Quantumlike description of the nonlinear and collective effects on relativistic electron beams in strongly magnetized plasmas

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In this paper, we want to study, in a self consistent way, some nonlinear and collective transverse effects due to the interaction of a relativistic electron/positron beam with a magnetoactive plasma in overdense regime, i.e., $n_0 >> n_b$ (n_0 and n_b being the unperturbed plasma and beam densities, respectively). To this end, in the long beam limit, we consider the self-interaction that is produced when a relativistic charged particle beam is travelling in the plasma exciting large amplitude plasma waves, namely plasma wake field (PWF) excitation [1].

Hereafter, we refer to our paper [2] appearing in this proceedings, as well. According to this paper, we assume that the plasma is collisionless and cold, with ions at rest forming a uniform background of positive charge. Furthermore, a strong constant and uniform external magnetic field is assumed to be acting along the z-axis, $B_0 = B_0\hat{\mathbf{e}}_z$. We also assume that the electron/positron beam is initially travelling along the direction of the magnetic field with a velocity $\mathbf{v}_b = \beta c\hat{\mathbf{e}}_z$ ($\beta \simeq 1$). We consider a fluid model, consisting of Lorentz-Maxwell system of equations for the *beam-plasma* system. From the perturbed Lorentz-Maxwell system, we obtain an equation that governs the evolution of the *plasma wake potential* driven by the charged particle beam density. On the other hand, by ignoring the longitudinal beam dynamics, we write the equation that governs the *spatio temporal* evolution of the charged particle beam given by the thermal wave model (TWM) [3, 4] (For details, see Ref.[2]). These pair of equations can be cast as a *quantumlike Zakharov system of equations* which governs the self consistent *spatio temporal* evolution of the *PWF self-interaction* of the electron/positron beam, viz.,

$$i\varepsilon\frac{\partial\Psi}{\partial\xi} = -\frac{\varepsilon^2}{2}\nabla_\perp^2\Psi - \frac{i\varepsilon k_c}{2}\hat{z}\cdot(\mathbf{r}_\perp\times\nabla_\perp)\Psi + U_w(\mathbf{r}_\perp,\xi)\Psi + \frac{1}{2}Kr_\perp^2\Psi, \quad (1)$$

$$\left(\nabla_\perp^2 - \frac{\omega_{pe}^2}{\omega_{UH}^2}\frac{\omega_{pe}^2}{c^2}\right)U_w = \frac{\omega_{pe}^2}{\omega_{UH}^2}\frac{\omega_{pe}^2}{c^2}\frac{\rho_b}{n_0\gamma_0}, \quad (2)$$

where ω_{UH} is the electron upper hybrid frequency and $\Psi = \Psi(\mathbf{r}_\perp,\xi)$ is the beam wave function (BWF), so that its squared modulus is proportional to the beam density, i.e., $\rho_b(\mathbf{r}_\perp,\xi) = (N/\sigma_z)|\Psi(\mathbf{r}_\perp,\xi)|^2$, where N and σ_z are the total number of particles and the beam length, respectively, $K \equiv (\omega_c/2\gamma_0 c)^2 \equiv \left(qB_0/2m_0\gamma_0 c^2\right)^2 \equiv (k_c/2)^2$, $\nabla_\perp^2$ is the transverse part of the gradient operator, $U_w = (A_{1z} - \phi_1)/m_0\gamma_0 c^2$ is the dimensionless *wake potential* with $A_{1z} = A_{1z}(\mathbf{r}_\perp,\xi)$ and $\phi_1 = \phi_1(\mathbf{r}_\perp,\xi)$ the longitudinal vector potential perturbation and electric potential perturbation, respectively. Here, $\mathbf{r}_\perp$ is the transverse position vector, $\xi = z - \beta ct \simeq z - ct$ plays the role of time-like variable, m_0 and γ_0 are the electron/positron rest mass and the unperturbed relativistic gamma factor of the single particle of the electron/positron beam, respectively. In cylindrical coordinates, $r_\perp$, φ, ξ, we look for a solution of the Zakharov-like system of the form $\Psi(r_\perp, \varphi, \xi) = \exp[im(\varphi - k_c\xi/2)]\,\psi_m(r_\perp, \xi)$ with m integer, taking the limiting case $\left|\nabla_\perp^2\right| \ll \omega_{pe}^4/c^2\omega_{UH}^2$. Let us define the transverse beam size in the form of r.m.s., i.e., $\sigma_m^2(\xi) = 2\pi\int_0^\infty r_\perp^2 |\psi_m|^2 r_\perp dr_\perp$. Under the above assumptions and definitions, from the Zakharov-like system, we easily obtain the following 2D Gross- Pitaevskii-type equation, viz.,

