On the stability of electron drift turbulence with respect to excitation of large-scale magnetic fields

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I. In electrostatic drift wave theory, the generation of large-scale structures with additional symmetry, so called zonal flows and streamers, is well-known and active field of studies. These structures are spontaneously generated and sustained by small-scale drift type fluctuations via Reynolds stress using the free energy stored in density and temperature gradients. The mechanism behind can be attributed to the well-known inverse cascade guaranteed in quasi-two-dimensional fluids by the conservation of energy and enstrophy.

The present work investigates the generation of large-scale magnetic fields in magnetic electron drift mode (MEDM) turbulence via modulation instability. The underlying modes are of interest in, e.g., laser fusion experiments, where they are thought to be responsible for the very strong self-generated magnetic fields which have been observed since the 1970s. These experiments showed clearly that strong magnetic fields can be generated even in unmagnetized plasmas. The modulation instability arises in the presence of a small-scale pump wave and its sidebands. The large-scale filed generation occurs via triad interactions, i.e. with $\mathbf{k}+\mathbf{k}'+\mathbf{q}\sim 0$, and is intrinsically nonlocal interaction in k space, since $|\mathbf{k}|\sim|\mathbf{k}'|>>|\mathbf{q}|$. Here $\mathbf{k}$ and $\mathbf{k}'$ denote the small-scale and $\mathbf{q}$ the large-scale wave vector. Thus, if one assumes the presence of a pump wave with wave vector $\mathbf{k}$, two sidebands with wave vectors $\mathbf{k}_{\pm}=\mathbf{k}\pm\mathbf{q}$ will interact with the original pump wave and, as we will show, these interactions can lead to a growth of large-scale fields. This is the modulation instability. Elucidating of these interactions and expression for the resulting increment is the main goal of the present studies. Finally, the evolution of nonlinearly interacting MEDM is illustrated by a simulation study of the model equations for the different set of parameters.

II. We consider a nonuniform unmagnetized plasma, and fluctuations on a space scale much smaller than that of the equilibrium density and temperature inhomogeneities, which are taken to be in the *x*-direction. The time scale is faster than the ion and slower than the electron plasma frequency, and hence we consider an unpolarized electron fluid and immobile ions. We confine our analysis to two-dimensional solutions where all quantities are independent on *z*. The temperature and density gradients of the fluctuations are in general not collinear, and this generates a vorticity in the electron fluid. The consequent motion generates a perpendicular magnetic field, $B(x,y)\mathbf{z}$ say, which actually plays role of a stream function. Then, the simplest energy and momentum equations along with Faraday's and Ampere's laws can be reduced to a pair of coupled non-linear equations for B and perturbed electron temperature T

$$\frac{\partial}{\partial t}\left(B-\lambda^2\nabla^2 B\right)+\beta\frac{\partial T}{\partial y}-\lambda^4\frac{e}{mc}\left\{B,\nabla^2 B\right\}=0\,, \tag{1a}$$

$$\frac{\partial}{\partial t}T + \alpha\frac{\partial B}{\partial y} + \lambda^2\frac{e}{mc}\{B,T\} = 0\,, \tag{1b}$$

where $\alpha = \lambda^4\frac{eT_0}{mc}(\kappa_T - \frac{2}{3}\kappa_n)$, $\beta = \frac{c}{e}\kappa_n$, $\lambda = \frac{c}{\omega_{pe}}$, and κ_n, κ_T being the inverse length scales of the equilibrium density and temperature inhomogeneities. The Jacobian, or Poisson bracket, $\{a,b\}$ is defined as $(\nabla a\times\nabla b)\cdot\mathbf{z}$. The system (1) resembles models describing various low frequency electrostatic modes in magnetized plasmas, as well as shallow water model. Linear analysis for small perturbations, $(B,T)\sim\exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$ yields $\omega_k = k_y\left(\frac{\alpha\beta}{1+k^2\lambda^2}\right)^{1/2}$, $\alpha\beta = V_T^2\lambda^2\kappa_n\left(\kappa_T - \frac{2}{3}\kappa_n\right)$. It is clear that there is a purely growing solution for $\alpha\beta < 0$ and the growth rate vanishes for modes with $k_y = 0$. However, linear theory can only predict strong magnetic fields (exponential growth) and is not capable of describing the wave-wave interactions needed for the generation of large scale magnetic fields. Here we will consider ω_k to be real, i.e. $\alpha\beta > 0$, so that we can concentrate on the nonlinear interactions.

III. To describe the evolution of the coupled system (wave turbulence + large scale plasma flows) we represent the perturbed magnetic field *B* as a sum of a large-scale flow $\bar{B}$ quantity and a small-scale turbulent part $\tilde{B}$. The large-scale plasma flow varies on longer time scale compared to the small-scale turbulent fluctuations, so we may employ a multiple scale expansion, thus assuming that there is a sufficient spectral gap separating large scale and small scale motions.

