On the mechanism of electron acceleration in "pre-plasma"

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I. Introduction

In last decade the generation of intense electron beams in laser-plasma interactions has attracted significant attention of plasma community due to potentially interesting and useful applications. However, the performance of intense electron beam in these applications can strongly depend on the energy spectrum of the beam. Recently it has been shown that the presence of pre-plasma in front of a solid target plays very important role in the establishing energy spectrum of the beam and has a trend to significantly increase average beam energy. For example, in recent experiments with planar targets, [1] high-energy tail of fast electrons with energies much greater than the ponderomotive potential was observed in presence of pre-plasma with a density scale-length of approximately 10 μm. Also, the experiments with cone shaped targets [2] have shown that the fast electron generation is significantly modified by the presence of long scale pre-plasma inside the cone. In case of proton acceleration experiments, increase in maximum proton energy was reported with increase in pre-pulse energy [3]. However, the underlying physics of the impact of pre-plasma had not clear. Our study has suggested [4] that the synergetic effects of large electrostatic potential well formed in pre-plasma and relativistic laser radiation are responsible for the generation of very energetic electrons with the energy well above so-called ponderomotive scaling. But, the mechanism of the synergy was not identified in Ref. 4.

Here we present the results of both analytic and numerical studies of the impact of electrostatic potential well on electron dynamics in and acceleration by linearly polarized relativistic laser radiation. We show that the presence of electrostatic potential well results in a stochastic heating of the electron even in the presence of just one plane laser wave and resembles the Fermi acceleration mechanism [5].

II. Equations and analytic estimates

We consider the relativistic electron dynamics in the fields of plane laser wave propagating along z-coordinate and electrostatic electric field directed along z:

$$\frac{d(\gamma v_z)}{dt} = -\frac{1}{2\gamma}\frac{\partial a^2}{\partial z} - \varepsilon, \qquad \frac{d\gamma}{dt} = \frac{1}{2\gamma}\frac{\partial a^2}{\partial t} - \varepsilon v_z, \tag{1}$$

where v_z is the electron velocity component along z, $a(t,z)$ is the normalized vector potential of the laser wave, $\varepsilon(z)$ is the normalized electrostatic electric field, γ is the relativistic factor, $\gamma \equiv \gamma_A \gamma_z$, $\gamma_A(t,z) = \sqrt{1+(a(t,z))^2}$, and $\gamma_z = 1/\sqrt{1-v_z^2}$. For the case of just one propagating wave $a(t,z) \equiv a(t-z)$ from Eq. (1) we find

$$\frac{d}{dt}\left\{\gamma_A \gamma_z (1-v_z)\right\} = \varepsilon(1-v_z). \tag{2}$$

For constant electric field Eq. (2) can be integrated and we have

$$\gamma_A \gamma_z (1-v_z) = \delta_0 + \varepsilon(t - t_0 - z), \tag{3}$$

where t_0 is a time at which electron crossing boundary z=0, and $\delta_0 = \gamma_A \gamma_z (1-v_z)\big|_{t=t_0}$. For simplicity we consider V-shaped

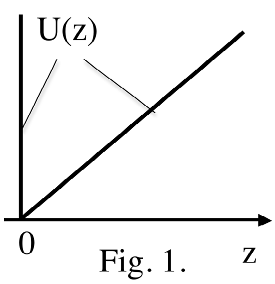

Fig. 1.

normalized electrostatic potential U(z) (see Fig. 1), which is characterized by constant normalized electric fields, $\varepsilon = \text{const.}$ at z>0 and "potential wall" at z=0 where electron is just reflected back preserving its energy.

The trajectory of electron, $z(t)$, at positive z can be found by introducing a local time $\tau = t - z$ and using velocity v_z from Eq. (3):

$$v_z(\tau) \equiv \frac{dz}{dt} = \frac{f^2(\tau)-1}{f^2(\tau)+1}, \qquad \text{or} \qquad \frac{dz}{d\tau} = \frac{f^2(\tau)-1}{2}, \tag{4}$$

where $f(\tau) = \gamma_A(t_0+\tau)/(\delta_0+\varepsilon\tau)$.

