

Modeling of spatial harmonic transfer functions and its application to the decoupling of the RFX-mod active control system

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Introduction The RFX-mod machine is equipped with an active control system of the MHD instabilities. It consists of 192 active coils, their independent power supply and 192 wide radial field sensors. Active coils and sensors lie on toroidal surfaces and are positioned to form regular grids ($M \times N$, with $M=4$ and $N=48$) in the θ, ϕ coordinates plane. In addition several conducting structures, with many non axial-symmetric features such as portholes, poloidal and toroidal cuts, are placed in the neighborhood of the active coils. Their presence shapes the dynamic relation between the coils currents and the magnetic field.

In particular, the control system is extremely useful in the study of the new helical equilibrium which spontaneously appears in the RFP configuration at plasma currents above 1 MA [1]. In such conditions it may be useful to improve the quality of the radial field spatial spectrum produced by the active control system using a current distribution which reduces the $m=1, n=7$ side-harmonics produced by the system.

The problem of designing such a decoupler in the space of spatial Fourier components has proven to be non-trivial and rich of interesting points to be addressed. Most of them derive from the consideration that the kernel describing the linear relation between two complex signals (input current harmonics and output flux harmonics) is itself complex. In order to address this fact, a systemic description of the active control system has been given, making use of the Unified Signal Theory (UST) developed by Prof. G. Cariolaro at the Padova University [2]. This conceptual framework to perform abstract Fourier analysis is based on the Haar integral, an integral which can be defined over any locally compact topological group [3, 4]. The objective of the work is to study the control system couplings in the Fourier space in order to develop a real-time dynamic decoupler.

Systemic description The system input (currents) and output (fluxes) quantities are described as real signals over the domain $I_\theta \times I_\phi \times \mathbb{R} \rightarrow \mathbb{R}$ (denoted by $u, y: I_\theta \times I_\phi \times \mathbb{R} \rightarrow \mathbb{R}$) where $I_\theta = \mathbb{Z}(1/M)/\mathbb{Z}(1)$ and $I_\phi = \mathbb{Z}(1/N)/\mathbb{Z}(1)$ are quotient groups and $\mathbb{Z}(T)$ denotes the algebraic group consisting of the integral multiples of the real number T . The first two domains are used to describe the poloidal and toroidal spatial dimensions, respectively, while the third is used to represent the time. The relation between input and output is described by the Haar

integral

$$y(i, j, t) = \int_{I_9 \times I_\varphi \times \mathbb{R}} g(i, j, h, k, t - \tau) u(h, k, \tau) dh dk d\tau \quad (1)$$

with $i, h \in I_9$, $j, k \in I_\varphi$ and $t, \tau \in \mathbb{R}$. The kernel function $g: I_9 \times I_\varphi \times I_9 \times I_\varphi \times \mathbb{R} \rightarrow \mathbb{R}$ describes a space-variant time-invariant linear system. The spatial variance of the system arises from its toroidal geometry and from the presence of local features such as the poloidal gaps in the shell and in the support structure.

The Fourier analysis of this system can be performed spatially, temporally or as a whole. Consequently three different operators are applied to variable names: $\hat{\cdot}_s$, $\hat{\cdot}_t$ and $\hat{\cdot}$, respectively. Fourier-transformed quantities are defined over domains which are easily derived by applying the rules of the UST. For example, the signal $\hat{u}(m, n, f)$ is defined over the (dual) domain $\hat{I}_9 \times \hat{I}_\varphi \times \mathbb{R}$ and takes values into \mathbb{C} . Other examples are $\hat{u}_s: \hat{I}_9 \times \hat{I}_\varphi \times \mathbb{R} \rightarrow \mathbb{C}$, $\hat{u}_t: I_9 \times \hat{I}_\varphi \times \mathbb{R} \rightarrow \mathbb{C}$ or $\hat{g}_s: \hat{I}_9 \times \hat{I}_\varphi \times \hat{I}_9 \times \hat{I}_\varphi \times \mathbb{R} \rightarrow \mathbb{C}$. The dual domains \hat{I}_9 and \hat{I}_φ are $\mathbb{Z}(1)/\mathbb{Z}(M)$ and $\mathbb{Z}(1)/\mathbb{Z}(N)$, respectively.

