On offset toroidal rotation in NTV

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Introduction

Tokamak has toroidal symmetry and symmetry breaking via application of non-axisymmetric field produces dissipation of toroidal momentum by NTV [1] and produces offset toroidal rotation[2]. Callen explained NTV physics and origin of offset toroidal rotation without impurity [3] and with impurity [4]. In Callen [4], same toroidal rotation for ion and impurity is assumed, which is not always correct as shown by Kim [5]. I have revisited this subject [6] and refined formulas of offset toroidal rotation of bulk ion and impurity are derived.

Offset Toroidal Rotation

One important property of axisymmetric system is the conservation of total toroidal momentum.Total toroidal angular momentum balance equation is given as follows [7],

$$\sum_a m_a \langle n_a R \frac{du_{a\zeta}}{dt} \rangle = \sum_a \langle R^2 \nabla\zeta \cdot (\nabla \cdot \mathbf{\Pi}_a + \boldsymbol{M}_a) \rangle \tag{1}$$

Noting viscous tensor $\mathbf{\Pi}_a$ is a symmetric tensor for axisymmetric plasma and $\nabla(R^2\nabla\zeta)$ is antisymmetric tensor [8], flux surface averaged toroidal viscous force is shown to be zero, $\langle R^2\nabla\zeta \cdot \nabla \cdot \mathbf{\Pi}_a \rangle = 0$. When symmetry is broken by the application of non-axisymmetric field, $\langle R^2\nabla\zeta \cdot \nabla \cdot \mathbf{\Pi}_a \rangle$ becomes non zero. This drag force in the toroidal direction is called Neoclassical Toroidal Viscous Force. The 0-th order ion force balance equation is give by,

$$0 = eZ_i n_i (\boldsymbol{E} + \boldsymbol{u}_i \times \boldsymbol{B}) - \nabla P_i \tag{2}$$

In the flux coordinates (ψ, θ, ζ), the magnetic field is expressed as $\boldsymbol{B} = \nabla\psi \times \nabla(q\theta - \zeta)$. The radial component of above equation can be obtained by taking inner product with tangent vector $\partial \boldsymbol{x}/\partial\psi$ and using the identity $\partial \boldsymbol{x}/\partial\psi \cdot \nabla\psi = 1$, where $\boldsymbol{x}$ is position vector.

$$\boldsymbol{u}_i \cdot \nabla\zeta = -\left[\frac{d\Phi}{d\psi} + \frac{1}{eZ_i n_i}\frac{dP_i}{d\psi}\right] + q\boldsymbol{u}_i \cdot \nabla\theta \tag{3}$$

, where q is the safety factor. In the tokamak plasma with symmetry breaking, the electrostatic potential Φ is determined so that non-umbipolar flux $\langle \mathbf{\Gamma}_{na} \cdot \nabla V \rangle$ is zero. The non-ambipolar flux $\langle \mathbf{\Gamma}_{na} \cdot \nabla V \rangle$ is related to toroidal viscous force as $\langle \mathbf{\Gamma}_{na} \cdot \nabla V \rangle = (V'(\psi))^2/e_a q)\langle \boldsymbol{B}_t \cdot \nabla \cdot \mathbf{\Pi}_a \rangle$ in Hamada coordinates [9] (extra $\langle \boldsymbol{B}_t \cdot \nabla P_a \rangle$ term appears in other coordinates as shown by Shaing [10]). In the collisionless regime, ion viscous force is larger than that for electron by a factor of $(m_i/m_e)^{1/2}$. The zero non-ambipolar flux condition is then given by $\langle \boldsymbol{B}_t \cdot \nabla \cdot \boldsymbol{\Pi}_i \rangle = 0$ for electron-ion plasma. Shaing [11] derived following relation for the ion in collisionless regime.

$$\frac{d\Phi}{d\psi} + \frac{1}{eZ_i n_i}\frac{dP_i}{d\psi} = -\frac{\lambda_2}{eZ_i\lambda_1}\frac{dT_i}{d\psi} \tag{4}$$

Here λ_1 and λ_2 are numerical constants in [11]. If we include impurity, we may be able to assume impurity is in a Pfirsh-Schluter regime (namely impurity parallel viscosity is nearly zero) while ion is deeply in collisionless regime as in next section. Then (4) is still zero non-ambipolar flux condition. Substituting equation (4) into (3), we obtain following expression of offset toroidal rotation,

$$\boldsymbol{u}_{i0} \cdot \nabla\zeta = \frac{\lambda_2}{eZ_i\lambda_1}\frac{dT_i}{d\psi} + q\boldsymbol{u}_{i0} \cdot \nabla\theta \tag{5}$$

,where offset poloidal rotation $\boldsymbol{u}_{i0} \cdot \nabla\theta$ is given by the equation (15) in Appendix A. Therefore, offset toroidal rotation $u_{i\zeta 0} = R\boldsymbol{u}_{i0} \cdot \nabla\zeta$ is given as follows,

$$u_{i\zeta 0} = R\left[\frac{\lambda_2}{eZ_i\lambda_1} - \frac{qK_1F(\boldsymbol{B}\cdot\nabla\theta)}{eZ_i\langle B^2\rangle}\right]\frac{dT_i}{d\psi} \tag{6}$$

