

## Extended cold-source Tonks-Langmuir-type model with non-Boltzmann-distributed electrons

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### Abstract

A general formalism for calculating the potential distribution  $\Phi(z)$  in the quasineutral region of a Tonks-Langmuir (TL)-type model allowing for arbitrary cold ion sources and arbitrary electron distributions is presented. The kinetic concept of “collision/sink/source (CSS)-free electron trajectories (characteristics)” is extensively used. Two types of electron populations are distinguished: the “type-t” ones (populating the “trapped” characteristics, which do not intersect the walls and close on themselves) and the “type-p” ones (populating the “passing” ones, which start out at one of the walls and end at the other). The potential in the plasma region satisfies a “plasma equation” of the form  $\underline{n}^i\{\Phi\} = \underline{n}^e(\Phi)$ , with the electron density  $\underline{n}^e(\Phi)$  given and the ion density  $\underline{n}^i\{\Phi\}$  expressed in terms of trajectory integrals of the ion kinetic equation. While previous TL-type models, including the “classical” TL model [1], were approximated by Maxwellian [2] or bi-Maxwellian [3] electron velocity distribution functions (VDFs), which imply zero CSS terms (“Vlasovian electrons”) and electron currents, we here propose a more general class of electron VDFs allowing for non-zero CSS terms (“non-Vlasovian electrons”) and electron currents inside the plasma region. The sheath-edge and floating-wall potentials are calculated by balancing the ion and electron current densities at the sheath-edge singularities. In a first detailed application, Vlasovian electrons are assumed for which the type-t and type-p VDFs are “inner” and “outer” cut-off Maxwellians, respectively, with different amplitudes and “formal” temperatures. For the special case of equal amplitudes and formal temperatures, the classical Boltzmann electron distribution is formally retrieved. Special cases with other amplitude and formal-temperature ratios show significant deviations from the classical Maxwellian case. This work is a first attempt at introducing electron VDFs different from Maxwellian or bi-Maxwellian VDFs, leading to the conclusion that substantial efforts will be required in the future to arrive at more realistic electron VDFs.

## 1. Model description

We start out from the TL model described in [1]. Instead of the Maxwellian VDF, we propose an ad-hoc model for the electron VDF based on the electron phase space shown in Fig. 1. Two types of electron populations are distinguished: The “type-t” electrons (populating the “trapped” characteristics, which do not intersect the walls and close on themselves) and the “type-p” ones (populating the “passing” ones, which start out at one of the walls and end at the other).

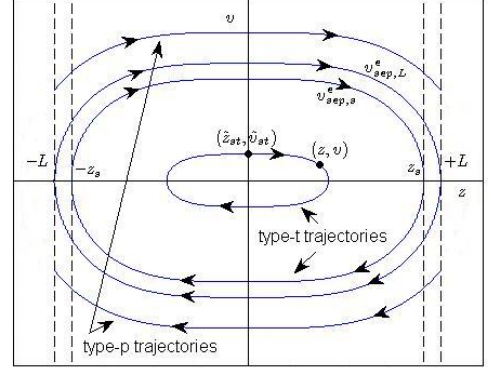


Figure 1:

## 2. Type-t electron VDF

In our ad-hoc model, we propose to approximate the type-t electron VDF by an “inner” cut-off Maxwellian given by

$$f^{et}(z, v) = A^{et} e^{\frac{e\Phi}{k_B T_f^{et}}} e^{-\frac{m^e v^2}{2k_B T_f^{et}}} H\left(\left|v_{sep,L-}^e(z)\right| - |v|\right). \quad (1)$$

where  $T_f^{et}$  is the “formal” temperature (which in general is different from the effective temperature) of the type-t electrons,  $\left|v_{sep,L-}^e(z)\right| := \sqrt{\left[v_{sep,L}^e(z)\right]^2 - \frac{2}{m^e} \delta W}$  (with  $\delta W \rightarrow 0_+$ ) is the maximum speed for the type-t electrons, which is infinitesimally less than the “wall-separatrix speed”  $|v_{sep,L}^e(z)|$ , (cf. Fig. 1), and  $A^{et}$  is the amplitude constant.

