Constraints on the turbulent relaxation of braided magnetic fields.

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Taylor relaxation

The hypothesis of Taylor relaxation [1] was initially developed to explain relaxation in the Reversed Field Pinch (RFP). Here the assumption is that in a turbulent resistive evolution all ideal invariants excepting the magnetic helicity are destroyed. The system relaxes towards the minimum energy state compatible with the boundary conditions and containing the same total magnetic helicity. The result, as shown via a variational problem, is a linear force-free field, i.e. one satisfying $\nabla \times \vec{B} = \alpha \vec{B}$ where α is spatially uniform (by contrast in a non-linear force-free field α is constant only along field lines but may vary between field lines). It has been demonstrated experimentally (e.g. [2]) that Taylor relaxation satisfactorily describes the final state in the RFP (except near the walls of the device) but fails in certain other cases.

Relaxation in the solar corona.

The solar corona is another domain in which relaxation is of great importance. Here the geometry is rather different since, in coronal loops for example, both ends of the flux tube are effectively anchored at the photosphere. Nevertheless, generalizations to the Taylor hypothesis have been made showing that relaxation to a linear force-free state may be viable in such domains [3]. Here relative (rather than total) helicity is conserved [4]. Taylor's hypothesis has since been widely applied to local domains in the solar coronal case (e.g. [5, 6, 7]), not least because of the simplicity of the model (although the corona cannot globally be linearly force-free). Indeed some numerical simulations in simple geometries (e.g. [8, 9]) do follow a Taylor relaxation. However, others with slightly greater complexity (e.g. [10, 11]) as well as observations [12] have shown significant differences, suggesting the Taylor hypothesis does not provide a complete description of coronal relaxation. In the next section we present a simulation of the turbulent relaxation of a model braided coronal loop to test this and guide further theory development.

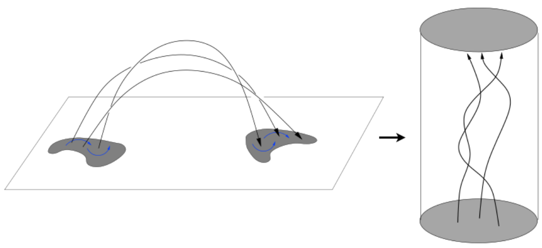

Figure 1: For modelling purposes a coronal loop is straightened out to lie between parallel plates.

Example: relaxation of a braided flux tube.

Here we consider resistive MHD simulations into the turbulent relaxation of a field designed to model a solar coronal loop. In our idealised model the loop is straightened out to lie between two parallel plates (both representing the photosphere), as shown schematically in Figure 1. The magnetic field may be built up via a sequence of both left- and right-handed rotational footpoint motions acting on an initially uniform vertical field. The result, illustrated in Figure 2, is a field with zero net relative helicity but containing braided magnetic flux, with the mapping of field lines from lower to upper boundary being complex. This is illustrated by the lower panel of Figure 2 where field lines are traced from the lower boundary at $\mathbf{x}_0 = (x_0, y_0)$ to the upper boundary $(f_x(\mathbf{x}_0), f_y(\mathbf{x}_0))$ and coloured red if $f_x > x_0 \wedge f_y > y_0$, yellow if $f_x < x_0 \wedge f_y > y_0$, green if $f_x < x_0 \wedge f_y < y_0$ and blue if $f_x > x_0 \wedge f_y < y_0$. Additionally, the field is constructed in such a way [13] as to be close to a (nonlinear) force-free field, since the corona is considered a largely force-free environment.

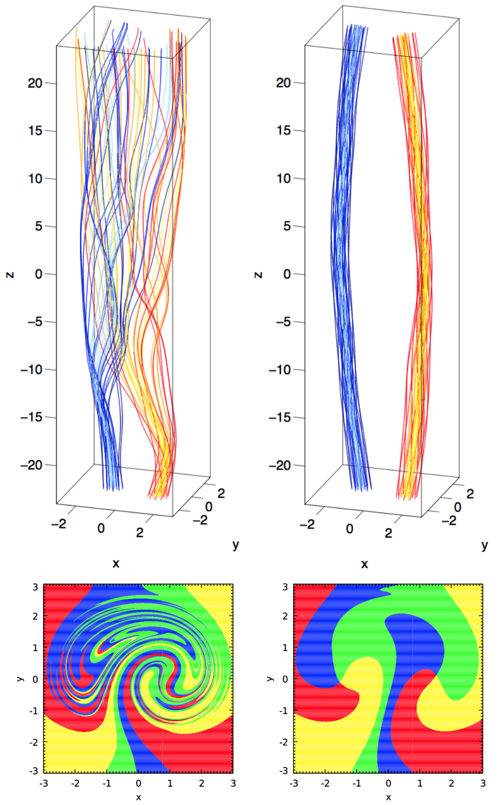

Figure 2: Upper panel – Illustrative field lines traced from fixed locations on the lower boundary in the initial (left) and final (right) states. Lower panel – Connectivity map of field lines as they intersect the lower boundary, with colouring as described in the main text.

