Absolute Parametric Decay Instability in ECRH experiments at tokamaks

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Introduction. Electron cyclotron resonance heating (ECRH) at power level of up to 1 MW in a single microwave beam is often used in present day tokamak and stellarator experiments and planed for application in ITER for neoclassical tearing mode control. This power range is substantially lower than the parametric decay instability (PDI) threshold value predicted by standard theory [1] and therefore the EC wave propagation and damping in toroidal plasmas are believed to be well described by linear theory and predictable in detail. However, during the last decade a “critical mass” of observations [2-4] has been obtained evidencing presence of anomalous phenomena in ECRH experiment. The non local electron transport effect [2], fast ion generation [3] and anomalous backscattering phenomenon [4] accompanying ECRH are not explained in linear theory and therefore appeal to further theory development accounting for the nonlinear effects. A possible explanation of these observations by the backscattering PDI threshold lowering due to suppression of convective losses of ion Bernstein (IB) decay wave was proposed recently [5]. It was shown that due to non-monotonic density profile and toroidal inhomogeneity of magnetic field IB waves can be trapped both in radial and poloidal direction. Due to this effect the convective PDI can be excited in typical ECRH experiment at the pump power less than 100 kW [6].

In the present paper we demonstrate the possibility of total 3D trapping of IB waves in the tokamak equatorial plane in the vicinity of the local density maximum produced by electron pump-out-effect and, as result, excitation of the absolute parametric decay instability (APDI). The threshold and the growth rate are obtained.

The basic equations. To elucidate the physics of the APDI excitation we use the most simple but nevertheless relevant to the experiment cylindrical co-ordinate system (ρ,ϑ,φ), where ρ is a radius of the magnetic surface, ϑ and φ are the poloidal and toroidal radii, respectively. For the sake of simplicity in a vicinity of the density maximum we consider the profile as $n \approx n_0(\rho/a)\left[1+\delta\exp(-(\rho-\rho_m)^2/l_\rho^2)\right]$, where δ relates to the amplitude of the density bump $\sim n_0(\rho_m/a)\cdot\delta$, ρ_m is a radial position of the density maximum, a and l_ρ are a minor radius and a spatial size of the local maximum, which obeys inequality $l_\rho \ll a$. We also assume the pump frequency exceeding both the electron cyclotron and plasma frequency so that the following strong inequality holds: $\omega_i^2 \gg \omega_{pe}^2, \omega_{ce}^2$, which is typical for the second harmonic X-mode ECRH experiments. We neglect also a weak dependence of the high frequency wave numbers on coordinate that allows us to introduce the pump wave in a vicinity of the density maximum at $(\rho_m, 0, 0)$ as $E_{i\vartheta} = a_i / 2\exp\left(ik_{i\rho}x + ik_{iz}z - i\omega_i t - \left(y^2+z^2\right)/(2w^2)\right) + c.c.$, where $a_i = \sqrt{8\pi P_i / \left(\pi w^2 c\right)}$, P_i is the pump wave power, w is the beam radii, $r = \rho - \rho_m$, $z = R_0\varphi$, $y = \rho_m \vartheta$, R_o is a major radius. The basic equations describing in a vicinity of the density maximum $(\rho_m, 0, 0)$ the backscattered wave generation and its convective losses from the decay region as well as the low-frequency, $\Omega = \omega_i - \omega_s \ll \omega_i$, electrostatic IB wave $\vec{E} = -\vec{\nabla}\phi\exp(i\Omega t)$ are as follows:

$$\left(\frac{\partial^2}{\partial r^2} + k_{s\rho}^2\right)E_{s\vartheta} = i\frac{4\pi\omega_s}{c^2}j_{s\vartheta};\int_{-\infty}^{\infty} d\vec{r}'\hat{D}\left(\vec{r}-\vec{r}', (\vec{r}+\vec{r}')/2\right)\phi(\vec{r}') = 4\pi\rho_\Omega \qquad (1)$$