$$i\frac{\partial\psi_m}{\partial\xi} = -\frac{1}{2r_\perp}\frac{\partial}{\partial r_\perp}\left(r_\perp\frac{\partial\psi_m}{\partial r_\perp}\right) - \delta_m \mid \psi_m \mid^2 \psi_m + \left(\frac{1}{2}K_b r_\perp^2 + \frac{m^2}{2r_\perp^2}\right)\psi_m, \tag{3}$$

where we have introduced the following dimensionless quantities: $\xi \to \xi/\beta_0$, $r_\perp \to r_\perp/\sigma_0$, $\psi_m \to \sqrt{\pi m!\sigma_0^2}\,\psi_m$, $K_b = K\sigma_0^4/\varepsilon^2$, $\delta_m = n_b\sigma_0^2/n_0\gamma_0\varepsilon^2 m!$, σ_0 and ε being the initial transverse beam spot size and the transverse emittance, respectively. We use the virial equation associated with eq. (3)

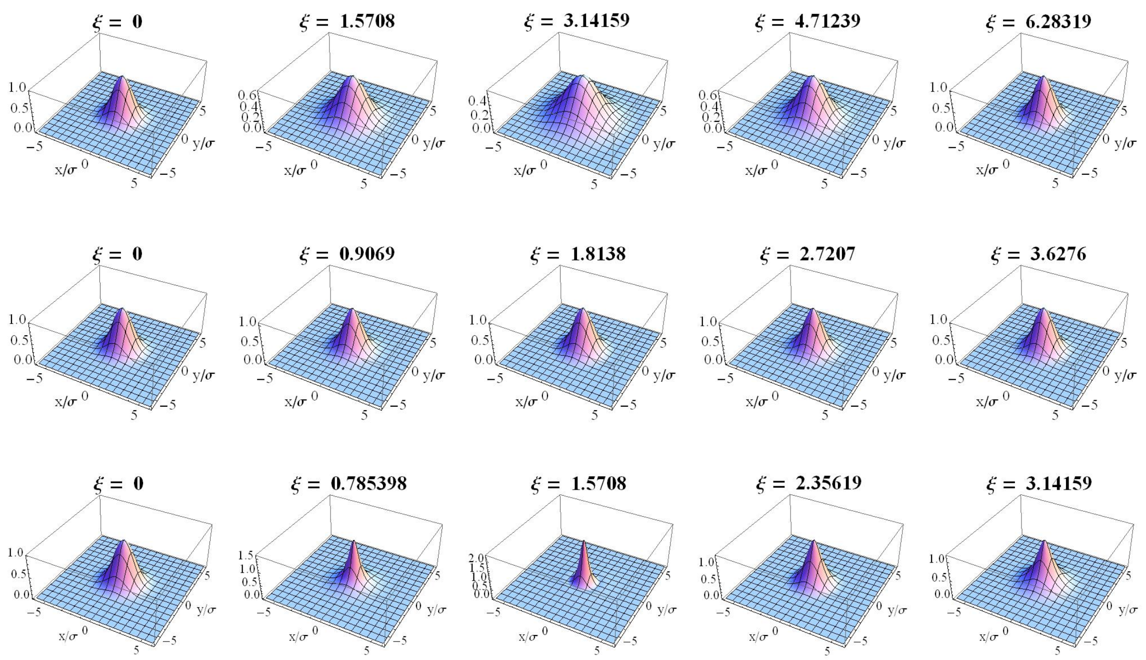

Figure 1: 3D plots of $|\psi_m|^2$ as function of x/σ and y/σ for different values of the dimensionless time ξ for $m = 0$: $K_b = 0.25$, $\delta_m = 0.5$, $\mathscr{A}_m = 0.5$ (first row); $K_b = 0.75$, $\delta_m = 0.5$, $\mathscr{A}_m = 0.75$ (second row); $K_b = 1.0$, $\delta_m = 1.5$, $\mathscr{A}_m = 0.625$ (third row).

to get the envelope equation $d^2(\sigma_{r_\perp}^m)^2/d\xi^2 + 4K_b(\sigma_{r_\perp}^m)^2 = 4\mathscr{A}_m$, where $\sigma_{r_\perp}^m \to \sqrt{m!}\,\sigma_{r_\perp}^m/\sigma_0$ and $\mathscr{A}_m = \frac{1}{2}(m+1)!(1+K_b) - \delta_m(2m)!2^{-2(m+1)}$ is constant of motion. Note that, from the envelope equation, the matching condition for the equilibrium transverse beam spot size, σ_{eq}^m, is $K_b(\sigma_{eq}^m)^2 = \mathscr{A}_m$.

