Evaluation of Magnetic Diagnostics for MHD Equilibrium Reconstruction of LHD Discharges*

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Introduction: Equilibrium reconstruction is the process of determining the set of parameters of an MHD equilibrium that minimize the difference between expected and experimentally observed signals. This is routinely performed in axisymmetric devices, such as tokamaks, and the reconstructed equilibrium solution is then the basis for analysis of stability and transport properties. The V3FIT code [1] has been developed to perform equilibrium reconstruction in cases where axisymmetry cannot be assumed, such as in stellarators. The present work is focused on using V3FIT to analyze plasmas in the Large Helical Device (LHD) [2], a superconducting, heliotron type device with over 25 MW of heating power that is capable of achieving both high-beta (~5%) and high density (>1 x $10^{21}/m^3$). This high performance as well as the ability to drive tens of kiloamperes of toroidal plasma current leads to deviations in the equilibrium state from the vacuum flux surfaces. This initial study examines the effectiveness of using magnetic diagnostics as the observed signals in reconstructing experimental plasma parameters for LHD discharges.

V3FIT uses the VMEC [3] 3D equilibrium solver to calculate an initial equilibrium solution with closed, nested flux surfaces based on user specified plasma parameters. This equilibrium solution is then used to calculate the expected signals for specified diagnostics. The differences between these expected signal values and the observed values provides a starting χ^2 value. V3FIT then varies all of the fit parameters independently, calculating a new equilibrium and corresponding χ^2 for each variation. A quasi-Newton algorithm [1] is used to find the path in parameter space that leads to a minimum in χ^2.

Effective diagnostic signals must vary in a predictable manner with the variations of the plasma parameters and this signal variation must be of sufficient amplitude to be resolved from the signal noise. Signal effectiveness can be defined for a specific signal and specific reconstruction parameter as the dimensionless fractional reduction in the posterior parameter variance with respect to the signal variance [4]:

$$S_j^i = \frac{\sigma_i^{sig}}{\sigma_j^{param}} \frac{d\sigma_j^{param}}{d\sigma_i^{sig}} \tag{1}$$

Here, σ_i^{sig} is the variance of the *ith* signal and σ_j^{param} is the posterior variance of the *jth* fit parameter. The sum of all signal effectiveness values for a given reconstruction parameter is normalized to one. This quantity will be used to determine signal effectiveness for various reconstruction cases. The next section will examine the variation of the expected signals with changes in plasma pressure and the following section will show results for reconstructing model plasmas using these signals.

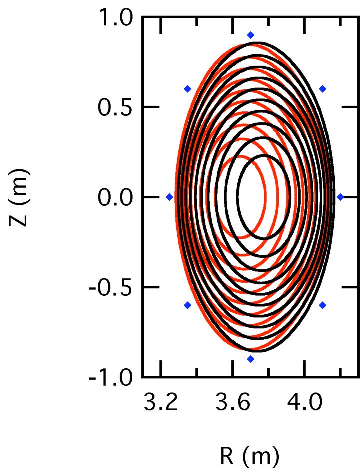

Figure 1: Flux surfaces for the initial vacuum state (red) and the final 2% β state (black).

Magnetic signal variation with pressure: An initial vacuum equilibrium provides the basis for the equilibrium calculations in this study. The vacuum magnetic field structure at the start of one field period is shown by the red flux surfaces in Figure 1. This configuration has zero pressure, zero current and a volume averaged magnetic field of 2.8 T. Increasing pressure is added to this initial vacuum state, with the internal pressure profiles shown in Figure 2 versus the normalized toroidal flux parameter, *s*, that serves as the radial coordinate in VMEC. The highest pressure case has a volume averaged β of 2.3%. The increasing pressure results in a small change in the position of the edge, but the magnetic axis undergoes a significant Shafranov shift as shown in Figure 1. The black surfaces correspond to the β = 2.3% case shown in Figure 2.

