Effect of radiation on one dimensional electromagnetic Solitons in an electron- positron Plasma

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The set of relativistic hydrodynamic equations describing propagation of arbitrary electromagnetic (EM) wave in fully radiative electron-positron plasma in an arbitrary temperature and velocity is derived. The numerical solutions are obtained. The existence of one-dimensional localized solitonlike EM distribution in a radiative e^{-}-e^{+} plasma is represented. The dispersion relation turns out to be dependent on the incoming electromagnetic wave amplitude. The amplitude of the wave shows a saturating type behavior in case of temperature changes. We calculated the range of the background temperatures in which stable solitary waves can exist. Our results should be useful in understanding the nonlinear propagation of intense laser beams in a radiative electron-positron plasma.

I. Introduction

The electron–positron (e^--e^+) plasma is created in the presence of strong electric or magnetic fields (high intensity laser-matter interaction) or at extremely high temperatures. In these conditions, e^--e^+ can be created by temperature radiation. Therefore, this plasma consists of electron-positron and photon gas. The presence of the radiation has a tremendous effect on propagation of the wave in this kind of plasma. In these relativistic regimes, novel nonlinear properties of the laser–plasma interaction come into play [1,2]. 30%-40% of the laser pulse energy can be carried by a relativistic soliton [3,4]. In the present paper, we investigate the propagation of electromagnetic (EM) waves in a plasma with e^--e^+ and photon gas which has an arbitrary temperature. The equations are extended in order to investigate the existence of soliton-like EM distributions in one-dimensional geometry. We aim to find localized stable structures sustained by this plasma.

II. General System of equations and extension to the one-dimensional case

We consider a relativistic two-fluid plasma composed of electron and positron moving under the influence of EM field. The conservation of the particles number can be considered as Ref. [5]. The equation of motion for relativistic ideal fluids with consideration of electromagnetic field is the following

$$\frac{d\vec{p}_s}{dt}=\frac{\partial\vec{p}_s}{\partial t}+(\vec{v}_s.\vec{\nabla})\vec{p}_s=-\frac{\vec{\nabla}P_s}{\gamma_s n_s}+Q_s\left(\vec{E}+\frac{1}{c}\left(\vec{v}_s\times\vec{B}\right)\right)=-\frac{1}{\gamma_s}\vec{\nabla}w_s+\frac{T}{\gamma_s}\vec{\nabla}S_s+Q_s\left(\vec{E}+\frac{1}{c}\left(\vec{v}_s\times\vec{B}\right)\right)\quad s=e,p \qquad (1)$$

where T, W_s, S_s and P_s are temperature, heat function per particle, entropy per particle and pressure for an e^--e^+ plasma and photon gas, respectively. All of them defined in the inertial rest frame, $\gamma_s=(1-v_s^2/c^2)^{-1/2}$ is relativistic factor, $p_s=v_s(\gamma w_s/c^2)$ is the generalized momentum per each fluid particle and v_s is the associated three-component velocity vector. Applying Maxwell equations [6] to Eq.(1) in an adiabatic process gives

$$\frac{\partial\vec{P}_s}{\partial t}=\vec{v}_s\times\vec{\nabla}\times\vec{P}_s-\vec{\nabla}(\gamma_s w_s+Q_s\phi), \qquad (2)$$

where $\vec{P}_s=\vec{p}_s+\frac{Q_s}{c}\vec{A}$ is generalized momentum. Supplementing this equation by the law of generalized vorticity conservation [4], the relativistic factor will be

$$\gamma_s=\left[1+\frac{c^2}{{w_s}^2}{p_{s\parallel}}^2+\frac{{Q_s}^2A_\perp^2}{{w_s}^2}\right]^{\frac{1}{2}} \qquad (3)$$

The field equations for the scalar (ϕ) and the vector ($\vec{A}$) potential can be written

$$\nabla^2A-\frac{1}{c^2}\frac{\partial^2A}{\partial t^2}-\frac{\partial}{\partial t}(\vec{\nabla}\phi)=-\frac{4\pi}{c}J=-\frac{4\pi}{c}\sum_s Q_s n_s\gamma_s\vec{v}_s, \qquad (4)$$

$$\nabla^2\phi=4e\pi(\gamma_e n_e-\gamma_p n_p), \qquad (5)$$

In which $\vec{J}$ is the current density. We consider the propagation of circularly polarized EM wave in one dimension along the x axis with mean frequency ω. In order that localized solution in such geometry exist, we have $A_x=0$. The vector potential can be shown as $A_\perp(x,t)=A_y(x,t)+iA_z(x,t)$. It is now convenient to introduce new variables $\xi=x-Vt$ and $\tau=t$, where V is the group velocity of the EM wave packet; The vector potential can be represented as

$$A_\perp(\xi,\tau)=a(\xi)\exp\left[i\omega\{(1-(V/c)^2)\tau-(V/c)^2\xi\}\right] \tag{6}$$