By using Eq. (4) becomes rather straightforward to study the dynamics of electron motion due to impact of both laser wave and V-shape electrostatic potential. From Eq. (4) one finds that two consecutive times t_0 and t_1 at which electron is rejected from the boundary z=0 are related by the following equation

$$\int_0^{t_1-t_0} d\tau \frac{\left(\gamma_A(t_0+\tau)\right)^2}{\left(\delta_0+\varepsilon\tau\right)^2} = t_1 - t_0, \tag{5}$$

and corresponding parameter $\delta_1 = \gamma_A\gamma_z(1-v_z)\big|_{t=t_1}$ can be expressed as following

$$\delta_1 = \frac{\left(\gamma_A(t_1)\right)^2}{\delta_0+\varepsilon(t_1-t_0)}. \tag{6}$$

For the case of linearly polarized wave $a(t,z) = a_w\cos(t-z)$ and $\delta_0 << 1$ (which corresponds to large electron energy) integral (5) can be taken analytically with required accuracy $\sim O(\delta_0) << 1$. As a result, introducing parameter $\hat{E}_{(...)} = \left(\gamma_A(t_{(...)})\right)^2/\varepsilon\delta_{(...)}$, which in the limit $\delta_0 << 1$ is proportional to E/ε ($E = \gamma = \gamma_A\gamma_z >> 1$ is the dimensionless electron energy), we find

$$\hat{E}_1 = \hat{E}_0 - \left(\frac{a_w}{\varepsilon}\right)^2\left\{\frac{\pi}{2}\cos(2t_0) + \left[\ell n\left(\frac{\hat{E}_0\varepsilon^2}{2\left(\gamma_A(t_0)\right)^2}\right) - C\right]\sin(2t_0)\right\}, \tag{7}$$

$$t_1 = t_0 + \hat{E}_1. \tag{8}$$

where C is the Euler constant. Mapping (7, 8) is very similar to the Chirikov standard map [6]. As a result, we conclude that stochastic heating of electron caused by the synergetic effects of laser radiation and electrostatic field occurs for $a_w \tilde{>} \varepsilon$.

For a deep stochastic regime, $a_w > \varepsilon$, electron heating can be described by diffusion [6] in energy space, E ($E >> 1$). For this case from Eq. (7) it follows that the elementary "steps" in the energy space, δE, and time, δt, can be estimated, correspondingly, as $\delta E \sim a_w^2\Lambda/\varepsilon$ and $\delta t \sim E/\varepsilon$, where $\Lambda \approx \text{const.}$ is a slowly varying logarithmic function in the right hand side of Eq. (7). As a result the energy diffusion coefficient can be estimated as $D_E \sim (\delta E)^2/\delta t \sim (a_w^2\Lambda)^2/E\varepsilon$. With such diffusion coefficient the asymptotic time evolution of the averaged electron energy, $\langle E\rangle$, and the electron distribution function, $f(E,t)$, is given by the following expressions:

$$\langle E\rangle \propto \left(D_E t\right)^{1/3}, \quad \text{and} \quad f(E,t) \propto t^{-1/3}\exp\left(-E^3/9D_E t\right). \tag{9}$$

III. Numerical modeling

To verify our analytic estimates we solve electron equation of motion in the form of Eq. (4) with a proper boundary conditions at z=0. We find that the analytical predictions in the high-energy limit obtained from equations (7) and (8) are in good agreement with the numerical calculations. The Fig. 2 shows the comparison of Poincare map in (γ, ϕ) space through $z = 0$, where the "phase" $\phi = t_{...} - \pi\,[t_{...}/\pi]$, where [x] is the integer part of x.

