VLASOV SIMULATION OF WAKE-FIELD ACCELERATION USING A RELATIVISTIC ELECTRON BEAM

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Large amplitude wake fields in a plasma can be produced by a relativistic electron beam which drives plasma waves, which in turn support acceleration gradients much greater than those obtained in conventional linear accelerators. In the plasma wake-field accelerator concept, a correctly placed trailing electron bunch can be accelerated to high energies by the longitudinal electric field of the wake plasma waves [1]. We present a one-dimensional (1D) simulation for this problem using a Vlasov code in which the relativistic beam is evolving and treated self-consistently by following the time evolution of the total distribution function (the plasma and the beam) in the self-consistent field. Direct simulation via the Vlasov equation is an ideal way for examining the details of the low density region of the phase-space where electron bunches are accelerated, and of the thermal and trapping effects, because the artificially high temperatures associated with typical particle simulations codes are avoided.

The relevant equations and results.

The initial plasma distribution function is a Gaussian in momentum $\exp(-p_x^2/2\upsilon_{the}^2)$ with a temperature $T_e = 10$ keV. The plasma is located in a domain with a vacuum gap of length $L_{vac} = 1.183$ on each side (see Fig.(1)). Time t is normalized to ω_{pe}^{-1}, length is normalized to $l_0 = c\omega_{pe}^{-1}$, momentum is normalized to $M_e c$. The scale length of the jump in the plasma density at the plasma edge is $L_{edge} = 1.414$ on each side of the plasma. The length of the flat top plasma density is $L_p = 35.16$, for a total length of the domain of 40.355. A similar density profile is used for the background ions, with an initial Maxwellian distribution with $T_i = 1$ keV. The beam is injected with a relativistic velocity υ_b and a relativistic factor $\gamma_b = 1/\left(1-\upsilon_b^2\right)^{1/2}$ =25 (the beam velocity υ_b is normalized to the velocity of light c, so that $\upsilon_b^2 = 0.9984$), and an initial distribution function which is a Gaussian in momentum $\exp(-(p_x - p_b)^2/2\upsilon_{thb}^2)$ centered at $p_b = \gamma_b \upsilon_b$, with a thermal temperature of $T_b = 10$ keV. The beam is injected at the left boundary. We assume for simplicity that it is initially located in space at the left boundary with a spatial profile $\sin(\pi x / L_b)$ for $0 < x < 2\pi$ with $L_b = \lambda_p = 2\pi c / \omega_{pe}$ (see Fig.(1)). In our normalized units $\upsilon_{the,b} = (T_{e,b} / 510.984)^{1/2}$. The beam density n_b and the plasma density n_p are such that $n_b / n_p = 0.2$. We study this problem by using an Eulerian Vlasov code for the numerical solution of the 1D relativistic Vlasov equations for electrons and ions (similar to the one previously applied in [2] to study the wake-field acceleration produced by a laser pulse):

$$\frac{\partial f_{e,i}}{\partial t} + \frac{p_{xe,i}}{m_{e,i}\gamma_{e,i}} \frac{\partial f_{e,i}}{\partial x} \mp E_x \frac{\partial f_{e,i}}{\partial p_{xe,i}} = 0 \ ; \qquad \frac{\partial E_x}{\partial x} = (n_i - n_e) \qquad (1)$$

Subscripts *e* and *i* refer to electrons and ions, $f_{e,i}(x, p_{xe,i}, t)$ is the total distribution function for electrons (plasma and beam) and ions. $n_{e,i}$ is the density of electrons (beam and plasma) and ions. $\gamma_{e,i} = (1 + p_{xe,i}^2)^{1/2}$.

$$n_{e,i}(x,t) = \int f_{e,i}(x, p_{xe,i}, t) dp_{xe,i} \qquad (2)$$

In our normalization $m_e = 1$ for the electrons, and $m_i = M_i / M_e$ for ions. Hydrogen ions are used. The code applies a numerical scheme based on a two-dimensional advection technique, of second order accuracy in time-step, where the value of the distribution function is advanced in time by interpolating in two dimensions along the characteristics, using a tensor product of cubic *B*-splines [2]. We include a background ion population which is also treated with a kinetic equation. We note however that for the parameters used in the present simulation, ions were essentially immobile and had no effect on the evolution of the system. The Eulerian Vlasov code has very low noise level and provides detailed representation of the phase-space of the distribution function, including the fine structure of the accelerated electrons and the evolution of the initially injected beam.

