The limiting regime of the universal mode in rotating plasma and its new defining parameter: effective Larmor radius

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The effective Larmor radius. This quantity is suggested by the usual drift wave theory. The equations

$$\begin{aligned}\frac{\partial \mathbf{v}}{\partial t}+\nabla\cdot\mathbf{v} &= -\frac{e}{m}\nabla\phi+\Omega_{ci}\mathbf{v}\times\widehat{\mathbf{n}}\\ \frac{\partial n}{\partial t}+\nabla\cdot(n\mathbf{v}) &= 0 \text{ and } n=n_0\exp\left(\frac{e\phi}{T_e}\right)\end{aligned}$$

The result of these three equations is the Ertel's theorem $\frac{d}{dt}\left(\frac{\omega+\Omega_{ci}}{n}\right)=0$ where the vorticity is $\omega\equiv\nabla\times\mathbf{v}$ directed in $2D$ along z. It is assumed that the plasma rotates with the velocity u in poloidal direction. The solution of the general equation involves an arbitrary function F

$$\ln\left(1+\varepsilon_n\nabla_\perp^2\phi\right)=\ln F\left(\phi-ux\right)+\ln n_0+\frac{\varepsilon_n\phi}{T\left(x\right)}$$

where $\frac{\rho_s}{L_{n0}}\equiv\varepsilon_n$, with the profile of the density taken exponential $n\left(x\right)=n_0\exp\left(-\varepsilon_n x\right)$. Choosing the *arbitrary* function as$F\left(\phi-ux\right)\equiv\exp\left[\varepsilon_n\left(x-\frac{\phi}{u}\right)\right]$ then to the lowest order in ε_n we have

$$\nabla_\perp^2\phi=\left(\frac{1}{T\left(x\right)}-\frac{v_{dia0}}{u}\right)\phi$$

The parameter $k_0^2\equiv\frac{1}{T}-\frac{v_{dia}}{u}$ appears currently in the theory of drift waves (**Petviashvili**, **Horton, Spatschek, Nycander, etc**). This suggests to introduce

$$\frac{1}{\left(\rho_s^{eff}\right)^2}\equiv\frac{1}{\rho_s^2}\left(1-\frac{v_{dia}}{u}\right)$$

Effect on the drift waves. When the *effective* Larmor radius goes to infinity $\rho_s^{eff}\rightarrow\infty$ due to the fact that the diamagnetic velocity becomes very close to the rotation velocity, the effect is to re-define the diamagnetic velocity, which becomes an *effective* diamagnetic velocity: $v_*\rightarrow v_*^{eff}=\frac{\rho_s^{eff}c_s}{L_n}$. If we assume that in this regime the transversal wavenumbers $k_\perp$ remains finite, the diamagnetic frequency tends to infinity $\omega_*^{eff}=k_\perp\frac{\rho_s^{eff}c_s}{L_n}\rightarrow\infty$. This means that the basic resonance that leads to excitation of drift waves $\omega-\omega_*\rightarrow\omega-\omega_*^{eff}$ can only be realised for very high frequency of the perturbation, $\omega\rightarrow\infty$, which is incompatible with the

H-mode flow. Then we have to assume that when $v_* \to u$ the perpendicular (here poloidal) wavenumber cannot remain finite and actually must approach 0, and we have: (1) $k_\perp \to 0$ i.e. the poloidal wavelength becomes very large; and (2) $\rho_s^{eff} \to \infty$ i.e. the product $\left(k_\perp \rho_s^{eff}\right) \to 0$ which means $\omega_*^{eff} \to 0$. Then for the drift wave we have: very large poloidal wavelengths (quasi-zonal flow), slow poloidal oscillations. This is compatible with flow and does not use argument of the suppression by finite shear. Physically, radially elongated structures will form and on them secondary perturbations can propagate very rapidly in the radial direction, possibly explaining non-local transport events. The quantity $\rho^{eff} = (1 - v_{*e}/u)^{-1/2} \rho_s$ (which we call *effective Larmor radius*) and the effective sound speed $c_s^{eff} = \rho^{eff} \Omega_i$ are the parameters which characterize this regime, $\rho^{eff} \to \infty$, $c_s^{eff} \to \infty$. The same parameters govern the existence of vortical structures. Emission of drift waves (Cherenkov-type radiation) when u approaches from above v_{*e} makes that the regime identified is priviledged and is an attractor for the system ([1]).

The state with $\rho_s^{eff} \to \infty$ is privileged The Field Theory shows us that a state with suppressed compressibility of the polarization drift is Euler type, conformal invariant, no internal space scale. The density is decoupled from the vorticity. It is known from studies of stationary distribution of vorticity that the system evolves to a profile of rotation with zero vorticity over almost all radial domain with the exception of a small region: either in the centre (for cuasi-singular vortices like typhoons) or ring-type distribution, at the edge, a version of dipole. We take this as a working hypothesis.