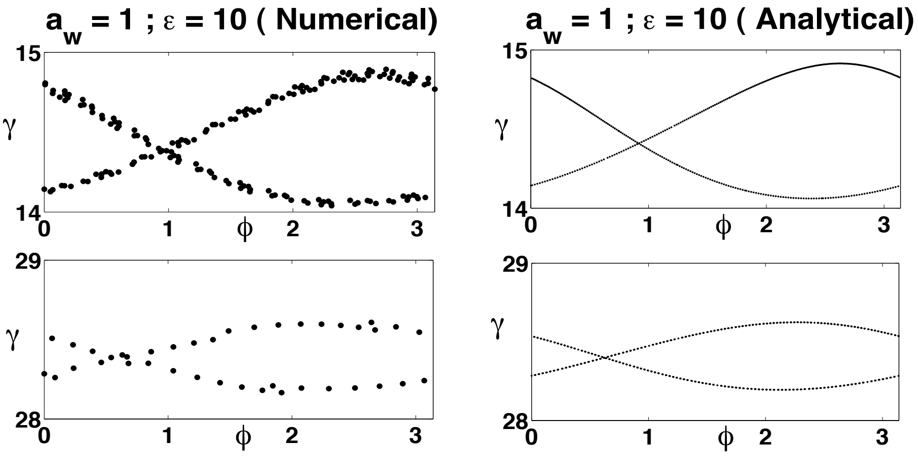

Fig. 2. Comparison of numerical calculations with analytic results obtained from Eq. (7,8).

As a result, to model electron dynamics for large energy we can use mapping equations (7) and (8) rather than solving more complex equation of motion (4). By using Eq. (7, 8), the transition from regular (Fig. 3a, 3b) to stochastic (Fig. 3d) motion, with increasing a_w/ε ratio and effective "threshold" $a_w/\varepsilon \sim 1$ is clearly seen in Fig. 3.

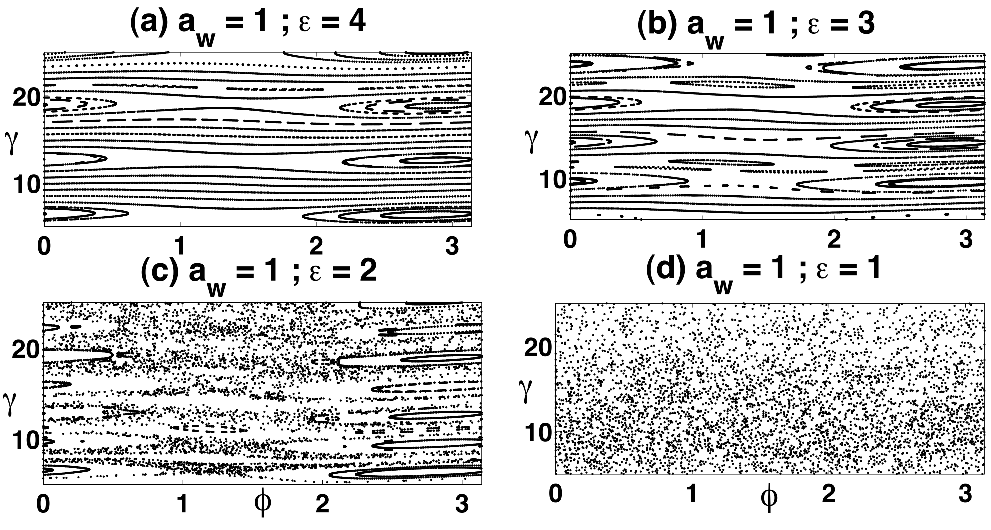

Fig. 3. Transition from regular to stochastic motion of electron with increasing ratio a_w/ε.

