Simple 1D Fokker-Planck modeling of ICRF heating at arbitrary harmonics accounting for non-Maxwellian plasma populations

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INTRODUCTION

To rigorously assess the impact of ion cyclotron resonance frequency (ICRF) heating on the distribution functions of the various plasma constituents, a set of coupled Fokker-Planck equations needs to be solved, ideally in conjunction with a powerful wave equation solver and a transport code. As doing so while accounting for all details of the wave-particle interaction is time consuming, simpler models are often used to get a crude impression of how the absorbed RF power affects the plasma. Stix [1,2] proposed a method to analytically compute the isotropic (pitch angle averaged) distribution function of a population heated by electromagnetic waves. To ensure that the solution can be found in analytical form, the applicability of the expression provided by Stix was somewhat limited: It was assumed that the particles are heated at their fundamental cyclotron frequency and that a not too energetic minority tail is formed. Moreover, the background plasma particles were assumed to be Maxwellian.

Allowing for numerical rather than analytical integration and adopting the general Coulomb collision operator for arbitrary distributions proposed by Karney [3], Stix's method can immediately be extended to describe ICRF heating of not only small minorities but also of large populations at any cyclotron harmonic, fully accounting for their Coulomb collisional interaction by solving a set of coupled Fokker-Planck equations in which none of the species is assumed to be Maxwellian.

GENERALIZATION OF STIX'S MODEL

In case there are no particle sources nor losses, the isotropic (pitch angle averaged) part of the Fokker-Planck equation for a particle distribution F_0 can be written as

$$\frac{1}{\mathrm{v}^2}\frac{\partial}{\partial \mathrm{v}}\left[G_1F_0+G_2\frac{\partial F_0}{\partial \mathrm{v}}+G_3\mathrm{v}^2\frac{\partial F_0}{\partial \mathrm{v}}+\frac{\mathrm{v}^3}{\tau_E}F_0\right]=0 \qquad (1)$$

in which $G_1=2\mathrm{v}^2F_\mathrm{v}$ and $G_2=2\mathrm{v}^2D_\mathrm{vv}$ are the drag F_v and diffusion D_vv coefficients connected to the Coulomb collisions, G_3 represents the RF diffusion and τ_E is the local energy loss time. In particular for a background of thermal particles and for heating at the fundamental cyclotron frequency [1,2]

$$G_1 = \sum_f C_f \frac{m\mathrm{v}^2}{kT_f} \mathrm{G}(l_f \mathrm{v}), \quad G_2 = \sum_f C_f \mathrm{v}\,\mathrm{G}(l_f \mathrm{v}), \quad G_3 = \frac{2P_{RF}}{3nm} \tag{2}$$

in which P_{RF} is the RF power density, n and m are the density and mass of the examined (test) species, C_f and l_f are constants related to various parameters of the background species *f*. The sum is on all background species and the function $\mathrm{G}(x)$ is defined by $\mathrm{G}(x) = [\Phi(x) - 2\Phi'(x)]/2x^2$, in which $\Phi(x)$ is the error function.

Integrating Eq.1 over velocity space (and reminding that both F_0 and its derivative drop to zero when $\mathrm{v} \to \infty$) yields a simple expression for the test species' distribution

$$F_0(\mathrm{v}) \propto \exp\left(-\int_0^{\mathrm{v}} d\mathrm{v}' \frac{G_1 + \mathrm{v}'^3/\tau_E}{G_2 + G_3 \mathrm{v}'^2} \right) \tag{3}$$

The expression found by Stix can immediately be extended to model heating at higher cyclotron harmonics by describing the velocity dependence of the RF diffusion term G_3 via the Kennel & Engelmann operator T_N [4]. Since the T_N operator for fundamental ICRH ($N = 1$) reduces to $|E_+|^2$ in the limit taken by Stix, generalization merely requires to substitute the function G_3 by

$$G_3 \to G_3 T_N = G_3 \left| J_{N-1} E_+ + J_{N-1} E_- + J_N E_{//} \right|^2 \tag{4}$$

Knowledge of the RF field polarizations $(E_+, E_-, E_{//})$ is now needed and the function G_3 is no longer related to the power density by a simple proportionality relation.