Relevant Symmetries A condition on, or property of, the kernel or the signals above can always be described in the primal (anti-transformed) space or in the dual (transformed) space. This greatly improves the understanding how the system nature. For example, equation (1) has an equivalent in the dual space which consists in a generalization of the well known theorem on the spectral representation of linear time-invariant systems. In the framework of the UST, the derivation is straightforward [5] and gives the following result.

$$\hat{y}(m, n, f) = \int_{\hat{I}_9 \times \hat{I}_\varphi} \hat{g}(m, n, -l, -r, f) \hat{u}(l, r, f) dl dr \quad (2)$$

With the given choice of domains, f represents time-frequency, l and r input poloidal and toroidal harmonic numbers, respectively and m and n output poloidal and toroidal harmonic numbers, respectively. The fact that the output spectrum is a result of a convolution explains the presence of coupling between different harmonics in the dual space, as shown in Fig. 1.

In the following we will consider other properties, or symmetries, which may or may not be possessed by the kernel g and give their interpretation in the dual domain.

The first one is the fact that the kernel is real ($g(i, j, h, k, t) = \overline{g(i, j, h, k, t)}$ when \cdot is used to denote the complex conjugation). It is a fundamental property of

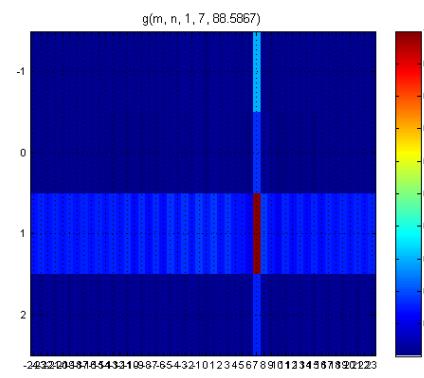


Fig. 1: Normalized spectrum of the $l=1$, $r=7$, $f=88$ Hz harmonic.

the abstract Fourier operator that the transforms of real signals exhibits Hermitian symmetry. Therefore all the Fourier operators defined above show this symmetry, and the transformed input and output signals with them, for example

$$\hat{g}_s(m, n, l, r, t) = \overline{\hat{g}_s(-m, -n, -l, -r, t)} \quad (3)$$

The second property, herein referred to as **strong geometrical symmetry**, is described by the equation $g(i, j, h, k, t) = g(-i, -j, -h, -k, t)$. This kind of symmetry can be possessed by a system only if the underlying physics, geometry and frame of reference allows it. The MHD control system shows that symmetry only if the dynamic response of the sensor (k, h) to an impulse on coil (i, j) is the same as the response of sensor ($-k, -h$) to an impulse on coil ($-i, -j$), for any choice of i, j, h and k . Combining the spatial dual of this equation with (3), turns out that the spatial dual of the kernel also must be real and therefore the following equation must be satisfied.

$$\hat{g}(m, n, l, r, f) = \overline{\hat{g}(m, n, l, r, -f)} = \hat{g}(-m, -n, -l, -r, f) \quad (4)$$

It is legitimate to suspect that this property is rarely possessed by systems which have not been explicitly designed to show it. This condition is relevant because couplings that satisfy this equation can be modeled with a reduced set of states, being the imaginary part of $\hat{g}_s(m, n, l, r, t)$ equal to zero.

The third property, referred to as **weak geometrical symmetry**, exists if there is a translation $(i_0, j_0, h_0, k_0) \in I_\vartheta \times I_\varphi \times I_\vartheta \times I_\varphi$ so that $\tilde{g}(i, j, h, k, t) = g(i - i_0, j - j_0, h - h_0, k - k_0, t)$ shows the strong geometrical symmetry. In this case, a simplified expression of a couplings transfer function exists only if the harmonic numbers of the coupling satisfy the following equation

$$mi_0 + nj_0 - lh_0 - rk_0 = p/2 \quad (5)$$

with $p \in \mathbb{Z}$. For example, the FEM model of the RFX-mod machine used to perform the development of the dynamic decoupler shows this property with the translation $(i_0, j_0, h_0, k_0) = (0, 15/N, 0, 15/N)$, which means that real couplings occurs when the difference of the output and input toroidal harmonic numbers is an integral multiple of 8.