To address the issue of the energy of electrons in the beam generated in pre-plasma (see Ref. 4) we need to consider the potential well of finite depth and analyze the distribution function of electrons coning out from the well. To be able to carry such analysis but still be compatible with our previous approach we adopt potential well shown in Fig. 4. We will assume that the "depth" of the potential well, $U(L)$, is larger than the energy space diffusion step-size, $U(L) = \varepsilon L > \delta E \sim a_w^2/\varepsilon$, and electron motion in the well is stochastic $a_w > \varepsilon$. For this case one can expect that the kinetic energy of electrons, E_{kin}, at z=L will be bounded by $E_{kin} \lesssim \delta E$. As a result, the beam energy can be estimated as $\gamma_{beam} \sim 1 + \alpha a_w^2/\varepsilon$, where $\alpha \sim 1$ is the numerical factor. For relativistic laser intensity this energy is much larger than the energy following from the ponderomotive scaling $\gamma_{ponder} \sim \sqrt{1 + a_w^2}$ (recall that $a_w > \varepsilon$), which is in agreement with simulation results from Ref. 4. To verify our estimates we solve electron equation of motion (4) for potential well from Fig. 4. The distribution function of electrons coming out of potential well for different values of a_w, ε, and L, but keeping $\delta E \sim a_w^2/\varepsilon$ constant is plotted in Fig. 5. We find that within the error bar both $<\gamma>$ and $<\gamma_z>$ depend only on a_w^2/ε, which confirms our qualitative consideration.

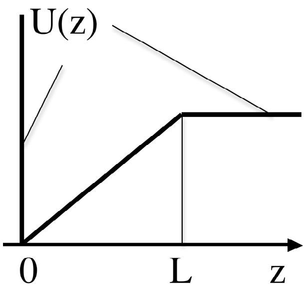

Fig. 4. Potential well with finite depth.

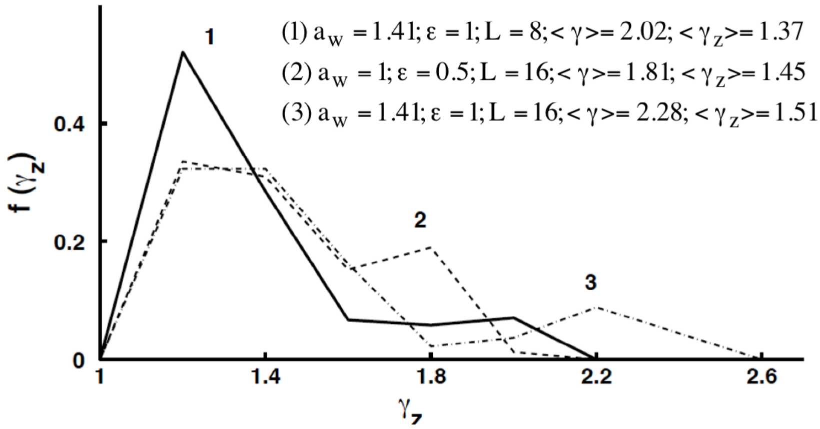

Fig. 5. Energy distribution and averaged energies of electrons coming out of the well. Notice that δE is kept constant in all cases.

IV. Conclusions

We analyze the heating mechanism of pre-formed plasma electrons due to synergetic effects of relativistic laser radiation and longitudinal electric field. We consider V-shaped electrostatic potential well which allows to use the integrals of the motion available for electron motion in laser and constant electric fields. This approach significantly simplifies our numerical simulation and allows making some theoretical predictions. Based on our theoretical results and numerical simulations, which are in agreement with each other, we conclude that: i) For the ratio of the normalized wave vector potential a_w to electric field ε roughly exceeding unity ($a_w/\varepsilon \gtrsim 1$) electron motion in the laser and electrostatic potential fields becomes stochastic. ii) For $a_w/\varepsilon > 1$ electron dynamics can be considered as a diffusion in the energy space with the characteristic energy step-size $\sim a_w^2/\varepsilon$. iii) The energy of electrons escaping from the well (beam electrons) can be estimated as $\gamma_{beam} \sim 1 + \alpha a_w^2/\varepsilon$ which is (in agreement with Ref. 4) much larger than the ponderomotive energy.

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