Role of the plasma anisotropy in the "missing power" problem

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1. Introduction. This is an attempt to find a better explanation of the T-10 experimental results [1], where a considerable part of the injected power was 'missing' just after the switching on the electron cyclotron resonance heating (ECRH) pulse. Similar effect has also been observed during ECRH on the TEXTOR tokamak [2]. In the both cases, only one third of the launched EC power was experimentally found in the plasma in the heating region. In [3, 4] it was shown that, in the frame of ideal MHD with isotropic plasma pressure, the plasma interaction with the magnetic field cannot explain such a deficiency in the energy balance, though the magnetic field (disregarded in [1]) provides some fast redistribution of the injected power. Therefore, assuming that the experimental results are basically correct, we have to look for the model better suited to describe the plasma equilibrium evolution at fast plasma heating. Here the equilibrium with anisotropic plasma pressure (see, for example, [5]) is considered as a natural extension of the models used in [3, 4]. The anisotropy affects not only the energy balance, but also the diagnostic signals. We analyse the both aspects. Magnetic measurements are considered as a diagnostic tool for the plasma energy increase.

2. Formulation of the problem. Two states are compared, before and after the fast heating of the plasma. Fast means that the time interval of interest is smaller than the time of the magnetic field diffusion through the wall, and the resistive losses in the wall can be disregarded. In such case, the injected energy absorbed in the plasma must be somehow redistributed inside the volume bounded by the vessel wall. In [3, 4], the energy acceptors were the plasma and the magnetic field. The energy loss through other channels (for example, thermal conduction, radiation, ionization of the neutral gas) is disregarded.

Our assumptions mean that the energy balance is

$$\delta E_H = H - \delta W_m^{pl} - \delta W_m^{g}, \tag{1}$$

where δ denotes the increment, E_H is the thermal energy of the plasma, H is the energy deposition from the outer sources (heating if $H > 0$) and W_m^{pl} and W_m^{g} are, respectively, the magnetic energies in the plasma and in the plasma-wall vacuum gap. Precisely,

$$E_H \equiv \frac{3}{2}\int_V \frac{2p_\perp + p_\parallel}{3} dV, \qquad W_m \equiv \int_V \frac{\mathbf{B}^2}{2} dV, \tag{2}$$

with different V for W_m^{pl} and W_m^g. Here $p_\parallel$ and $p_\perp$ are the pressures parallel and perpendicular to the magnetic field $\mathbf{B}$.

Analysis of the energy balance (1) at $p_\perp = p_\parallel = p$ is presented in [3, 4]. Here with $p_\perp \neq p_\parallel$ we have to calculate W_m^{pl} and W_m^g in equilibrium with the force balance

$$0 = -\nabla \cdot \vec{\vec{p}} + \mathbf{j} \times \mathbf{B}, \tag{3}$$

where $\mathbf{j} = \nabla \times \mathbf{B}$ is the current density and $\vec{\vec{p}}$ is the pressure tensor ($\vec{\vec{\mathbf{I}}}$ is the unit dyadic),

$$\vec{\vec{p}} = p_\parallel \frac{\mathbf{BB}}{\mathbf{B}^2} + p_\perp \left(\vec{\vec{\mathbf{I}}} - \frac{\mathbf{BB}}{\mathbf{B}^2} \right). \tag{4}$$

As in [3, 4], we supplement this by the constraints that the toroidal magnetic fluxes in the plasma and in the plasma-wall gap remain unchanged during the fast heating.

3. Equilibrium and the magnetic energy. Anisotropic plasma equilibrium in toroidal systems is described in detail in [5]. In the cylindrical approximation, the perpendicular component of (3) gives us

$$\frac{dp_\perp}{d\rho} - \frac{p_\parallel - p_\perp}{\mathbf{B}^2} \frac{B_\theta^2}{\rho} = \mathbf{e}_\rho \cdot [\mathbf{j} \times \mathbf{B}], \tag{5}$$

where $\mathbf{e}_\rho = \nabla \rho$, ρ, θ, z are the cylindrical coordinates with radius ρ, poloidal angle θ and coordinate z along the main axis. For plasmas in tokamaks and stellarators, the term with $p_\parallel - p_\perp$ can be disregarded. Then the only difference of (5) from the equilibrium equation for isotropic plasma is that it contains $p_\perp$ instead of p. This means that the consequences of isotropic version of (5) must be valid for anisotropic case too if p is replaced by $p_\perp$ there.

Therefore, calculations of the magnetic energy terms in (1) at the conditions similar to those in [3, 4] will give us the same result with $p \to p_\perp$: $\delta W_m^{pl} + \delta W_m^g \approx 0$ at $\mathbf{B}^2 >> p_\parallel, p_\perp$. This means that, finally, at $p_\perp \neq p_\parallel$ we have (disregarding corrections of the order of $p / \mathbf{B}^2$)

$$\delta E_H \approx H \,. \tag{6}$$

With almost unchanged magnetic energy during the fast transition of the anisotropic plasma from one equilibrium state to another we come to the conclusion that, as in the case with isotropic plasma [3, 4], all the ‘missing’ power must be perfectly confined in the plasma.

