Large Eddy Simulations for Gyrokinetics

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Anomalous transport due to micro-turbulence is known to play an important role in stability properties of magnetically confined fusion plasma devices such as ITER. Plasma microturbulence is described by gyrokinetic equations [1]. Due to the various scales characterizing realistic experimental conditions, Direct Numerical Simulations (DNS) of gyrokinetic microturbulence remain close to the computational limit of actual supercomputers [2], so that any alternative is welcome to decrease the numerical effort. In particular, Large Eddy Simulations (LES) are a good candidate for such a decrease. LES techniques have been devised for simulating turbulent fluids at high Reynolds number. In these simulations, the large scales are computed explicitly while the smallest scales are filtered out and their influence is modelled [3].

The gyrokinetic formalism is based on the presence of a strong background magnetic field, that allows to filter out the cyclotron motion of particles around their guiding centers [4]. In the Gyrokinetic Electromagnetic Numerical Experiment code (GENE) [5], the dimensionless distribution function is splitted into an equilibrium part $F_0 = e^{-v_\parallel^2 - \mu B_0}$ and the unknown δf (where $v_\parallel$ is the normalized velocity parallel to the magnetic field B_0 and $\mu = mv_\perp^2/(2B_0)$ is the magnetic moment). The gyrokinetic equation for the guiding centers distribution function reads:

$$\partial_t \delta f_k = L[\delta f_k] + N[J_0\phi_k, \delta f_k] - D[\delta f_k] \tag{1}$$

where $L[\delta f_k]$ represents the linear terms. $N[J_0\phi_k, \delta f_k]$ stands for the nonlinear $\mathbf{E}\times\mathbf{B}$ advection, while $D[\delta f_k]$ are the dissipations. In these expressions, δf_k represents the Fourier transform of the distribution δf.

Since we consider electrostatic ion turbulence, the dimensionless electrostatic potential ϕ_k is given by the quasi-neutrality equation, where electrons are assumed adiabatic:

$$\beta_k\phi_k - \langle\phi_k\rangle_{FS} = \pi B_0 \int dv_\parallel d\mu J_0 \delta f_k\,, \tag{2}$$

where the gyroaverage J_0 links the fields and guiding centers coordinates, and β_k is a fixed prefactor taking into account ion to electron charge and temperature ratios (J_0 as well as β_k are *even* functions of k).

In the study of plasma micro-turbulence, the most demanding directions in terms of grid points are the two spatial ones perpendicular to the background magnetic field. In GENE, thanks to the local fluxtube geometry [6], these two directions are Fourier transformed, and they correspond to k_x and k_y wave vectors. It is then natural to focus the LES filtering effort onto k_x and k_y. In the Fourier space, the decrease of resolution corresponds to a Fourier cutoff filter, if we symbolize by an overline $\overline{\cdots}$ its action on the unknowns, we obtain the filtered gyrokinetic equation:

$$\partial_t \overline{\delta f}_k = L[\overline{\delta f}_k] + N[J_0\overline{\phi}_k, \overline{\delta f}_k] - D[\overline{\delta f}_k] + T\,, \tag{3}$$

where the sub grid term $T = \overline{N[J_0\phi_k, \delta f_k]} - N[J_0\overline{\phi_k}, \overline{\delta f}_k]$ appears from the filtering. This term still contains non resolved information ϕ_k and δf_k, and needs to be modelled.

In order to characterize further the role played by the sub grid scales, we base our analysis on the free energy balance, that is recognized to be especially relevant for δf, fluxtube, gyrokinetic solvers [7, 8]. Such a quantity is constructed by multiplying the gyrokinetic equation (1) by $\delta h_{-k}/F_0 = \delta f_{-k}/F_0 + J_0\phi_{-k}$, and then integrating over the whole phase space $d\Lambda_k$. When considering the filtered gyrokinetic equation (3) and the associated phase space $d\overline{\Lambda}_k$, one obtains:

$$\partial_t \mathscr{E}_{\overline{f}} = \mathscr{G}_{\overline{f}} - \mathscr{T} - \mathscr{D}_{\overline{f}}\,, \tag{4}$$

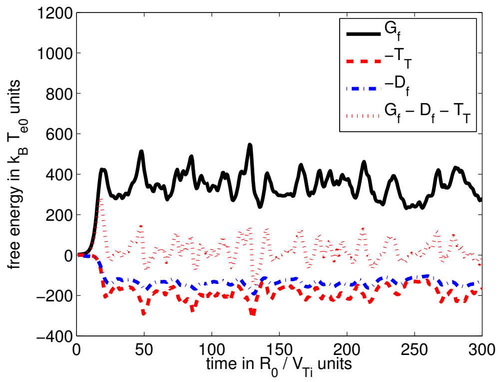

Figure 1: *Injection, dissipation and sub grid scales contributions to the filtered free energy balance (CBC parameters).*

where $\mathscr{E}_{\overline{f}} = \int d\overline{\Lambda}_k \overline{\delta h}_{-k} \overline{\delta f}_k/F_0$ is the free energy, $\mathscr{G}_{\overline{f}} = \int d\overline{\Lambda}_k \overline{\delta h}_{-k} L[\overline{\delta f}_k]/F_0$ is the free energy injection term resulting from the fixed background gradients, $\mathscr{D}_{\overline{f}} = \int d\overline{\Lambda}_k \overline{\delta h}_{-k} D[\overline{\delta f}_k]/F_0$ is the free energy dissipation term. Integration of the resolved nonlinearity cancels and the sub grid scales contribution reduces to: $\mathscr{T} = -\int d\overline{\Lambda}_k \overline{\delta h}_{-k} \overline{N[J_0\phi_k, \delta f_k]}$.