An equilibrium e^--e^+ plasma (e=p) includes the properties $n_e=n_p$ and $\gamma_e=\gamma_p$. Since the inertia of the two plasma species is equal, there is not any charge imbalance in the system, so $\phi=0$. As a consequence, we obtain from Eqs. (4) and (6)

$$a_{\xi\xi}+(\omega/c)^2a=\frac{8\pi\ e^2N_e}{w_e\gamma_e(1-(V/c)^2)}a, \tag{7}$$

III. Localized Solitons in a radiative e^--e^+ plasma

Our investigation is restricted to immobilized EM patterns; V=0, in an equilibrium e^--e^+ plasma (e=p). Then Eq. (7) can be written as

$$a_{\xi\xi}+\left(\frac{\omega}{c}\right)^2a=\frac{8\pi\ e^2N}{w\gamma}a=\frac{8\pi\ e^2n}{w}a; \qquad \gamma=\left[1-\frac{e^2a^2}{{w_0}^2}\right]^{-1/2} \tag{8}$$

where $w\gamma=w_0$. The specific enthalpy is $w=(w_0^2-e^2a^2)^{1/2}$. Considering the first law of thermodynamics in adiabatic case (s=const.), allows us to write Eq.(8) as

$$a_\xi^2+(\omega/c)^2a^2=-16\pi(P_s(w)-P_s(w_0)), \tag{9}$$

where the pressure P_s (w) is considered as a function of w. Eq. (9) shows that there is localized solution around $\xi=0$ only if the amplitude becomes lower than a certain value a_{max}

$$a < a_{\max}=\frac{1}{e}(w_0^2-w^2(n=0))^{1/2}, \tag{10}$$

The plasma temperature and density tends down to zero ($n\rightarrow 0$ *and* $T\rightarrow 0$) in the region of field localization ($\xi=0$). Appearance of zero temperature is not surprising since the corresponding region is the "plasma vacuum". The trajectory in the phase space can be written in the form

$$F(a)=\left\{-16\pi\left[P((w_0^2-e^2a^2)^{1/2})-P(w_0)\right]-(\frac{\omega}{c})^2a^2\right\}^{1/2}, \tag{11}$$

There are two conditions to have single-humped solutions around $\xi=0$ which exponentially vanish at infinity. One is that, if $F(a_+=a_{max})=0$ be a real quantity, therefore there is an intersection of the trajectory at $a_\xi=0$, then Eq. (11) gives

$$w_0^2-w^2=-\frac{16\pi\ c^2e^2}{\omega^2}\left[P(w_{(n=0)})-P(w_0)\right], \tag{12}$$

In the case of a Maxwellian plasma, the total entropy of the system consisting of e^--e^+ and photon gas is [7]

$$S = -K_B \ln\left[\frac{nx}{K_2(x)}\exp(-xG(x) - \frac{D_0}{nx^3})\right] + const, \tag{13}$$

where $D_0=2\pi^2/45\lambda_0^3$ ($\lambda_0=\hbar/m_0c$ is the Compton wavelength of electron), $G(x) = K_3(x)/K_2(x)$, (K is McDonald function), and $x= m_0c^2/K_BT$. The expression $\frac{x\,n}{K_2(x)}\exp\left\{-\frac{D_0}{nx^3}\right\}$ is connected with the photon gas entropy. The conservation of entropy gives the relation

$$\frac{\overline{T}_0\overline{n}}{\overline{T}K_2(\frac{1}{\overline{T}})}\exp(-G(\frac{1}{\overline{T}})\frac{1}{\overline{T}} - \frac{1}{2}\mu\frac{\overline{T}^3}{n}) = \frac{1}{K_2(\frac{1}{\overline{T}_0})}\exp(-\frac{1}{\overline{T}_0}G(\frac{1}{\overline{T}_0}) - \frac{1}{2}\mu\ \overline{T}_0^{\,3}). \tag{14}$$