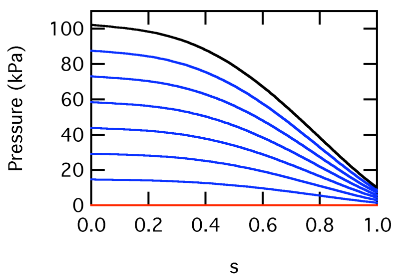

Figure 2: Pressure profiles for diagnostic signal calculation.

LHD has a significant magnetic diagnostic set that is described in detail in [5]. Rogowski coils are used to measure the coil currents, currents flowing in support and vacuum structures as well as the plasma current. Poloidal flux loops measure the loop voltage and a diamagnetic loop measures the plasma stored energy. Helical and toroidal arrays of magnetic pickup coils provide localized measurement of the magnetic field and saddle loops mounted inside the vacuum vessel provide localized measurements of the magnetic flux. For this study, the as-installed saddle loops will be examined as well as a model diamagnetic loop and magnetic pickup coils. The locations of these model magnetic probes are shown by the blue diamonds around the perimeter of the last flux surface in Figure 1. At each of these locations, magnetic probes have been modeled measuring the field in the x, y and z directions.

Diagnostic signals are calculated for equilibria for each profile shown in Figure 2. The results for a magnetic probe located at the outboard midplane measuring the field in the z-direction is shown in Figure 3. The measured value is the total signal the probe would measure and the value labeled plasma is the contribution due to the plasma, which is zero at zero pressure. The two signals have scales of equal amplitude, so the traces overlay since the only variation with increasing pressure is due to the plasma contribution. The plasma contribution to the magnetic probe signal is ~4%, while the plasma contribution to the saddle loops is ~20% and the plasma contribution to the diamagnetic loop is only ~0.6%. All of the signals exhibit nearly linear variation with increasing pressure, indicating that they have the required variation to guide the fitting algorithm to a minimum in χ^2.

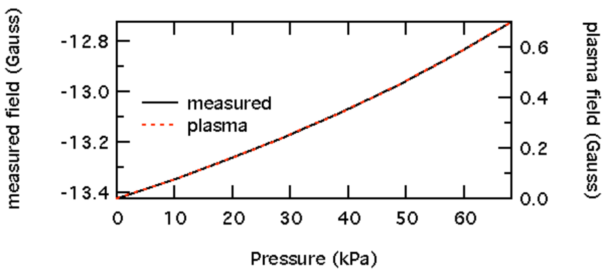

Figure 3: Magnetic probe signal versus pressure.

Signal effectiveness: Initial studies have been performed to determine the signal effectiveness when reconstructing the equilibrium parameters. The first case examined is the reconstruction of the external coil currents with no plasma pressure or current. This case is examined to determine if the code algorithm is capable of solving the simplest equilibrium reconstruction problem. Also, depending on the accuracy of the coil current measurements or if there are significant currents flowing in external structures, it may be necessary to fit the external currents during plasma reconstruction.

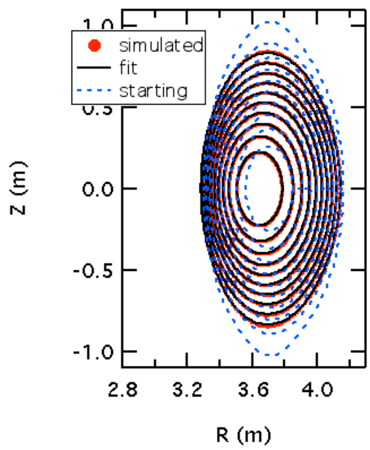

Figure 4: Vacuum reconstruction showing the simulated (red), starting (blue dashed) and reconstructed (black) surfaces.

Ideal measurements are used during the fit with a 1% σ on each signal. The six coil currents are the only variable fit parameters. The reconstructed flux surfaces are shown in Figure 4. The flux surfaces shown in red are the simulated surfaces that represent the ideal reconstruction. The dashed blue surfaces are the surfaces from the initial coil currents, and the black surfaces are the reconstructed surfaces. The flux surface shape was fit well by the code, with the black surfaces nearly overlaying the red. The signal effectiveness for one of the helical coil currents is shown in Figure 5 (a) and the effectiveness for one of the poloidal field coil currents is shown in Figure 5 (b). The helical coil current is most strongly coupled to one of the magnetic field probes, since this probe is very near the coil. The poloidal field coil current is most strongly coupled to four of the saddle loops.