The field is taken as an initial condition in a resistive MHD simulation, using the Copenhagen Stagger Code. A spatially uniform resistivity (of $\eta = 10^{-3}$) is taken with 512^3 nodes over the domain $x, y \in [-6, 6]$ and $z \in [-24, 24]$. Field lines are line-tied and the plasma velocity fixed as $\vec{v} = \vec{0}$ on the $z = \pm 24$ boundaries, with periodic boundary conditions on the x, y boundaries. Time is such that an Alfvén wave travels around one spatial unit in one unit of time.

The system is initially approximately stationary but at around $t = 8$ an instability occurs which results in the formation of two thin current layers. These layers fragment into a myriad of weaker current layers which have a volume filling effect, increasing in complexity until $t \approx 80$, after which their number begins to decrease. Finally the system becomes approximately stationary with two large-scale current systems running through the domain. This evolution

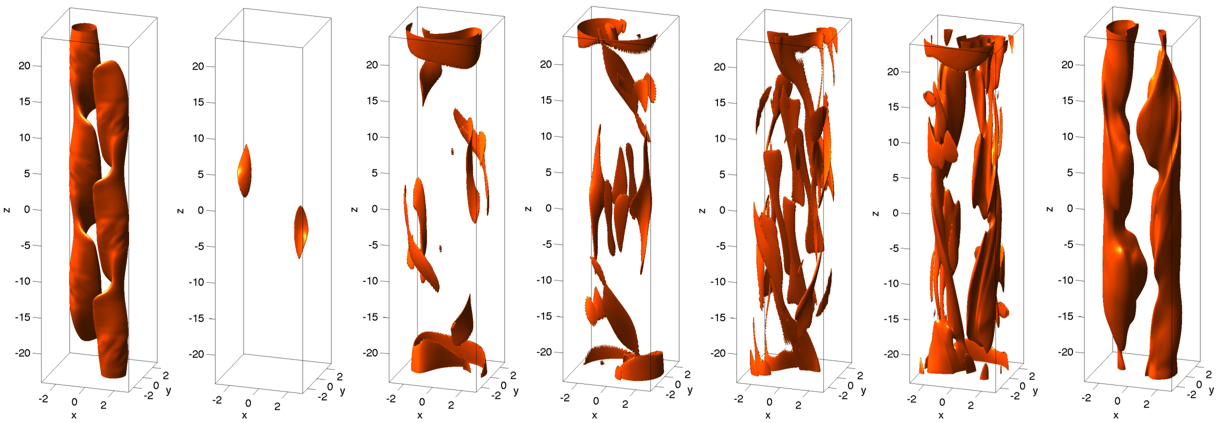

Figure 3: Isosurfaces of current at 25% of the domain maximum in the initial state (far left) and at 50% of the domain maximum at $t = 15, 27.5, 35, 50, 140, 290$ (remaining images).

is illustrated in Figure 3. We refer to $t = 290$ as the final state of relaxation, since only slow diffusive changes to the system occur after this time.

Illustrative field lines in the final state are shown in the right-hand panel of Figure 2. The field has separated, via multiple magnetic reconnection events, into two distinct flux tubes (corresponding to the two current fingers) of opposite helicity signs embedded in a non-linear force free field (for further details on relaxation see [14, 15, 16]). This finding contradicts the state predicted by a simple Taylor relaxation where the homogeneous field would be attained. One consequence is that the energy released in the relaxation is only around 60% of that predicted by a Taylor relaxation. The total (relative) helicity is well-conserved during the relaxation and with reconnection taking place throughout the volume the separation is not due to a lack of fully developed turbulence [9] or non-interacting flux systems [4], the previous best suggestions for non-constant α profiles in the solar case.

Additional constraint: Topological Degree

We have identified an additional constraint which can act to prevent a Taylor relaxation in any magnetic configuration and does so in this case. This constraint is the topological degree (T), which is a topological invariant in any flux tube with a non-zero field and is a property of the fixed points of the field line mapping from one boundary of the flux tube to the other. Each generic fixed point of this mapping has of Poincaré index ± 1 (corresponding to hyperbolic and elliptic orbits) and the fixed points can only be created or destroyed in pairs of opposite index. The topological degree is the total index of all fixed points inside the domain and is conserved in any (ideal or resistive) continuous evolution of the magnetic field [17]. Accordingly it is only in cases where the topological degree of the Taylor state is identical to that of the initial flux tube that a Taylor relaxation can potentially occur.

In the lower panel of Figure 2 these fixed points lie where all four colours intersect. In the initial state we have $T = 2$ (with 12 elliptic periodic orbits and 10 hyperbolic periodic orbits) and so the final state of relaxation must also have $T = 2$. This is confirmed by the right-hand lower panel of Figure 2 where the two fixed points seen correspond to two elliptic periodic orbits at the center of each of the two linear force-free flux tubes. However, the requirement $T = 2$ is incompatible with the homogeneous field, explaining why the Taylor state cannot be reached in this case.

Conclusions

Turbulent relaxation is a key process in both laboratory and astrophysical plasmas. We have presented a simulation, initially designed to model a coronal loop, in which the end state of relaxation differs greatly from that predicted by the Taylor hypothesis. This finding may be explained by noting that the topological degree of a flux tube acts as a constraint above total helicity conservation, a notion which may be extended also to a toroidal domain. Hence for any given magnetic field configuration the Taylor state can only possibly be reached when its topological degree is the same as that of the initial magnetic field itself. This limits the energy that can be released in relaxation which is of interest for the coronal heating problem.

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