Although ion cyclotron heating schemes aim at heating a specific ion species, the other plasma constituents are usually directly or indirectly heated as well. As a result, aside from writing down a Fokker-Planck equation with the proper RF heating term for each of the species, the Coulomb collision operator adopted by Stix should be upgraded to an operator that describes collisional interaction with species away from thermal equilibrium, including the self-collisions among species of the same kind. The collisional diffusion and drag coefficients for arbitrary distributions are given explicitly by Karney [3]. It can readily be verified that for each of the species *f*, the $\mathrm{G}(x)$ function in G_1 and G_2 should be substituted, respectively by

$$\mathrm{G}_f \to \frac{8\pi}{n_f} \frac{kT_f}{m_f} \frac{1}{\mathrm{v}^2} \int_0^{\mathrm{v}} d\mathrm{v}' \mathrm{v}'^2 F_{0,f} \quad \text{and} \quad \mathrm{G}_f \to \frac{8\pi}{3n_f} \frac{1}{\mathrm{v}^2} \left[\int_0^{\mathrm{v}} d\mathrm{v}' \mathrm{v}'^4 F_{0,f} + \mathrm{v}^3 \int_{\mathrm{v}}^{\infty} d\mathrm{v}' \mathrm{v}' F_{0,f} \right]. \tag{5}$$

Because the collision operator for the self-collisions is a non-linear operator, the solution of the FP equation can no longer be found analytically. In the procedure described here, the solution is found by setting up an iterative numerical scheme similar to the one proposed by Louche [5]: In each iteration loop, the Fokker-Planck equation for all the plasma constituents is solved, taking the distributions from the previous time step to compute the collision operators. At the end of a series of Fokker-Planck evaluations of the distribution functions, a convergence check is made. If the newly found distributions are not yet close enough to the ones found in the previous iteration a next iteration step is initiated, otherwise the computation is stopped.

EXAMPLE

An example that illustrates all the different ingredients described here to extend the applicability of Stix's method is the $\omega = \Omega_{3He} = 2\Omega_T$ ICRF heating scenario proposed for ITER's D-T phase, where simultaneous N=2 'majority' T and N=1 minority ^{3}He ICRF heating (together with significant fast wave electron damping) will take place. Because the tritons and the electrons absorb RF power both directly and indirectly, accounting for their self-collisions is crucial in this case. Fig.1 shows the evolution of several quantities during the first 600 iterations of the numerical procedure. A typical ITER plasma composed of equal amounts of D and T with 3% of ^{3}He was considered. The total RF power density is P_{RF}=0.5MW/m^3, of which 20%, 20% and 60% were assumed to be absorbed by the electrons, by the Tritons and by the ^{3}He ions, respectively [6]. A loss term corresponding to τ_E=2.1s was adopted for all species and the RF field polarizations estimated with the 1D TOMCAT code [7] were used.