Since measurement of toroidal rotation is made using impurity toroidal rotation, actual measurement of offset toroidal rotation of the impurity can be expressed as follows using the expression in the Appendix B.

$$u_{I\zeta 0} = R\left[\frac{\lambda_2}{eZ_i\lambda_1} - \frac{qK_1F(\boldsymbol{B}\cdot\nabla\theta)}{eZ_i\langle B^2\rangle} - \frac{1.5K_2B_\zeta^2}{eZ_i\langle B^2\rangle}\right]\frac{dT_i}{d\psi} - \left[1 - \frac{B_\zeta^2}{\langle B^2\rangle}\right]R\left(\frac{1}{eZ_In_I}\frac{dP_I}{d\psi} - \frac{1}{eZ_in_i}\frac{dP_i}{d\psi}\right) \tag{7}$$

In the large aspect ratio cylindrical plasma, offset toroidal rotations of bulk ion and impurities are given as follows,

$$u_{i\zeta 0} = \frac{3.54}{eZ_iB_\theta}\frac{dT_i}{dr} - \frac{(r/R)K_1}{eZ_iB_\theta}\frac{dT_i}{dr} \tag{8}$$

$$u_{I\zeta 0} = \frac{3.54 - 1.5K_2 - (r/R)K_1}{eZ_iB_\theta}\frac{dT_i}{dr} \tag{9}$$

Summary

Refined formulas of offset toroidal rotation of ion and impurity due to NTV, (8) and (9), are derived by correctly including parallel flow effect on residual poloidal flow and impurity effect. Due to $1/B_p$ dependence, this offset toroidal rotation is especially important in high poloidal beta regime, where plasma current is carried largely by the bootstrap current and relevant for steady state operation of tokamak.

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Appendix A : Offset Poloidal Rotation

As discussed by Shaing [11], offset poloidal rotation can be obtained from standard neoclassical theory of axisymmetric plasma. We can use Kim's analysis [5] as follows. Since electron inertia is small, electron contribution to ion and impurity momentum balance can be neglected and flow relations of ion and impurity are given as follows [7].

$$\hat{\boldsymbol{\mu}}^I \cdot \boldsymbol{u}^I_\theta = \hat{\boldsymbol{L}}_{II} \cdot \boldsymbol{u}^I_\parallel + \hat{\boldsymbol{L}}_{Ii} \cdot \boldsymbol{u}^i_\parallel \tag{10}$$

$$\hat{\boldsymbol{\mu}}^i \cdot \boldsymbol{u}^i_\theta = \hat{\boldsymbol{L}}_{iI} \cdot \boldsymbol{u}^I_\parallel + \hat{\boldsymbol{L}}_{ii} \cdot \boldsymbol{u}^i_\parallel \tag{11}$$

, where $\hat{\boldsymbol{u}}^a_\theta$, $\boldsymbol{u}^a_\parallel$, $\boldsymbol{V}^a$, $\hat{\boldsymbol{\mu}}^a$, $\hat{\boldsymbol{L}}_{ab}$ are defined as follows,

$$\hat{\boldsymbol{u}}^a_\theta = \begin{bmatrix} \langle B^2\rangle \hat{u}_{a\theta} \\ \frac{2\langle B^2\rangle \hat{q}_{a\theta}}{5P_a} \end{bmatrix}, \boldsymbol{u}^a_\parallel = \begin{bmatrix} \langle B u_{\parallel a}\rangle \\ \frac{2\langle B q_{\parallel a}\rangle}{5P_a} \end{bmatrix}, \boldsymbol{V}^a = \begin{bmatrix} BV_{1a} \\ BV_{2a} \end{bmatrix}, \hat{\boldsymbol{\mu}}^a = \begin{bmatrix} \hat{\mu}_{a1} & \hat{\mu}_{a2} \\ \hat{\mu}_{a2} & \hat{\mu}_{a3} \end{bmatrix}, \hat{\boldsymbol{L}}_{ab} = \begin{bmatrix} \hat{l}^{ab}_{11} & -\hat{l}^{ab}_{12} \\ -\hat{l}^{ab}_{21} & \hat{l}^{ab}_{22} \end{bmatrix} \tag{12}$$

Here, continuity of 1st order flow on flux surface gives $\hat{\boldsymbol{u}}^a_\theta = \boldsymbol{u}^a_\parallel - \boldsymbol{V}^a$. Since impurity collisionality is given as $\nu^*_I = (n_I Z_I^4 / n_i Z_i^4)\nu^*_i$ considering fast equipartition between ion and impurity ($T_i \approx T_I$), impurity may be in the Pfirsh-Schluter regime (negligible impurity viscous force: $\hat{\boldsymbol{\mu}}^I \cdot \boldsymbol{u}^I_\theta \approx 0$) while bulk ion is deeply collisionless regime, $\nu^*_I \gg \nu^*_i$. Therefore, impurity parallel flow can be given as $\boldsymbol{u}^I_\parallel = -\hat{\boldsymbol{L}}_{II}^{-1} \hat{\boldsymbol{L}}_{Ii} \cdot \boldsymbol{u}^i_\parallel$. Using large impurity mass approximation $m_I \gg m_i$, substitution into (11) gives following ion momentum balance equation.