In [1], the ‘missing’ power problem was described as the fact that, in the T-10 tokamak, the absorbed ECRH power, determined by the change in time derivative of the electron temperature at the region of ECRH power input, and the absorbed ECRH power, determined by the magnetic measurements, were several times different (their ratio varied from 0.2 to

0.4). With (6) this result implies that responsible for the discrepancy may be interpretation of diagnostics. We consider measurements of the diamagnetic signal and of the pressure-induced changes in the poloidal magnetic flux ($\Delta\psi$) outside the plasma.

4. Magnetic signals. It is well known that the diamagnetic signal $\Delta\Phi$ reacts to the changes of $p_\perp$, which is illustrated by the formula (see [6] and references therein)

$$2\frac{\Delta\Phi}{\Phi_0} = \frac{B_J^2}{B_0^2} - \overline{\beta}_\perp + 2\frac{\Delta\Phi_{st}}{\Phi_0}, \tag{7}$$

where $\Phi_0 = B_0 S$ with $S = \pi b^2$ the transverse cross-section of the plasma column, b is its minor radius, B_0 is the toroidal field, B_J is B_θ at the plasma boundary, $\overline{\beta}_\perp \equiv 2\overline{p}_\perp / B_0^2$ is the ratio of the volume-averaged perpendicular pressure of the plasma to the magnetic field pressure $B_0^2/2$, and $\Delta\Phi_{st}$ is a 'stellarator' term (zero in tokamaks).

In [1], $\Delta\psi$ measurements were used to determine the absorbed ECRH power at the fast heating [1, 7]. This is equivalent to determining the first harmonic of the poloidal field $B_\theta = \sum H_n \cos n\theta$ at the plasma boundary [8]. Finding H_1 requires full-scale toroidal calculations because $H_1 = 0$ in the 'cylindrical' limit. As shown in [8], in tokamaks and stellarators with a major radius R we have

$$H_1 - \frac{b}{R}B_J = \frac{b}{R}B_0\left(\mu\Delta_b' + \mu_h'\Delta_b\right) + B^*\frac{\Delta_b}{b}, \tag{8}$$

where μ is the rotational transform, μ_h is the part of μ due to the helical fields ($\mu_h = 0$ in tokamaks), $B^* \equiv B_0\mu_h b/R$, Δ_b is the plasma shift and the prime means its radial derivative.

According to [9], in tokamaks the right hand side of (8) is proportional to $0.5(\overline{\beta}_\perp + \overline{\beta}_\parallel)$, where $\overline{\beta}_\parallel \equiv 2\overline{p}_\parallel / B_0^2$. As shown in [5], this result may be valid only if the poloidal asymmetry of $p_\perp$ is small. With this restriction, for anisotropic plasma in tokamaks and stellarators, H_1 is expressed by the 'isotropic' formula [8, 9] if we make a substitution there:

$$\beta \to 0.5(\beta_\perp + \beta_\parallel). \tag{9}$$

From diagnostic viewpoint, this is different from (7) which has exactly the same form as the 'isotropic' result with $\beta \to \beta_\perp$. If only $p_\perp$ is varied at the fast heating while $p_\parallel$ is fixed, the increase of $\Delta\psi$ must be two times smaller than that expected with $\overline{\beta}_\perp$ found from (7) and with disregard of anisotropy. In other words, measurements of $\Delta\Phi$ and $\Delta\psi$, both yielding $\delta\overline{\beta}$ for isotropic plasma, must give, respectively, $\delta\overline{\beta}_\perp$ and $\delta\overline{\beta}_\perp/2$ for anisotropic at $\delta p_\parallel = 0$.

5. Magnetic measurements and plasma energy. At fast change in the ECRH power we can expect larger increase in $p_\perp$ than in $p_\parallel$. Then

$$\delta E_H = (\delta\overline{p}_\perp + 0.5\delta\overline{p}_\parallel)V_{pl}, \qquad (10)$$

where V_{pl} is the plasma volume, can be reasonably estimated with $\delta p_\parallel = 0$. In the opposite limit, with $\delta p_\parallel = \delta p_\perp$, equation (10) gives us δE_H larger by factor of 1.5. This discrepancy can appear if $\delta\overline{p}_\perp$ is precisely known. However, there can be additional ambiguity related to interpretation of the measurements. Equation (9) shows that estimate of $\beta_\perp = 2\beta_{\Delta\psi} - \beta_\parallel$ with $\beta_{\Delta\psi}$ found from the measured $\Delta\psi$ and standard 'isotropic' formulas [8, 9], as was [7] the case in [1], can vary by a factor of two depending on the assumption on $\delta p_\parallel$. This and the presence of $\delta p_\parallel$ in (10) means that the integration of $2\beta_\perp + \beta_\parallel = 4\beta_{\Delta\psi} - \beta_\parallel$ in (2) will give us, if the anisotropy is neglected, only 3/4 of the correct value of δE_H at $\delta p_\parallel = 0$.

In [1, 2], the misbalance comes from interpretation of the measurements. We proved that the plasma anisotropy, if not properly treated, can contribute to the 'missing' power effect. With plasma heating affecting mainly the perpendicular pressure, the effect must be larger than predicted by the isotropic model [3, 4]. Accuracy of determining the absorbed energy can be improved by measuring both $\Delta\Phi$ and $\Delta\psi$ and using proper 'anisotropic' formulas. An additional magnetic diagnostics for the fast transient events is proposed in [10].

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