Fig.(2) show the density profiles at *t*=34.34 , at a time where the beam has reached the right boundary (see Fig.(4)). The full curve represents the total electron density (plasma+beam), and the dashed curve represents the ion density profile. Fig.(3) presents the longitudinal wake field excited by the beam (normalized to $E_0 = M_e c \omega_{pe} / e$). The peak value is about 0.42. The value calculated in [3] for a warm plasma, using a waterbag distribution function for the warm plasma electrons, with a non-relativistic Vlasov-Poisson system and $\mu = 3T_e / (M_e c^2) \approx$ 0.0587, is given by:

$$E_{\max} / E_0 = (1 - \mu / 3 - 8\mu^{1/4} / 3 + 2\mu^{1/2})^{1/2}$$

assuming $\upsilon_{ph} = \upsilon_b \approx c$. We get $E_{\max} / E_0 = 0.39$, in close agreement with the value 0.42 observed in Fig.(3).

Figs.(2,3) show a wavelength of $2\pi \approx 6.283$, which is the initial length of the beam in Fig.(1). Fig.(4) represents the phase-space contour plot of the total electron distribution function at *t*=34.34, in the domain containing the plasma (the small vacuum region on either side is eliminated). Half the beam has already left the domain where the plasma had initially a flat profile. We see the electrons accelerated in the wake field detaching from the plasma (the density of the accelerated electrons has been artificially enhanced to make them visible in the contour plot). In Fig.(5) we follow the evolution of the injected beam by concentrating the contour plot on this driving beam at three different times. The contour plot at the left in Fig.(5) represents the initially injected beam, a relativistic distribution in p_x with a half-sine profile in *x*, of length 2π as previously mentioned above. The contour in the middle represents the injected beam at *t*=16.16, and at the right is the beam contour at *t*=34.34, where half the beam profile has already left the domain where originally the plasma had a flat density of 1. We see the peak of the beam decelerating while two extended arms are formed. The contour plot of the ion distribution function shows the same modulation length of $2\pi \approx 6.283$ as in Figs.(2,3). However, if we except the small deformation at the left boundary due to the initial injection of the beam, ions show little effect on the results, as we verified by rerunning the code with immobile ions. Fig.(7) plots the temperature $T_e = P_e/n_e$ at *t*= 38.34 (after the injected beam has completely left the plasma domain), where P_e is defined by [1]:

$$P_e(x,t) = \int dp_{xe} (p_{xe} - \overline{p}_{xe})(\upsilon_{xe} - \overline{\upsilon}_{xe}) f_e(x, p_{xe}, t) \quad (3)$$

$\overline{p}_{xe}$ and $\overline{\upsilon}_{xe}$ are the average momentum an velocity defined by:

$$\overline{p}_{xe}(x,t) = \frac{1}{n_e} \int p_{xe} f_e(x, p_{xe}, t) dp_{xe} \;;\quad \overline{\upsilon}_{xe}(x,t) = \frac{1}{n_e} \int (p_{xe} / \gamma_e) f_e(x, p_{xe}, t) dp_{xe} \quad (4)$$

References

[1] J. Krall, G. Joyce, E. Esarey NRL Memorandum Report 6772 (1991)

[2] M. Shoucri Comm. Comp. Phys. 4, 703 (2008)

[3] T.P. Coffey, Phys. Fluids 14, 1402 (1971)

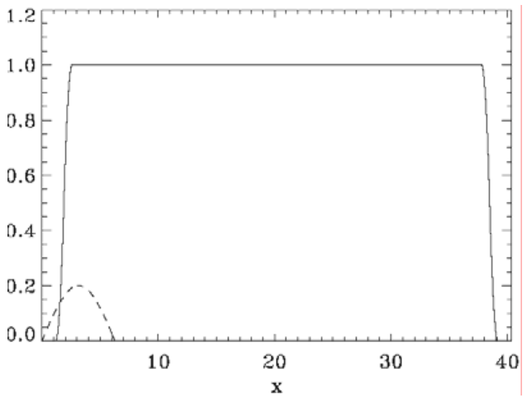

Fig.1 Initial density profile for the electrons and ions (full curve), and for the injected driving beam (dashed curve)