We note from the equation

$$\frac{dn}{dt} - \frac{n_0}{B_0 \Omega_{ci}} \frac{d}{dt} \nabla_\perp^2 \phi = 0$$

that the evolution of the vorticity $\nabla_\perp^2 \phi$ imposes a similar evolution of the density. This means that the tendency of the vorticity to collapse on the center and form a singular filament of vorticity will impose a similar behavior for the density $\frac{d}{dt}\omega \equiv \frac{d}{dt}\nabla_\perp^2 \phi$ has an evolution that reflects the pinch of ω toward the center. It results that n will also have a pinch toward the center $\frac{\partial n}{\partial t} = \frac{n_0}{B_0 \Omega_{ci}} \frac{\partial}{\partial t} \nabla_\perp^2 \phi$,

$$\frac{\partial}{\partial t} \frac{1}{n_0} \frac{\partial n}{\partial r} \simeq \frac{\partial}{\partial t} \frac{1}{c_s \rho_s} v_{dia} \simeq \frac{1}{B_0 \Omega_{ci}} \frac{\partial}{\partial r} \frac{\partial}{\partial t} \nabla_\perp^2 \phi$$

taking u slowly changing,

$$-\frac{\partial}{\partial t} \left(1 - \frac{v_{dia}}{u}\right) \simeq \frac{c_s \rho_s}{B_0 \Omega_{ci}} \frac{1}{u} \frac{\partial}{\partial t} \frac{\partial \omega}{\partial r}$$

The accumulation to the center of the vorticity means that $\frac{1}{u} \frac{\partial}{\partial t} \frac{\partial \omega}{\partial r} \to 0$ over almost all plane, with the exception of a small region around the center or at the periphery. Then

$$\frac{\partial}{\partial t}\left(1-\frac{v_{dia}}{u}\right)\to 0$$

which means that the factor $1-v_{dia}/u$ tends to become a constant in time, or : $v_{dia}\to u$.

Again we have invoked the fact that the vorticity evolves such as to vanish over almost all plane. This can be realized only if the density will evolve such that its diamagnetic velocity tends to become equal to the rotation velocity.

The presence of a maximum of the diamagnetic velocity. The H mode is characterized by the existence of a layer of small radial extension ($\sim 1\ cm$ in DIII-D) close to the Last Closed Magnetic Surface, where the plasma is in poloidal rotation. The poloidal rotation has strong shear and the suppression of the drift waves by the sheared velocity leads to the increase of the density. Such a layer has high magnitude of vorticity and induces a concentration of the density (due to the Ertel 's theorem) and of current density (at equilibrium the current density and the vector of the vorticity are aligned and their maxima must coincide). We can have the situation where the diamagnetic velocity has a local maximum $\frac{dv_i^{dia}}{dr}=0$ and a significant value of the curvature, $\frac{d^2v_i^{dia}}{dr^2}\sim\frac{c_s}{\rho_s L_n}$ with positive sign. We consider the state consisting of the increase of the poloidal rotation. Although it ultimately will be substantially higher than the diamagnetic velocity ($\sim 3\times v_i^{dia}$) in this phase the rotation velocity is slightly smaller than the diamagnetic velocity and we have the equation for the perturbation of the potential related to the drift wave or to a cuasi-coherent structure which can be generated on the background of the rotation

$$\Delta\phi=-\overline{k}_0^2\phi+\gamma\phi^3$$

since the term with ϕ^2, is zero due to $d\left(v_i^{dia}\right)/dr=0$. The coefficient in the first term is $\overline{k}_0^2=\frac{v_i^{dia}}{u}-1\geq 0$ and $\gamma\equiv\frac{\rho_s}{L_n}\frac{d^2v_0^{dia}}{dr^2}\frac{1}{6u^3}$. We make the substitution $y\to y'=\left(\frac{\gamma}{2}\right)^{1/2}y$ and introduce a parameter ν to be determined from the algebraic equation

$$(1+\nu)\left(\frac{2m}{1+\sqrt{\nu}}\right)^2\equiv\frac{\overline{k}_0^2}{\gamma/2}$$

where m is a parameter to be explained later. We consider the spatial variation along x fixed at the maximum of the diamagnetic velocity and the equation is re-written as

$$\frac{d^2\phi}{dy'^2}-2\phi^3+(1+\nu)\left(\frac{2m}{1+\sqrt{\nu}}\right)^2\phi=0$$

The solution is

$$\phi\left(y'\right)=\sqrt{\nu}\frac{2m}{1+\sqrt{\nu}}sn\left(\frac{2m}{1+\sqrt{\nu}}y';\nu\right)$$

where *sn* is the Jacobi elliptic function with real elliptic parameter $0 \leq \nu \leq 1$. The period of the elliptic function *sn* is $2\mathbf{K}(\nu)$

$$\mathbf{K}(\nu) \equiv \int_0^{\pi/2} \left(1 - \nu \sin^2(t)\right)^{-1/2} dt$$

Without fixing for the moment the parameter m we note that for $\nu = 1$ the solution is a simple *kink*

$$\phi(y) = m \tanh\left(m\sqrt{\frac{\gamma}{2}} y\right)$$

This is a isolated jump of the perturbed potential and generates a flow which is superimposed on the background rotation u. We note that the flow effectively consists of a localised radial structure and the amplitude of the radial flow together with the spatial width along the y direction measured by m. More interesting are the periodic solutions with $\nu \neq 1$, consisting of a sequence of localised pairs of positive and negative radial impulses of the same type as the previous one. This comes from

$$sn(x;\nu) = \frac{\pi}{2\sqrt{\nu}\mathbf{K}(1-\nu)} \sum_{n=-\infty}^{n=\infty} (-1)^n \tanh\left[\frac{\pi}{2\mathbf{K}(1-\nu)}(x - 2n\mathbf{K})\right]$$

and the region of periodicity is

$$x \in \left[-\frac{\mathbf{K}(\nu)\left(1+\sqrt{\nu}\right)}{2m}, \frac{\mathbf{K}(\nu)\left(1+\sqrt{\nu}\right)}{2m}\right]$$

When the radial positive and negative velocities are combined they create a sequence of periodic filaments distributed equidistant along the poloidal circumference.

References

[1] F. Spineanu, M. Vlad, http://arXiv.org/physics/0909.2583 (2009)