## 3. Type-p electron VDF

We furthermore propose the type-p electron VDF to be an “outer” cut-off Maxwellian of the form

$$f^{ep}(z, v) = g_{\sigma_v}(z) f_{vl}^{ep}(z, v), \quad (2)$$

where  $\sigma_v := \text{sign}(v)$ , the factor  $g_{\sigma_v}(z)$  is to be chosen such as to approximately account for the CSS processes involved, and

$$f_{vl}^{ep}(z, v) = A^{ep} e^{\frac{e\Phi}{k_B T_f^{ep}}} e^{-\frac{m^e v^2}{2k_B T_f^{ep}}} H\left(|v| - \left|v_{sep,L+}^e(z)\right|\right), \quad (3)$$

with  $A^{ep}$  and  $T_f^{ep}$  the type-p electron amplitude and formal temperature, respectively, and  $\left|v_{sep,L+}^e(z)\right| := \sqrt{\left[v_{sep,L}^e(z)\right]^2 + \frac{2}{m^e} \delta W}$  the minimum speed for the type-p electrons is, which is infinitesimally greater than the wall-separatrix speed.

The function  $g_{\pm}(z)$  must satisfy the following relations:

$$\begin{aligned} g_+(z) &= g_-(-z), \quad g_+(-L) = g_- (+L) = 0 \\ g_+(-z_s) &= g_- (z_s) = 0, \quad g_+(0) = g_-(0) = 1. \end{aligned} \quad (4)$$

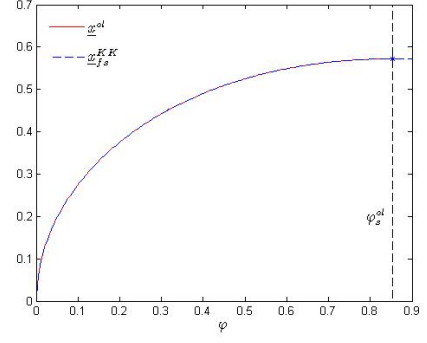
As a simple-first step model, we assume the function  $g_{\sigma_v}(z)$  to be of the form

$$g_+(z) = \begin{cases} 1 & \text{for } 0 \leq z < L \\ 0 & \text{for } z \geq L \end{cases} \quad (5)$$

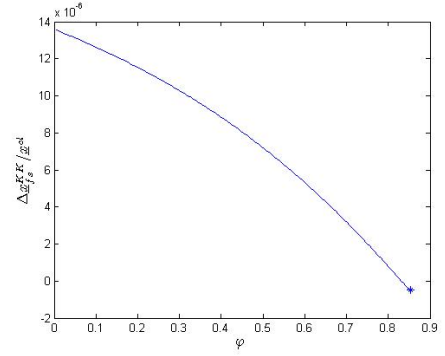
$$g_-(z) = \begin{cases} 1 & \text{for } 0 \leq z < z_s \\ 0 & \text{for } z_s \leq z < L \end{cases}$$

#### 4. Results and discussion

We now compare some of our results (in normalized form) with the corresponding ones based on kinetic solution of the “classical” TL model given in [2]. In Fig. 2(a) we compare (for the special case  $\tilde{A}^{ep} := A^{ep}/A^{et} = 1$  and  $\tilde{T}_f^{ep} := T_f^{ep}/T_f^{et} = 1$ , corresponding to the classical case of a Maxwellian electron VDF) the inverse potential profile  $\underline{x}_{fs}^{KK}$  (where the subscript “fs” indicates that the finite-sum solution scheme given in [4] is used, and the superscript “KK” denotes our model described in Sec. 1, 2 and 3 above) with the corresponding results based on the exact kinetic solution of [2]. The stars indicate the maxima of the curves, which correspond to the respective sheath edges. We see that our results are in excellent agreement with the ones of [2]. In Fig. 2(b) the relative deviation ( $\Delta \underline{x}_{fs}^{KK} / \underline{x}^{cl}$ , with  $\underline{x}_{fs}^{KK} := \underline{x}_{fs}^{KK} - \underline{x}^{cl}$ ) of our finite-sum curve from the classical one is given, and shown to be  $\sim 10^{-6}$ . A comparison of the inverse potential profiles for different

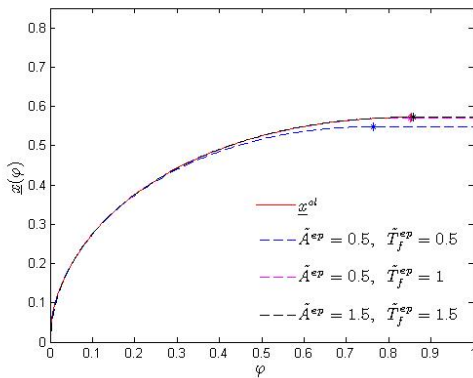


(a)

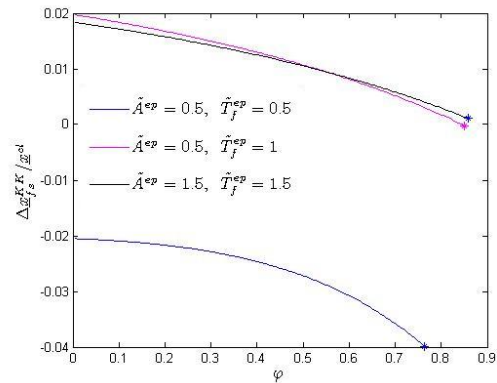


(b)