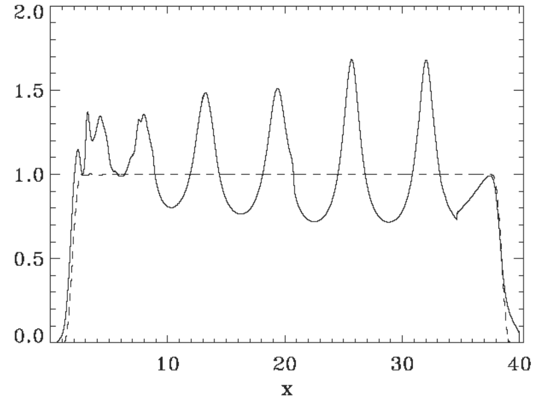

Fig.2 Density profile for the electrons (full curve) and ions (dashed curve)at *t*=34.34

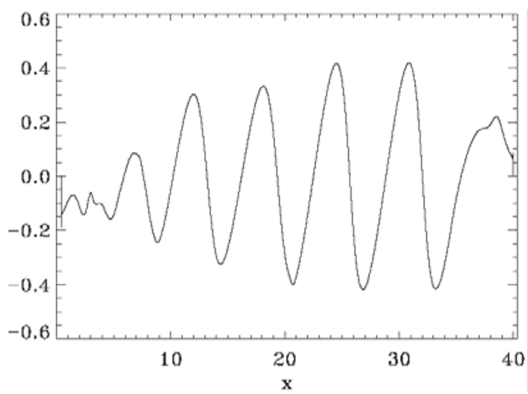

Fig.3 The wake electric field at *t*=34.34

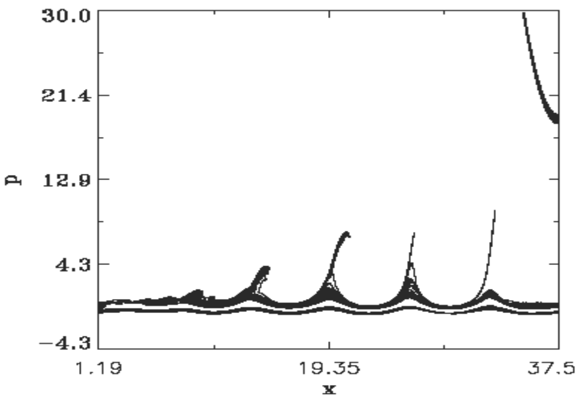

Fig.4 Contour plot of the total electron distribution function at *t*=34.34

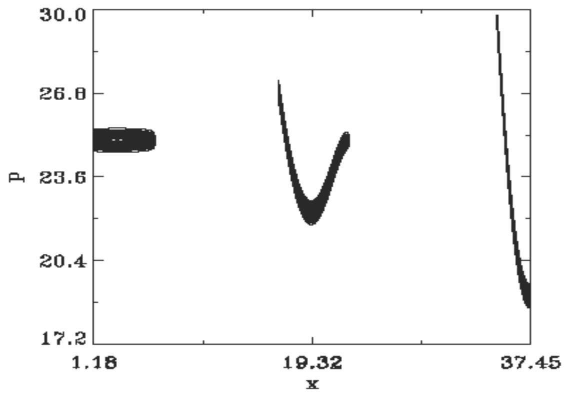

Fig.5 Contour plot of the injected driving beam at *t*=0 (left), *t*=16.16 (middle), *t*=34.34 (left)

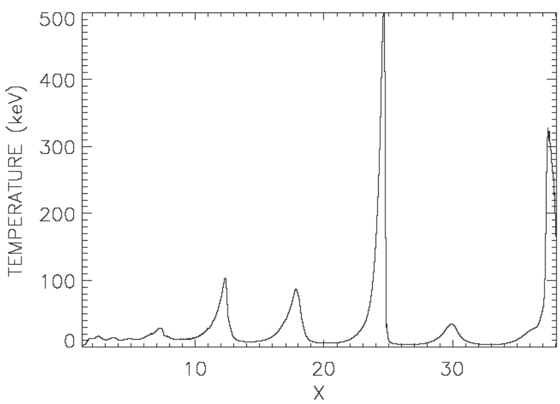

Fig.6 Temperature T_e of the plasma