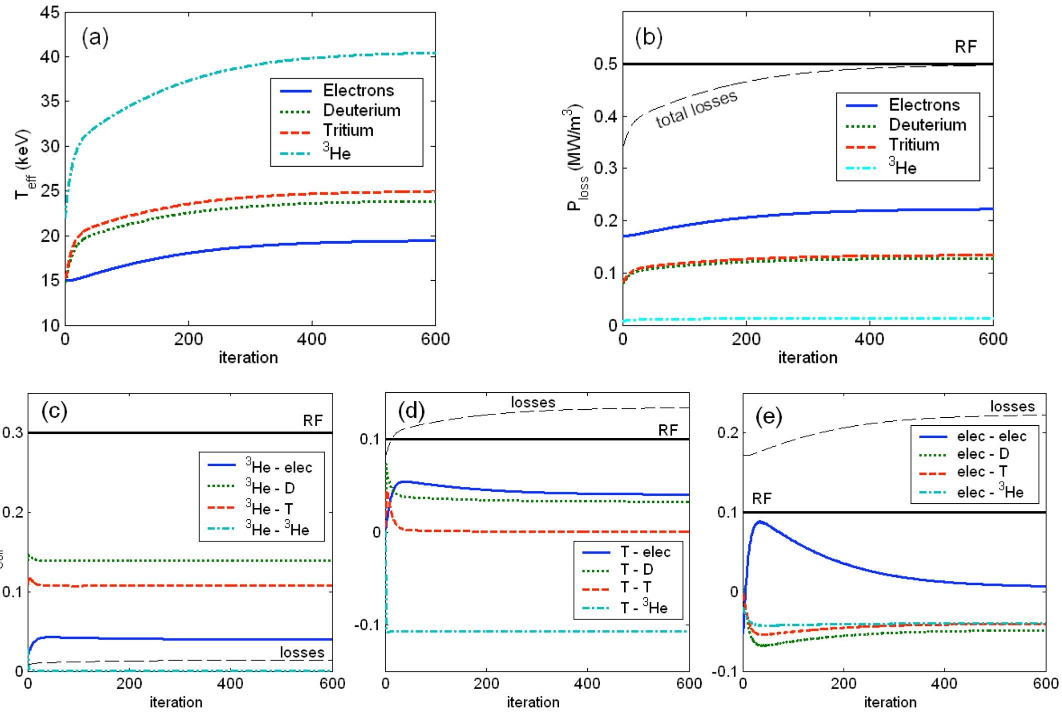

FIGURE 1: *(a) Effective temperatures and (b) power losses for various plasma species in the $\omega=\Omega_{3He}=2\Omega_T$ ICRF heating scheme foreseen for ITER; and evolution of the collisional power redistribution for (c) ^{3}He, (d) Tritium and (e) electrons during the iterative procedure.*

From (a) one sees that, for the considered parameters, the effective temperatures reached by the RF heated ions is not very large: $T_{3He}\approx$40keV and $T_{Trit}\approx$25keV for ^{3}He and T, respectively. Although not directly heated by ICRF, the deuterons achieve similar temperatures as the tritons while the electrons, which have the largest losses due to their higher density (b), reach a somewhat lower temperature, $T_e\approx$20keV. It is interesting to observe the two different time scales in the T_{eff} evolution: The ion dynamics is already 'settled' after 20-30 iterations and afterwards only responds to the slow changes in the background electron distribution. Also note that the total losses summed over species, which evolve in the time-scale of the electron dynamics, equal the RF input power when the simulation converges (b). The RF power redistribution is illustrated in more detail in Figs (c-e). The well absorbing ^{3}He ions (c) transfer most of the RF power to the bulk ions and only a small fraction to the electrons. The less efficiently heated T ions (d) redistribute the RF power absorbed (plus the power received via collisions with the ^{3}He ions) roughly equally to the electrons and to the deuterons but, because of their large concentration, exhibit considerable losses. In both cases the self-collision term drops to zero in a small number of iterations. Although the electrons get power from all species (on top of their direct RF power absorption), the T_{eff} reached is smaller because of their higher losses (despite the same τ_E considered). Their self-collisions evolve very slowly and eventually reach zero at the end of the simulation. To help achieving convergence on the electron distribution faster, several Fokker-Plank iterations on the electrons have been nested into each iteration step of the ions. Also note that, by definition, for each individual species the sum of the collisional power and the losses is equal to the RF input power.

CONCLUDING REMARKS

Adopting the philosophy proposed by Stix [1,2] to compute the 1D distribution function of the ICRF heated populations, a system of coupled Fokker-Planck equations for each of the plasma constituents is solved. Rather than exploring the possible analytical extensions of Stix's model as e.g. proposed by Anderson [8], the here described procedure is based on nested numerical iterations to upgrade the Stix model to describe RF tail formation of both minority and majority populations at any cyclotron harmonic by implementing the Kennel & Engelmann expression [4] for the wave-particle interaction and the Karney expressions [3] for dealing with Coulomb collisions on populations with arbitrary distribution functions. Including a constant energy loss time in the collision operator allows incorporating local transport losses in a crude way. Despite its simplicity, the model presented provides a clear picture of the local collisional power exchange among the plasma constituents in ICRF heated discharges and the effective temperatures inferred are representative of results obtained with more sophisticated (2D) numerical codes [5]. However, it goes without saying that the here presented model has a number of limitations. An in depth study of an RF heating scenario requires a model that enables accounting for the actual machine geometry, the non-uniformity of the plasma and the confining magnetic field, which gives rise to guiding center orbits drifting away from magnetic surfaces, trapped and passing subpopulations, etc. Two examples of simulation codes with a high degree of realism are due to Jaeger et al. [9] (coupling the AORSA wave code to the CQL3D Fokker-Planck code) and Brambilla et al. [10] (coupling the TORIC wave code to the SSFPQL Fokker-Planck code).

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