$$\hat{\boldsymbol{\mu}}^i \cdot \hat{\boldsymbol{u}}^i_\theta = -\begin{bmatrix} 0 & 0 \\ 0 & \gamma \end{bmatrix} \cdot \boldsymbol{u}^i_\parallel \tag{13}$$

Here, $\gamma = \sqrt{2} + \alpha$, $\alpha = n_I Z_I^2 / n_i Z_i^2$ (Here $\beta = O((m_i/m_I)^2)$ term of [5] is neglected). This equation indicates that $\hat{u}_\theta$-driven ion viscous force is balanced against $\hat{q}_\theta$-driven ion viscous force so that total parallel ion viscous force becomes zero even in the presence of impurity. Using the relation $\boldsymbol{u}^i_\parallel = \hat{\boldsymbol{u}}^i_\theta + \boldsymbol{V}^i$,we obtain following expression.

$$\hat{\boldsymbol{u}}^i_\theta = \begin{bmatrix} 0 & K_1 \\ 0 & -\frac{\hat{\mu}_{i1}}{\hat{\mu}_{i2}} K_1 \end{bmatrix} \boldsymbol{V}^i \tag{14}$$

where, $K_1 = \gamma \hat{\mu}_{i2}/D$, $D = \hat{\mu}_{i1}(\hat{\mu}_{i3} + \gamma) - \hat{\mu}_{i2}^2$. The equation (14) leads to $\langle B^2\rangle \hat{u}_{i\theta} = K_1 B V_{2i} = -K_1 F (dT_i/d\psi)/eZ_i$. Combination with the definition of $\hat{u}_{i\theta} = \boldsymbol{u}_{i0} \cdot \nabla\theta / \boldsymbol{B} \cdot \nabla\theta$ leads to following expression of residual poloidal flow.

$$\boldsymbol{u}_{i0} \cdot \nabla\theta = -\frac{K_1 F (\boldsymbol{B} \cdot \nabla\theta)}{eZ_i \langle B^2\rangle} \frac{dT_i}{d\psi} \tag{15}$$

Appendix B : Toroidal Rotation difference between Impurity and Ion

Substituting equation (14) into $\boldsymbol{u}^i_{\parallel} = \hat{\boldsymbol{u}}^i_{\theta} + \boldsymbol{V}^i$, we obtain following ion parallel flow relation.

$$\langle Bu_{\parallel i}\rangle = -F\left(\frac{d\Phi}{d\psi} + \frac{1}{eZ_i n_i}\frac{dP_i}{d\psi}\right) - \frac{K_1 F}{eZ_i}\frac{dT_i}{d\psi} \tag{16}$$

Using $\boldsymbol{u}^I_{\parallel} = -\hat{\boldsymbol{L}}^{-1}_{II}\hat{\boldsymbol{L}}_{Ii}\cdot\boldsymbol{u}^i_{\parallel}$, we obtain following impurity parallel flow relation.

$$\langle Bu_{\parallel I}\rangle = -F\left(\frac{d\Phi}{d\psi} + \frac{1}{eZ_i n_i}\frac{dP_i}{d\psi}\right) - \frac{(K_1 + 1.5K_2)F}{eZ_i}\frac{dT_i}{d\psi} \tag{17}$$

Here $K_2 = (\hat{\mu}_{i1}\hat{\mu}_{i3} - \hat{\mu}^2_{i2})/D$. Therefore, difference of parallel flows between impurity and ion is given as follows,

$$\langle B(u_{\parallel I} - u_{\parallel i})\rangle = -\frac{1.5K_2 F}{eZ_i}\frac{dT_i}{d\psi} \tag{18}$$

Local toroidal flow $u_{a\zeta 0} = R\boldsymbol{u}_{a0}\cdot\nabla\zeta$ is related to parallel flow $Bu_{\parallel a}$ as follows [7],

$$u_{a\zeta 0} = \frac{B_\zeta\langle Bu_{\parallel a}\rangle}{\langle B^2\rangle} - \left[1 - \frac{B^2_\zeta}{\langle B^2\rangle}\right]R\left(\frac{d\Phi}{d\psi} + \frac{1}{e_a n_a}\frac{dP_a}{d\psi}\right) \tag{19}$$

Therefore, difference in local toroidal rotation between impurity and ion is given as follows and its value is order of 50km/s in JT-60 [12],[6].

$$u_{I\zeta 0} - u_{i\zeta 0} = -\frac{1.5K_2 F B_\zeta}{eZ_i\langle B^2\rangle}\frac{dT_i}{d\psi} - \left[1 - \frac{B^2_\zeta}{\langle B^2\rangle}\right]R\left(\frac{1}{eZ_I n_I}\frac{dP_I}{d\psi} - \frac{1}{eZ_i n_i}\frac{dP_i}{d\psi}\right) \tag{20}$$

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