Since zonal magnetic fields, $\mathbf{q}(q,0,0)$/magnetic streamers, $\mathbf{q}(0,q,0)$ and small-scale turbulence interact via nonlocal triad interactions $\mathbf{q}+\mathbf{k}+\mathbf{k}'\sim 0$, some sidebands to the pump wave have to be involved in the interaction as well, satisfying $\mathbf{k}' = \mathbf{k}\pm\mathbf{q}$. The model representation of four interacting waves is done via Fourier expansion, i.e. for large scale fields we use $\bar{B}(\mathbf{r},t) = B_q\exp(i\mathbf{qr} - i\Omega t)$, and the small-scale turbulence is modeled as the sum of the pump wave and its two sidebands, $\tilde{B} = B_0 + B_+ + B_-$, where $B_0(\mathbf{r},t) = B_k\exp(i\mathbf{kr} - i\omega_k t) + c.c$ is the pump wave and $B_\pm = B_{k\pm}\exp(i\mathbf{k}_\pm\mathbf{r} - \omega_{k\pm}t) + c.c.$ are the upper/lower sidebands. The conditions defining the sidebands are $\omega_{k\pm}\equiv\omega_k\pm\Omega$, and $\mathbf{k}_\pm\equiv\mathbf{k}\pm\mathbf{q}$. A similar representation has been chosen for the electron temperature *T*. Averaging (1) over fast/small scales, we obtain the evolution equations for the large-scale flow:

$$\frac{\partial}{\partial T}\left(\bar{B} - \lambda^2\nabla_\perp^2\bar{B}\right) + \beta\frac{\partial}{\partial y}\bar{T} = \lambda^4\frac{e}{mc}\overline{\left\{\tilde{B},\nabla_\perp^2\tilde{B}\right\}}, \tag{2a}$$

$$\frac{\partial}{\partial T}\bar{T} + \alpha\frac{\partial}{\partial y}\bar{B} = -\lambda^2\frac{e}{mc}\overline{\left\{\tilde{B},\tilde{T}\right\}}. \tag{2b}$$

where $\partial/\partial T$ denotes the partial derivative with respect to the slow time variable. The nonlinear terms on the RHS of Eqs.(1) for small and (2) for the large-scales are determined using the resonance principle and the triad interactions. For zonal fields, the LHS of Eqs.(2) is proportional to $\exp(iqr)$, so that there are four possibilities for the RHS of Eq.(2) in order to be in resonance: $\exp(ik_+r)\times exp(-ikr)$, $\exp(-ikr)\times exp(ik_+r)$, $\exp(-ik_-r)\times\exp(ikr)$, and $\exp(ikr)\times\exp(-ik_-r)$, since these are the four possibilities to decompose q. It follows from these considerations that for determining the evolution of zonal fields/streamers, one has to find the expressions of the amplitudes of the sidebands B_{k+}, T_{k+}, B^*_{k-}, and T^*_{k-}. These can be found from Eqs.(1) with the same resonance arguments as before. Taking now into account the Fourier representation for large/small-scale fields in Eqs.(2) together with the expressions for sidebands yields the dispersion relation of the zonal magnetic fields/magnetic streamers as a function of the pump wave amplitude and small-scale eigenvalues. Anticipating a frequency of the form $\Omega = qv_g + i\gamma$, v_g is the group velocity of MEDM, (the imaginary part has, of course, to be added for instability to be possible) results in the increment of the modulation instability:

a) Zonal magnetic fields, $\mathbf{q}(q,00)$, $v'_g = \partial v_g/\partial k_x$

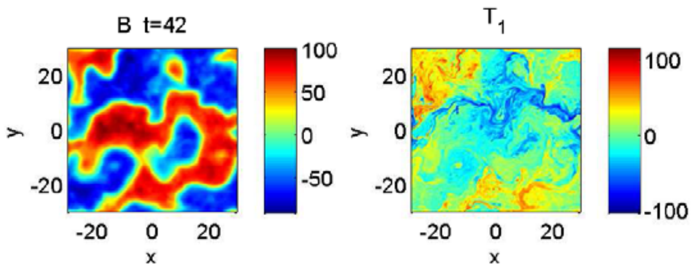
$$\gamma = \pm\sqrt{-\frac{v'_g}{\omega_k}\left(\lambda^2\frac{e}{m}(\mathbf{k}\times\mathbf{q})\cdot\mathbf{z}\right)^2\left(\frac{\lambda^2\left(k^2-q^2\right)}{1+k^2\lambda^2}+1\right)\left(1+k^2\lambda^2\right)^{3/2}|B_0|^2 q^2-\left(q^2v'_g/2\right)^2}$$

b) Magnetic streamers (the instability takes place in the limit $k\lambda >> 1$), $\mathbf{q}(0,q,0)$

$$\gamma = \pm\sqrt{-v'_g\frac{v_g}{\alpha\beta}\left(\lambda^2\frac{e}{m}(\mathbf{k}\times\mathbf{q})\cdot\mathbf{z}\right)^2 k_y q^2\lambda^2|B_0|^2-\left(q^2v'_g/2\right)^2},\qquad v'_g = \frac{\partial v_g}{\partial k_y}$$