The nonlinear current in (1) $j_{s\vartheta} \simeq -\dfrac{e}{4\pi m_e \omega_s}\dfrac{\omega_{pe}^2}{\omega_{ce}^2}\left(\dfrac{\partial^2}{\partial x^2}\phi\right)E_{i\vartheta}$ is given by a product of an electron density perturbation δn_Ω produced by a low-frequency small scale decay wave and the quiver electron velocity $u_{i\vartheta}$ associated with the pump wave. The nonlinear charge density ρ_Ω in (1), generated by the ponderomotive force is responsible for coupling of low and high frequency waves and takes in the LH frequency range a form $\rho_\Omega = \dfrac{1}{4\pi}\dfrac{e}{m_e}\dfrac{\omega_{pe}^2}{\omega_{ce}^2\omega_i^2}\dfrac{\partial}{\partial x}\left[E_{i\vartheta}^*\dfrac{\partial E_{s\vartheta}}{\partial x} + E_{s\vartheta}\dfrac{\partial E_{i\vartheta}^*}{\partial x}\right]$. In weakly inhomogeneous plasma the operator $\hat{D}$ in the integral equation (1), exhibiting much stronger dependence on the first argument $\vec{r}-\vec{r}'$ than on the second $(\vec{r}+\vec{r}')/2$, associated with the plasma inhomogeneity, can be represented as $\hat{D}[\vec{r}-\vec{r}', \vec{r}+\vec{r}'] = (2\pi)^{-3}\int D[\vec{q}, \vec{r}+\vec{r}']\exp[i\vec{q}(\vec{r}-\vec{r}')]d\vec{q}$. The kernel $D(\vec{q})$ contains real part and imaginary part, $D(\vec{q}) = D'(\vec{q}) + iD''(\vec{q})$ [7]:

$$D' = q_\perp^2\left(1 + \omega_{pe}^2/\omega_{ce}^2\right) - q_\parallel^2\omega_{pe}^2/\Omega^2 + 2\omega_{pi}^2/\upsilon_{ti}^2\left[1 - X\left(\Omega/(q_\perp\upsilon_{ti})\right) - \cot\left(\pi\Omega/\omega_{ci}\right)Y\left(\Omega/(q_\perp\upsilon_{ti})\right)\right], \quad (2)$$

$$D'' \simeq q_\perp^2\frac{\omega_{pe}^2}{\omega_{ce}^2}\frac{\nu_{ei}}{\Omega} + \frac{2\omega_{pi}^2}{\upsilon_{ti}^2}Y\left(\frac{\Omega}{q_\perp\upsilon_{ti}}\right)\left[\frac{\pi}{\omega_{ci}}\nu_{ii}\left(\frac{\Omega}{q_\perp}\right) + \frac{1}{\sqrt{\pi}}\frac{\omega_{ci}}{|q_\parallel|\upsilon_{ti}}\sum_{m=-\infty}^{\infty}\exp\left(-\frac{(\Omega - m\omega_{ci})^2}{q_\parallel^2\upsilon_{ti}^2}\right)\right] \qquad (3)$$

where $q_\perp = |\vec{q}\times\vec{b}|$, $q_\parallel = \vec{q}\cdot\vec{b}$, $\vec{b} = \vec{B}/|B|$, $X + iY = \pi^{-1/2}\xi\int_{-\infty}^{\infty}\exp(-t^2)(\xi - t)^{-1}dt$, the first and the second terms in (3) correspond to collisional damping and the last one is the ion cyclotron damping. We seek a solution of (1) in the vicinity of the density maximum in the tokamak mid-plane where localization of the IBW is possible. Assuming the IBW is close to the turning point we reduce the integral equation (1) , (2) in a vicinity of the frequency Ω_0, the wave vector $(q_{\rho 0}, 0, 0)$ and the coordinate $(\rho_0, 0, 0)$, $\rho_0 = \rho_m + \delta\rho$, $\delta\rho \ll \omega_{ci}a/\Omega_0$, which are a solution of the set of equations $D'|_{\rho_0, q_{\rho 0}} \doteq D'|_o = 0$ (dispersion relation of the IBW), $\partial D'/\partial q_\rho|_o = 0$ (the IBW turning point where its group velocity tends to zero), $\partial D'/\partial\rho|_o = 0$ (the condition of the local maximum of D'). The simplified IBW equation takes a form