The total enthalpy and total pressure can be written as

$$w = m_0c^2G(x) + \frac{4}{6}a_R\frac{T^4}{n}; \qquad P(w_0) = n_0K_BT_0 + \frac{1}{6}a_RT_0^4 \tag{15}$$

where $a_R = \frac{\pi^2}{15}\frac{K_B^4}{(\hbar c)^3}$ is Stefan Boltzmann constant. In the region of field localization the plasma temperature can be decreased considerably and the area of trapping region reduces as temperature increase. Moreover from Eqs. (15) and (12), we have

$$\left[G(\frac{1}{\overline{T}_{00}}) + \mu\ \overline{T}_{00}^4\right]^2 = 1 + 4(\frac{\omega_p}{\omega})^2\left[\overline{T}_{00} + \frac{1}{4}\mu\ \overline{T}_{00}^4\right], \tag{16}$$

where $\mu = \frac{2\pi^2}{45}\frac{(m_0c)^3}{\hbar^3 n_0}$, $\omega_p^2 = \frac{4\pi e^2 n_0}{m_0}$ is the plasma frequency and $\overline{T}_{00} = \left(\frac{K_B T}{m_0c^2}\right) = \frac{1}{x}$ is an equilibrium temperature of plasma to have localized solution ($a=a_{max}$) around $\xi=0$. The second condition to have soliton-like solution is that $a(\xi)$ vanish exponentially to zero at infinity. By linearizing Eq. (7), it becomes

$$a_{\xi\xi} + \overbrace{\left[(\frac{\omega}{c})^2 - 8\pi\ e^2\frac{n(w_0)}{w_0}\right]}^{\Delta\omega^2}a = 0, \tag{17}$$

The condition $\Delta\omega^2<0$ makes the solution zero at infinity, that is

$$\left[G(\frac{1}{\overline{T}_0}) + \mu\overline{T}_0^{\,4}\right] < 2(\frac{\omega_p}{\omega})^2 \quad \Rightarrow \left[G(\frac{1}{\overline{T}_{0E}}) + \mu\overline{T}_{0E}^4\right] < 2(\frac{\omega_p}{\omega})^2. \tag{18}$$

where $\overline{T}_{0E}$ is an equilibrium temperature of plasma which $a(\xi)\to 0$ at infinity. Eqs.(16) and (18) can be solved numerically and plotted in Fig.1.

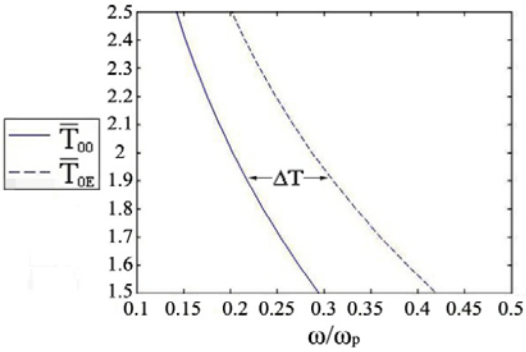

FIG.1.$\overline{T}_{00}$ (solid line) and $\overline{T}_{0E}$(dashed line) are plotted versus the normalized radiation frequency

Where, $\overline{T}_{00}$(solid l ine) a nd $\overline{T}_{0E}$ (dashed l ine) a re p lotted ve rsus the nor malized r adiation frequency. The area between these two lines (ΔT) indicates the region where stable solitons can exist. In addition, Eq.(12) for the behavior of wave amplitude leads to

$$\left[G(\frac{1}{\overline{T}})+\mu\overline{T}^4\right]^2=1-\overline{u}; \qquad \overline{u}=(\frac{e^2u}{c^4m_0^2})=(\frac{e^2}{c^4m_0^2})(a_+^2-a_m^2) \tag{19}$$

where $\bar{u}$ indicates how the amplitude changes with the change of temperature as demonstrated in Fig.2.

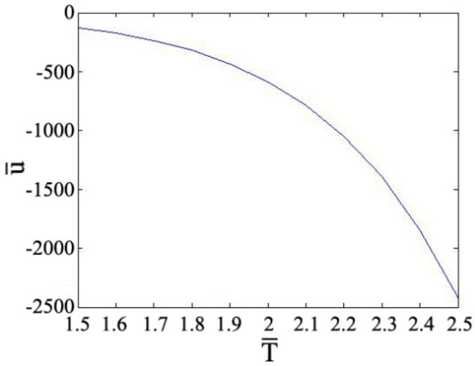

FIG.2. The variation of field amplitude from the a_{max} versus temperature.

IIII. Conclusion

Radiative energy transfer usually dominates over all other mechanisms. It exceeds the thermal conduction very much and at relativistic temperature the characteristic of the simple waves are essentially affected by photon pressure. In this paper, the possibility of EM radiation trapping in the form of solitonlike structures in such radiative plasma consisting of e^--e^+ and the photon gas has been investigated on the basis of relativistic, radiative and hydrodynamic approach, extended to a one-dimensional geometry. It is predictable that solitons can exist in an overdense plasma for $\frac{\omega}{\omega_p} \ll 1$. The existence of stable solitary wave has been exhibited for background temperatures ΔT. The amplitude of nonlinear wave exhibits a saturating behavior with respect to a_{max}.

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