Figure 2:



(a)



(b)

Figure 3:

values of  $\tilde{A}^{ep}$  and  $\tilde{T}_f^{ep}$  and their relative deviations from the classical curve is given in Figs. 3(a) and 3(b), respectively, showing significant deviations from the classical case. Figure 4 shows the total electron VDFs for different values of  $\tilde{A}^{ep}$  and  $\tilde{T}_f^{ep}$ . For (a)  $(\tilde{A}^{ep}, \tilde{T}_f^{ep}) = (1, 1)$  we obtain a full Maxwellian VDF. For (b)  $(\tilde{A}^{ep}, \tilde{T}_f^{ep}) = (0.5, 0.5)$  and (c)  $(\tilde{A}^{ep}, \tilde{T}_f^{ep}) = (0.5, 1)$

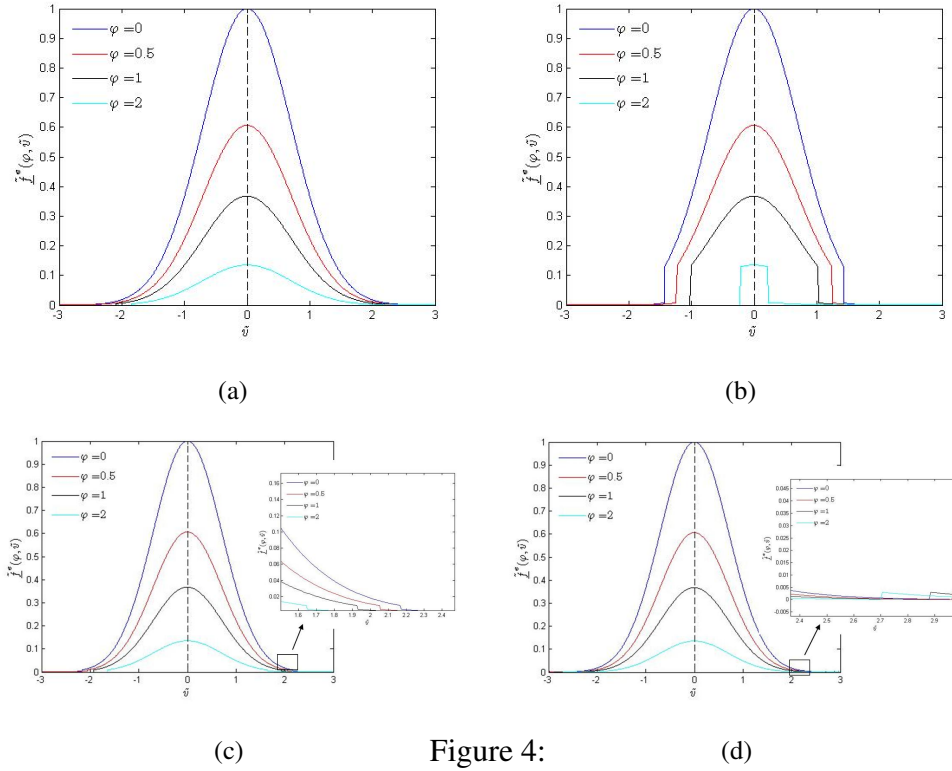


Figure 4:

we obtain cut-off Maxwellian VDFs, because for (b)  $(\tilde{A}^{ep} < 1) \wedge (\tilde{T}_f^{ep} < 1)$ , and for (c)  $\tilde{A}^{ep} < 1$ . For (d)  $(\tilde{A}^{ep}, \tilde{T}_f^{ep}) = (1.5, 1.5)$  we obtain a “peaked” Maxwellian VDF because  $(\tilde{A}^{ep} > 1) \wedge (\tilde{T}_f^{ep} > 1)$ .

More details and results of our KK model will be given in [5].

**Acknowledgments.** This work has been supported by the Higher Education Commission of Pakistan, the Austrian Science Fund (FWF) under project No. P22345-N16, the Georgian National Foundation under project grant 1-4/16 (GNSF/ST09\_305\_4-140), and the European Communities under the Contract of Association between EURATOM and the Austrian Academy of Sciences. It was carried out within the framework of the European Fusion Development Agreement. The views and opinions expressed herein do not necessarily reflect those of the European Commission. Valuable discussions with Dr. V. A. Godyak and Dr. K.-U. Riemann are gratefully acknowledged.

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