A preleminary numerical analysis has been carried out by solving eq. (3) assuming the initial normalized BWF (density profile) as $\psi_m(r_\perp, 0) = r_\perp^m \exp\left(r_\perp^2/2\right)$. The spatio-temporal evolution of $|\psi_m|^2$ has been

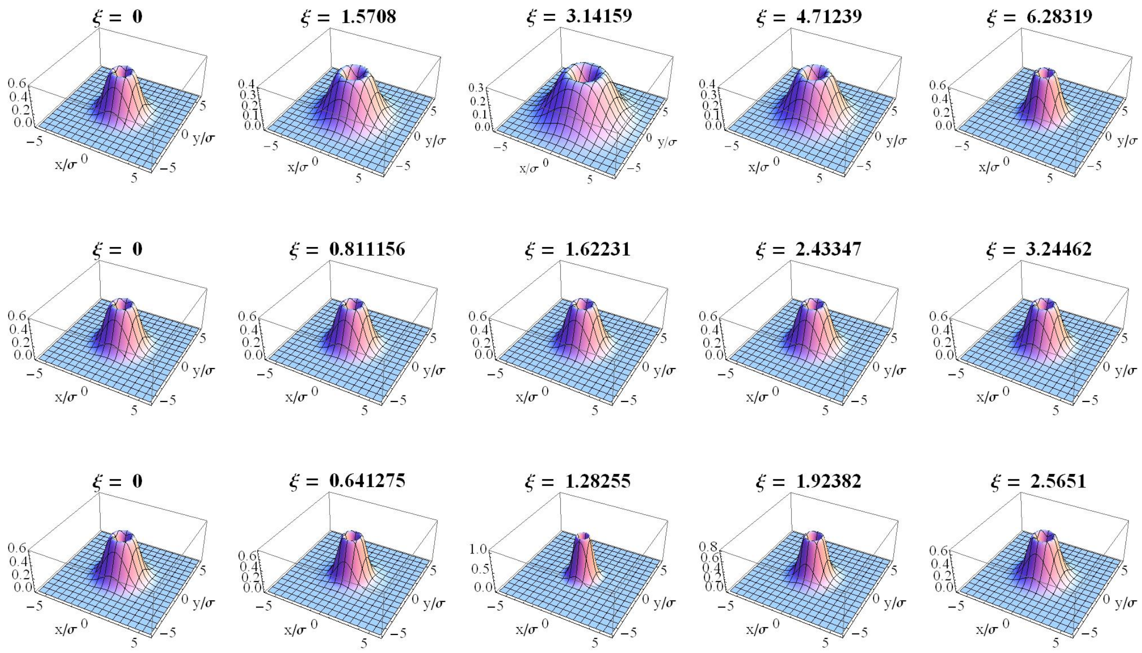

Figure 2: 3D plots of $|\psi_m|^2$ as function of x/σ and y/σ for different values of the dimensionless time ξ for $m = 1$: $K_b = 0.25$, $\delta_m = 0.5$, $\mathscr{A}_m = 1.1875$ (first row); $K_b = 0.9375$, $\delta_m = 0.5$, $\mathscr{A}_m = 1.1875$ (second row); $K_b = 1.5$, $\delta_m = 3.5$, $\mathscr{A}_m = 2.0625$ (third row).

