reached the transition line has moved from Q_{crit} to Q_{Maxwell} in which one finds that the position of the barrier is determined by an ‘equal area’ rule illustrating by the two equal shaded areas. The right panel shows

that transport barrier, represented by sudden increase in the pressure gradient, forms at the edge of the plasma. One other thing to be noticed here is that even though the heat flux is increased only from 2.4 in the previous case to around 3.2, the central pressure is raised significantly from 8 in *L*-mode to about 55 in *H*-mode, which is about 7 times increment. This confirms the experimental observation that the plasma performance like pressure or temperature can be increased by a large amount after the formation of the transport barrier.

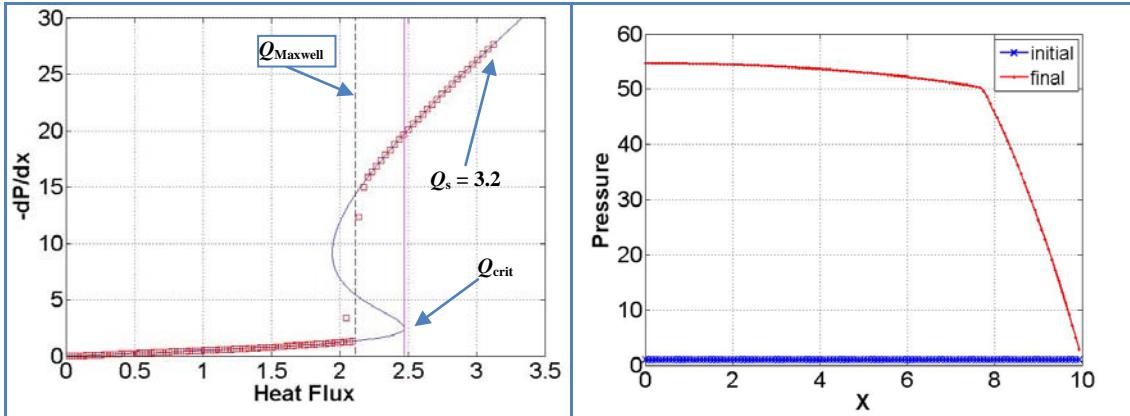


Figure 3 Bifurcation result (left) and pressure profile (right) when $Q_s > Q_{crit}$.

Conclusions

An analytical study shows that two conditions are necessary for plasma to make an *L-H* transition: firstly, the ratio of turbulent to neoclassical transport coefficients reaches a certain value and secondly the source heat flux injected into the system is higher than the critical value for the local maximum. Numerical results have confirmed that a source heat flux is needed to surpass the critical value in order for plasma to make a transition. When that happens, an edge transport barrier is formed as a steep pressure gradient, which leads to a significant increase in plasma pressure at the centre. Moreover, a transient hysteresis in the forward *L-H* transition are shown to exist which satisfies Maxwell's rule of equal areas at a phase transition.

Acknowledgments

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