Based on the filtered free energy, we can analyze the effect of sub grid scales in a fully resolved simulation, by simply applying a test filter. All simulations hereafter correspond to the Cyclone Base Case (CBC) set of parameters, that is the standard test case for the study of Ion Temperature Gradient (ITG) turbulence [9].

Fig. 1 illustrates that sub grid scales have a dissipative effect, that is comparable to the filtered resolved dissipations (the test filter width has been chosen to remove half of k_x and k_y domains).

A good model has also to dissipate correctly the free energy, as would have done the small scales that have been filtered out by the coarsening of the perpendicular grid. A first simple model could then be expressed:

$$M = -c_{\perp} k_{\perp}^4 \overline{\delta f}_k \approx T \tag{5}$$

The free parameter $c_{\perp}$ can be calibrated by a try and error process.

Fig. 2 represents the free energy $\mathscr{E}^{k_y}$ spectrum for different values of the free parameter, compared to the filtered spectrum obtained from an highly resolved DNS. A too small value of $c_{\perp}$ leads to an accumulation of free energy at smallest scales of turbulence, while a too high value tends to overestimate the largest scales of turbulence. An optimum is found for $c_{\perp} = 0.375$ in the considered case of Cyclone Base Case parameters. The perpendicular grid for LES is $48N_x \times 24N_y$, while the DNS grid is $128N_x \times 64N_y$. Along other directions grid is fixed to $16N_z \times 32N_{v_{\parallel}} \times 8N_{\mu}$, and the simulation box sizes $125\rho_i \times 125\rho_i$ in perp plane, $2\pi q R_0$ along z (with q the safety factor and R_0 the major radius), from -3 to 3 thermal velocities along $v_{\parallel}$ and 0 to 9 T_{i0}/B_0 along μ (T_{i0} being the ion equilibrium temperature).

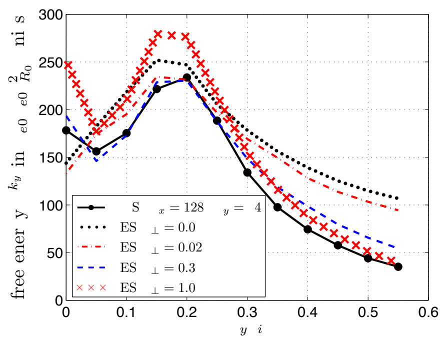

Figure 2: *Free energy $\mathscr{E}^{k_y}$ spectrum for various values of the free parameter $c_{\perp}$.*

The robustness of the optimized value $c_{\perp} = 0.375$ has been tested by comparing the results given by the LES to reference DNS, when varying the temperature gradient L_T, that is of prime importance regarding plasma micro-turbulence. Two values ($L_T = 6.0$ and $L_T = 8.0$) have been tested, corresponding respectively to a low turbulence case and a strong turbulence regime.

In Fig. (3), the model gives a good overall agreement with the DNS free energy spectrum, but the model accumulates a bit too much of free energy at the smallest scales.

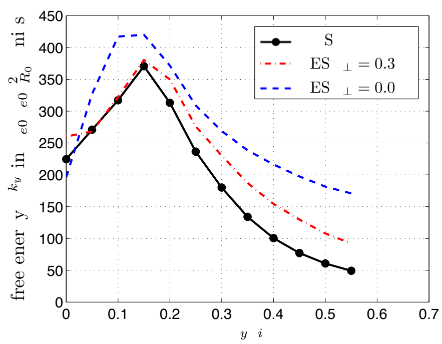

Figure 3: *Free energy $\mathscr{E}^{k_y}$ spectrum in the case of strong ITG turbulence ($L_T = 8.0$).*

On the other hand, comparing LES with DNS free energy spectrum in the low turbulence case (Fig. 4), the model gives very satisfying agreement at smallest scales but is found to overestimate the zonal flow component (associated to $k_y = 0$ mode).

Large Eddy Simulation techniques have been adapted for the first time in the study of plasma micro-turbulence, by using the Gyrokinetic Electromagnetic Numerical Experiment code. The effect of the sub grid scales is found to be clearly dissipative, motivating the choice of a very simple first Gyrokinetic LES hyper-diffusion model. This first model has been calibrated in the Cyclone Base Case set of parameters, and appears to be robust when varying the temperature gradient. A net gain of a factor ≈ 30 has been found by comparing the numerical cost of LES with the one of the DNS considered here.

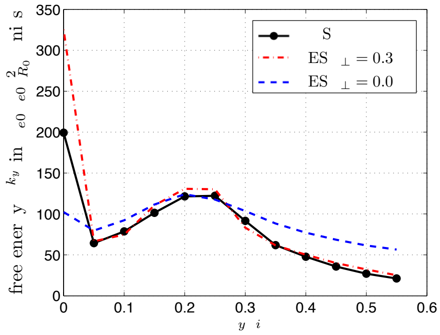

Figure 4: *Free energy $\mathscr{E}^{k_y}$ spectrum in the case of low ITG turbulence ($L_T = 6.0$).*

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