The next case studied is a finite β plasma case. Again, ideal measurements with 1% σ are used. The coil currents are held fixed while the pressure scale factor and the enclosed toroidal flux are the reconstructed parameters. The starting equilibrium is the vacuum case. The reconstructed pressure profile is shown in Figure 6. The plasma geometry and pressure are fit well. The simulated profile was generated with a 4th order polynomial, while the reconstructed profile was generated with a 2nd order polynomial, so an exact fit is not possible. The signal effectiveness for the reconstructed parameters is shown in Figure 7. Several of the saddle loops are effective for the pressure and the diamagnetic loop is most effective for the edge toroidal flux. In an ideal case, the diamagnetic loop is a direct measurement of the plasma toroidal flux, so this is expected.

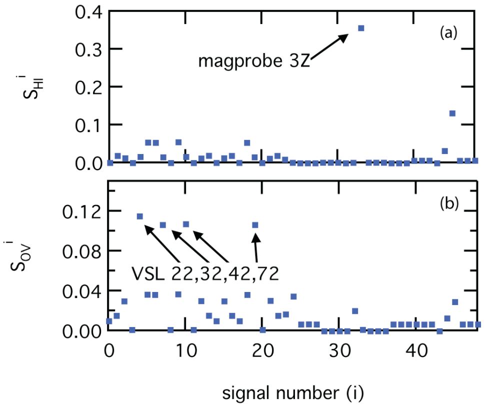

Figure 5: Signal effectiveness for vacuum reconstruction showing a helical coil (a) and a poloidal field coil (b).

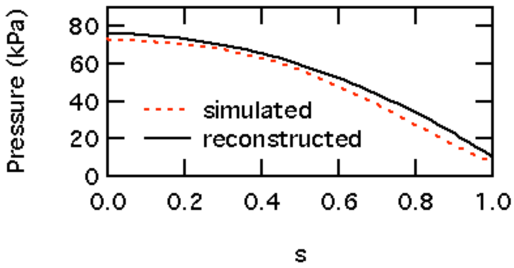

Figure 6: Reconstructed (black) and simulated (red) pressure profiles for a finite β case.

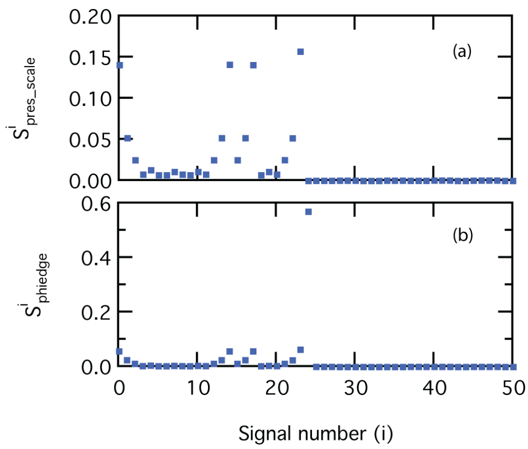

Figure 7: Signal effectiveness for the pressure scale (a) and the enclosed toroidal flux (b).

Summary and future work: Initial studies on the effectiveness of magnetic diagnostics for performing 3D equilibrium reconstruction on LHD indicate that the magnetic diagnostics are generally effective when fitting the pressure and external coil currents. Future work will include the effects of noise on the measurements and expand the scope to include plasmas with non-zero current profiles.

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[1] J.D. Hanson, *et al.*, *Nucl. Fusion* **49** (2009) 075031.
[2] A. Komori, *et al.*, *Fusion Sci. and Tech*. **58** (2010) 1.
[3] S.P. Hirshman and D.K. Lee, *Comput. Phys. Commun*. **39** (1986) 161.
[4] J.D. Hanson, *et al., Contrib. Plasma Phys.* **50** (2010) 724.
[5] S. Sakakibara, et al., *Fusion Sci. and Tech*. **58** (2010) 471.