$$\left\{D_\Omega \Delta\Omega - D_{qq}\frac{\partial^2}{\partial x^2} - 2D_{xq}x\frac{i\partial}{\partial x} - iD_{xq} + \eta_0\left(\cos\alpha\frac{\partial}{\partial z} + \sin\alpha\frac{\partial}{\partial y}\right)^2 + D_{xx}x^2 - D_{yy}y^2 + D''\right\}b = \\ -i\frac{q_{\rho0}^3}{4}\frac{\omega_{pe}^4}{\omega_i^2\omega_{ce}^2}\frac{|a_i|^2}{H^2}\exp\left(-\frac{y^2+z^2}{w^2}\right)\int_{-\infty}^{x} dx'\exp\left(i\Delta K\left(x-x'\right)\right)b\left(x',y,z\right), \tag{4}$$

where the coordinates x and y are redefined in a way $x = \rho - \rho_0$, $y = \rho_0(\vartheta - \vartheta_0)$, $\Delta\Omega = \Omega - \Omega_0$, $\eta_0 = \omega_{pe}^2/\Omega^2|_o$, $D_\Omega = \partial D'/\partial\Omega|_o$, $D_{qq} = \partial^2 D'/2\partial q_\rho^2|_o$, $D_{xq} = \partial^2 D'/2\partial q_\rho \partial\rho|_o$, $D_{xx} = \partial^2 D'/2\partial\rho^2|_o$, $D_{yy} = \partial^2 D'/2\partial y^2|_o$, $\tan(\alpha) = B_\vartheta/B_\varphi|_o$, $\Delta K = q_{\rho0} - k_{i\rho} - k_{s\rho}$, $b(x,y,z) = \phi(x,y,z)\exp\left[-iq_{\rho0}x + i\Omega t\right]$.

The PDI analyses and discussion. Assuming the IB wave damping and PDI pumping are small, we account for them using the perturbation theory approach [6]. In the zero order approximation we neglect D'' (*i.e.* damping) in (4) and r.h.s. (*i.e.* nonlinear pumping) of (4) and obtain equation which can be solved by separation of variables. The corresponding expression for the IB eigen mode trapped in radial and poloidal direction and possessing translation invariance in the toroidal direction is given by $b_{kln}(x,y,z) = \exp\left(-iD_{xq}/(2D_{qq})x^2 + in/R_o\left(z - y\cot\alpha\right)\right)\varphi_k(x)\varphi_l(y)$ with H_k being Hermitian polynomials, $\delta_x = \sqrt[4]{D_{qq}}\sqrt{L_x/q_{\rho0}}$, $\delta_y = \sqrt{L_y/q_{\rho0}}\sqrt{\sin\alpha\ \omega_{pe}/\Omega_0}$ being a size of the IB mode localization region in the radial and poloidal directions and $L_x \doteq \sqrt{2}q_{\rho0}\left[\partial^2 D'/\partial x^2 - \left(\partial^2 D'/\partial x\partial q_\rho\right)^2\Big/\left(\partial^2 D'/\partial q_\rho^2\right)\right]_0^{-1/2}$ and $L_y \doteq \sqrt{2}q_{\rho0}\,\partial^2 D'/\partial y^2\Big|_0^{-1/2}$. The exact value of the mode frequency $\Omega_{IB} = \Omega_0 + \delta\Omega_{kl}$ is determined by following quantization condition $\delta\Omega_{kl} = D_\Omega^{-1}\sqrt{D_{qq}}\frac{q_{\rho0}}{L_x}(2k+1) - D_\Omega^{-1}\sin|\alpha|\ \frac{\omega_{pe}}{\Omega_0}\frac{q_{\rho0}}{L_y}(2l+1)$. At the next step of the perturbation analysis procedure we account for IB wave damping and PDI pumping that yields for the growth rate $\gamma = i\left(\Omega - \Omega_{IB}\right)$ of the IBW mode the following expression (which we present here only for $k,l = 0$):