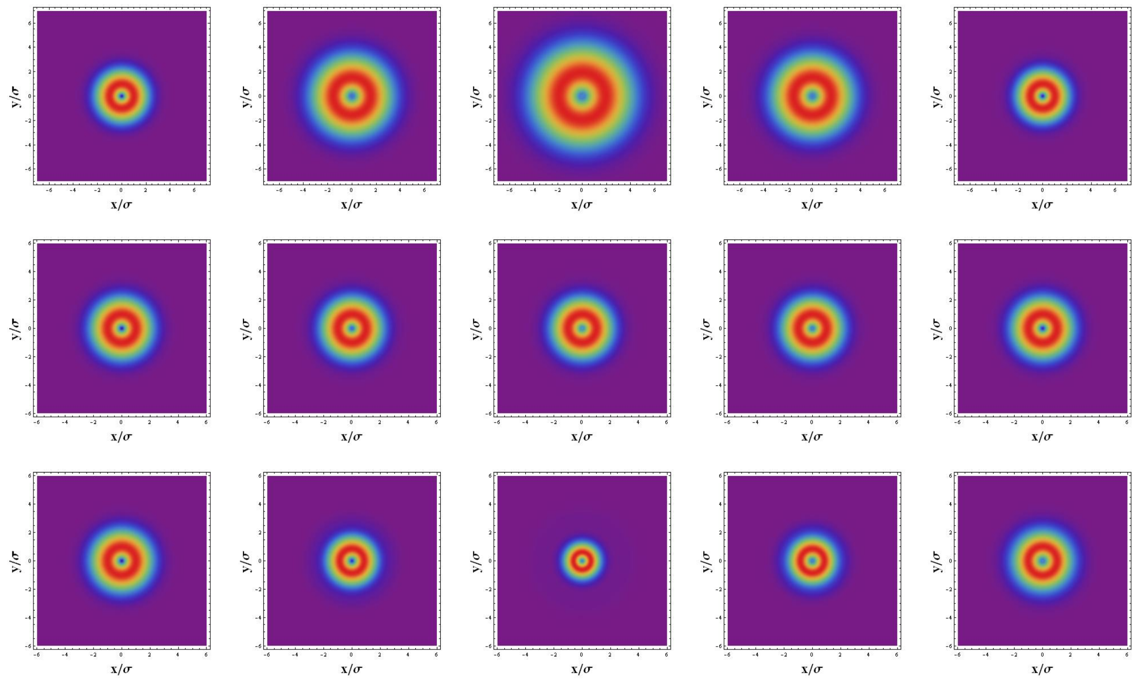

Figure 3: Density plots of $|\psi_m|^2$ in the x/σ, y/σ plane for different values of the dimensionless time ξ for $m = 1$. The choice of the parameters is the one corresponding to Figure 2: $K_b = 0.25$, $\delta_m = 0.5$, $\mathscr{A}_m = 1.1875$ (first row); $K_b = 0.9375$, $\delta_m = 0.5$, $\mathscr{A}_m = 1.1875$ (second row); $K_b = 1.5$, $\delta_m = 3.5$, $\mathscr{A}_m = 2.0625$ (third row).

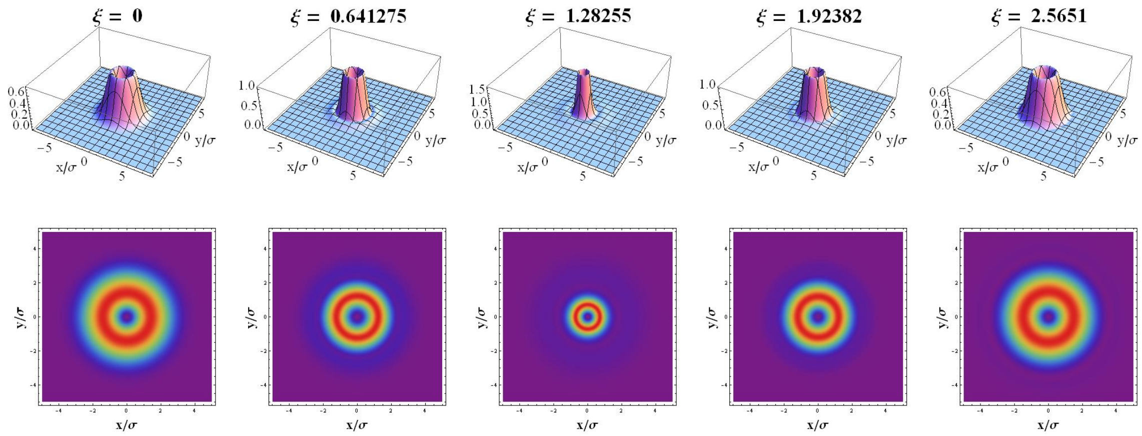

Figure 4: 3D plots (first row) and density plots (second row) of $|\psi_m|^2$ as function of x/σ and y/σ for different values of the dimensionless time ξ for $m = 2$: $K_b = 1.5$, $\delta_m = 5.0$, $\mathscr{A}_m = 5.625$.

investigated for different values of m, K_b, and δ_m, at $\xi = 0$, $0.25T$, $0.5T$, $0.75T$, T, where $T = \pi/\sqrt{K_b}$. For both $m = 0$ and $m = 1$, when the matching condition of the envelope equation is satisfied, the profile is practically unchanged (see the second row of Figures 1, 2 and 3, respectively). This predicts the existence of nonlinear coherent states (sometimes called 2D solitons). Furthermore, due to the strong nonlinearity, the effect of *beam halo* has been observed for $m = 2$, as displayed by both 3D and density plots in Figure 4. Due to the interplay between the strong transverse effects of the plasma wake field (collective and nonlinear effects) and the magnetic field, envelope oscillations with weak and strong focusing and defocusing have been observed for $m = 0$ (see the first and the third row of Figure 1), $m = 1$ (see the first and the third row of Figures 2 and 3, respectively) and $m = 2$ (see first and second row of Figure 4). Finally, the existence of vortices (effect of the orbital angular momentum due to the external magnetic field) are clearly shown in Figures 2,3 and 4, respectively.

The present investigation seems to be useful for the plasma-based focusing schemes to be employed in the final focusing stages of linear colliders as well as for manipulating relativistic electron/positron beams that suggests the new fields of nonlinear and collective singular electron optics.

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