Finally, the RFX-mod system shows the **spatial invariance along the toroidal direction at frequency zero**, because the source of toroidal non-invariance are currents flowing in passive structures with non-invariant

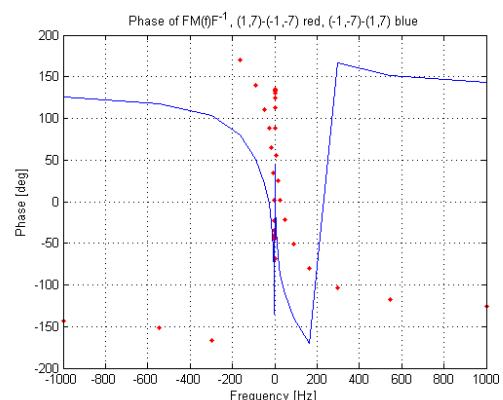


Fig. 2: Frequency dependence of the (1,7)-(-1,-7) and (-1,-7)-(1,7) couplings phase.

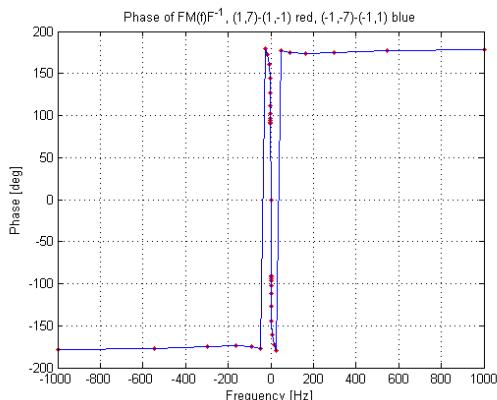


Fig. 3: Frequency dependence of the (1,7)-(1,-1) and (-1,-7)-(-1,1) couplings phase.

representations of the input-output relation. Fig. 2 proves that the RFX-mod machine does not exhibit the strong geometrical symmetry. In fact, the phase of the coupling between the $l=1$, $r=7$ and $m=-1$, $n=-7$ harmonics (denoted by (1,7)-(-1,-7)) differs from that of the (-1,-7)-(1,7) coupling. On the other hand, Fig. 3 proves that the model shows the weak geometrical symmetry and that the predictions of the theory are correct. Actually the (1,7)-(1,-1) and (-1,-7)-(-1,1) couplings are identical. Finally Fig. 1, makes evident the fact that the amplitude of the (1,7)-(1,-1) coupling vanish at low frequency, also as predicted by the abstract Fourier analysis. Apart these numerical validations, this analysis explains the reasons of quite an unexpected fact: that the dynamic relation between the output poloidal side-harmonics of a given input harmonic can be modeled with a real kernel. This is relevant for the design of the decoupler in the dual space because it means that the number of states of the real-time implementation can be halved.

Conclusions The use of abstract Fourier analysis has proven invaluable to improve the understanding of the behavior of the RFX-mod active control system of the MHD instabilities and a powerful tool to connect the primal and the dual descriptions of the system. This has been used to reduce the size of the modal decoupler, easing its real-time implementation.

References

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geometry. In the dual domain this means that at zero frequency the toroidal couplings must vanish.

Results Equation (2) is used to check the nature of couplings evaluating the transfer function matrix of the harmonic model of the system, which has been calculated taking advantage of the peculiar implementation of the Finite Elements Method present in the CARIDDI code [6], which supports state space

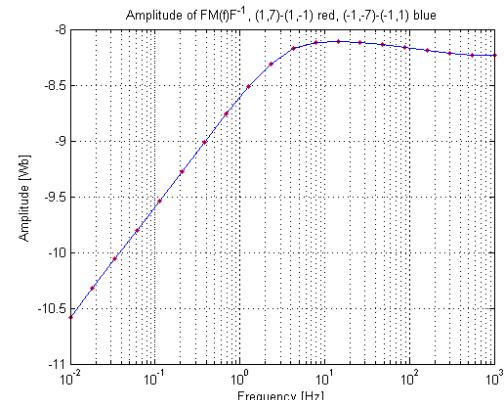


Fig. 4: Frequency dependence of the (1,7)-(1,-1) and (-1,-7)-(-1,1) couplings amplitude (log scale).