It is seen from these expressions that in order to have modulation instability, the system must satisfy $\left(v'_g/\omega_k\right)<0$, which is merely the well-known Lighthill criterion.

IY. A simulation study of the Eqs. (1) for different sets of parameters has been performed. The simulation code is based on a pseudospectral method to resolve derivatives in space with periodic boundary conditions, with random fluctuations as initial conditions.



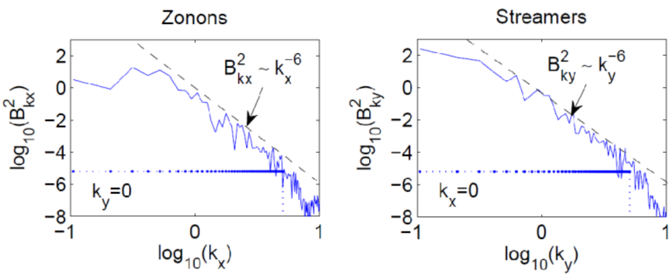


Figure 1: Linearly unstable regime. Top panels: Magnetic field and temperature fluctuations Bottom panels: Zonon and streamer energy spectra of the magnetic field.

In the unstable regime ($\alpha\beta < 0$), displayed in Fig. 1, we could observe magnetic field generation and the formation of large scale magnetic structures, accompanied by small-scale turbulence visible in the temperature fluctuations. The energy spectra are non-Kolmogorov and concentrated to streamers at small wave numbers.

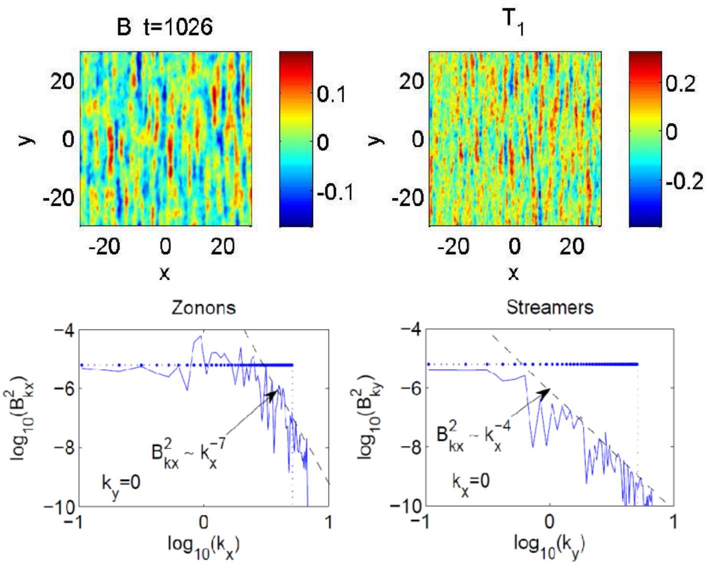


Figure 2: Linearly stable, small amplitude regime. Top panels: Magnetic field and temperature fluctuations Bottom panels: Zonon and streamer energy spectra of the magnetic field.

In the linearly stable regime ($\alpha\beta > 0$) in Fig. 2, we observe small-scale turbulence and the formation of zero-frequency zonal flows (zonons). The energy spectra are strongly anisotropic with magnetic wave energy concentrated at zonons.

Y. The properties of MEDM turbulence are studied and a mechanism of generation of large-scale magnetic fields is investigated. It is shown that in the presence of a small-scale pump wave with a wave vector $\mathbf{k}$, an upper and lower sidebands will interact with the pump wave due to nonlocal triad interactions with $\mathbf{k}+\mathbf{k}'+\mathbf{q} \sim 0$, where the wave vectors of the sidebands are $\mathbf{k}' = \mathbf{k} \pm \mathbf{q}$, and the condition $|\mathbf{q}| << |\mathbf{k}|$ is satisfied. These interactions are elucidated, an expression for the resulting increments of zonal fields and magnetic streamers is calculated, and a condition similar to the Lighthill criterion for instability is found. The obtained analytical results are illustrated by numerical studies of the model equations which exhibit the excitation of the large-scale magnetic structures by the MEDM turbulence.