$$\gamma = -\nu_{IB} + \frac{q_{\rho0}^3}{8}\frac{\omega_{pe}^4}{\omega_i^2\omega_{ce}^2}\frac{|a_i|^2}{H^2}\frac{\delta_x}{R_0}\frac{w^2}{\sqrt{w^2+\delta_y^2}}\frac{D_{qq}}{\sqrt{D_{qq}^2 + D_{xq}^2\delta_x^4}}\exp\left[-\frac{\Delta K^2\delta_x^2 D_{qq}^2}{D_{qq}^2 + D_{xq}^2\delta_x^4}\right]\frac{\partial D'}{\partial\Omega}\Bigg|_0^{-1} \tag{5}$$

where $\nu_{IB} = D''\big|_0 \partial D'/\partial\Omega|_o^{-1}$ is a damping rate of the IBW. Putting $\gamma = 0$ in (5) gives the threshold of the APDI in the form

$$\gamma_0^2\left(P_i\right)\frac{D_{qq}}{\sqrt{D_{qq}^2 + D_{xq}^2\delta_x^4}}\exp\left[-\frac{\Delta K^2\delta_x^2 D_{qq}^2}{D_{qq}^2 + D_{xq}^2\delta_x^4}\right] = \nu_{IB}\nu_{EC}\frac{V_{damp}}{V_{\mathrm{PDI}}},\quad \gamma_0\left(P_i\right) = \left[D_\Omega^{-1}\frac{q_{\rho0}^3 c}{4}\frac{\omega_{pe}^4}{\omega_i^2\omega_{ce}^2}\frac{|a_i|^2}{H^2}\right]^{1/2} \tag{6}$$

where $\gamma_o(P_i)$ is a maximal growth rate in homogeneous plasma, $\nu_{EC} = c/\delta_x$ is the convective loss rate of the high frequency daughter wave from the decay region and $V_{\mathrm{PDI}}/V_{damp}$ is a geometrical factor defined as a ratio of the decay volume and the IBW damping volume (V_{damp}). In the case of the dominating IBW collisional damping the factor is given by: $V_{\rm PDI} / V_{damp} = 2\pi R_0 \sqrt{1+\delta_y^2 / w^2} \Big/ \left(\sqrt{\pi} w\right)$.

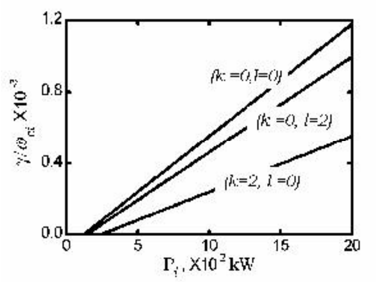

Fig.1. The dependence of the normalized APDI growth rate versus the pumping power for different modes of IBW for the JET experiment conditions.

The generalized dependence (6) in which we take account of parametric excitation of an arbitrary the radial and poloidal modes k, l can be illustrated with the example corresponding to the 2nd harmonic X-mode ECRH under discussion for application at JET. The dependence of the APDI growth rate versus the pumping power is shown in Fig. 1 for the typical conditions on JET ($n = 1 \cdot 10^{14} \text{cm}^{-3}$, $T_D = T_e = 3$ keV, $H = 23$ kGs, $R_o = 2.96m$, $a = 1.25m$, $b = 2.1m$, $B_\vartheta / B_\varphi \simeq 0.1$, $f_i = 170$ GHz, $\rho = 3$cm, $\delta = 0.1$ and width $l_\rho = 5$ cm). As we can see, the minimal threshold is realized for the fundamental mode $k = 0$, $l = 0$ and is about 200 kW which is more than three orders of magnitude lower than predicted by the standard theory [1]. Meanwhile, the APDI minimal threshold is much greater than the threshold of the fast convective PDI [6], which can be assessed for the same parameters of about 3.6 kW. This can be explained in such a way that the IBW in the case of the APDI is trapped in the radial and the poloidal direction in small region comparable to the microwave beam width and decay region, makes long excursion in the third, toroidal direction, intersecting the EC pumping beam (or decay region) many times and suffering from collisional damping. The most likely saturation mechanism for the APDI comes from stochastic (IB wave amplitude dependent) damping. It appears to be not very high and corresponds to the level of anomalous reflection of less than 10^{-3}. However when the parametric pumping exceeds the maximal possible stochastic damping this quasi-linear saturation fails leading to a stronger anomalous reflection, ion acceleration and possibly to reduction of ECRH efficiency.

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