Non-nested magnetic surfaces without toroidal-current reversal

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Tokamak operation with extreme magnetic-shear reversal allowed the possibility of equilibrium configurations with nearly zero toroidal current density flowing in the plasma core [1, 2], within a region usually termed a current hole and which is induced by strong off-axis current drive [3]. When dealing with such scenarios, it is commonly assumed that the toroidal current density inside the current hole can be lowered to values arbitrarily close to zero. Only after it reverses its sign, would some kind of current-clamp mechanism develop, possibly linked with the growth of axisymmetric modes and the resulting break of the nested disposition of magnetic surfaces [4]. However, it turns out that such breaking of the nested surfaces occurs even with positive toroidal current density flowing still throughout the plasma core. The transition limit is shown to be established by boundary conditions and other plasma parameters describing the magnetic configuration. Some specific examples of the predicted nested-surface breaking are also presented, using simple analytical solutions of the Grad-Shafranov (GS) equation.

The toroidal current density flowing at some magnetic axis z_0, where $\nabla\psi(z_0) = 0$, can be found using the axisymmetry condition and two relations of the ideal MHD equation set,

$$\nabla\cdot\mathbf{B} = 0 \quad \text{and} \quad \nabla\times\mathbf{B} = \mu_0\mathbf{J}, \tag{1}$$

yielding the full contraction

$$\mu_0 J_\phi(z_0) = T_\phi{}^{\mu\nu}(z_0)\,\partial^2_{\mu\nu}\psi(z_0), \tag{2}$$

with $T_\phi{}^{\mu\nu} = g_{\phi\alpha}\varepsilon^{\alpha\mu\beta}g_{\beta\gamma}\varepsilon^{\gamma\phi\nu}$, $\varepsilon^{\alpha\beta\gamma}$ the Levi-Civita symbol, and ψ the poloidal-field flux. Here, A_α, A^α, and $A_{(\alpha)}$ denote, respectively, the covariant, contravariant, and physical component of some vector $\mathbf{A}$, repeated Greek indices stand for the usual implicit summation over two coordinates in the poloidal plane, while ϕ designates the toroidal angle. Being real and symmetric, the matrix $\partial^2_{\mu\nu}\psi$ has two real eigenvalues, κ_1 and κ_2, and, after switching to a coordinate set where the contravariant basis matches two of its orthogonal eigenvectors, the condition (2) simplifies itself to the line

$$\kappa_1 + \kappa_2 = \mu_0 J_\phi(z_0). \tag{3}$$

It must be kept in mind that (3) is a local condition, valid at z_0 only, with the allowed eigenvalues being, in turn, determined by the set of parameters which define the magnetic configuration (boundary conditions, profile parametrization, etc.).

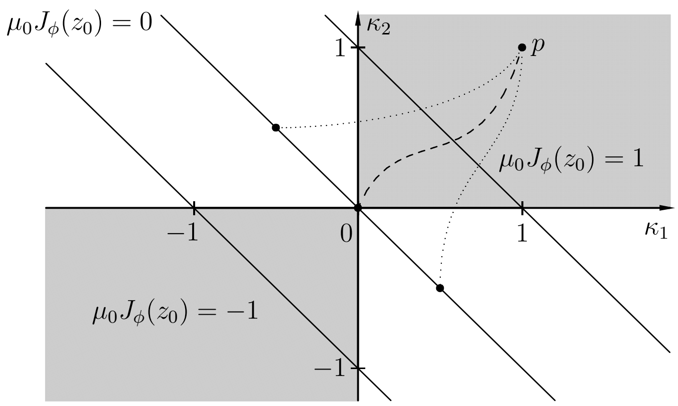

Figure 1: Solutions of (3) in κ^2 space for three different values of $\mu_0 J_\phi(z_0)$ (solid lines) and three possible routes (dashed and dotted lines) to take a configuration with positive curvature and current density into the line $\mu_0 J_\phi(z_0) = 0$.

On the other hand, the matrix $\partial^2_{\mu\nu}\psi(z_0)$ contains also all the information about how magnetic surfaces are arranged around z_0: if the curvature

$$K = \det \partial^2_{\mu\nu}\psi(z_0) = \kappa_1\kappa_2 \tag{4}$$

is positive, z_0 is elliptic with nested surfaces around it; otherwise, it is hyperbolic. Therefore, the graphical device illustrated in Figure 1 suffices to convince one that, in general, it is not possible to make the toroidal current density vanish (or reverse its sign) at some elliptic axis z_0 without turning it first into an hyperbolic one, with no nested magnetic surfaces within its close vicinity. Indeed, a configuration $p \in \kappa^2$ with both $K(p)$ and $\mu_0 J_\phi(z_0)$ positive can only be taken continuously into the line $\mu_0 J_\phi(z_0) = 0$, while keeping z_0 elliptic, if the path connecting them never leaves the shaded area ($K > 0$) and forcibly crosses the origin (dashed line). Any other path not crossing the origin (dotted lines) must step first into the light area ($K < 0$) and thus turn z_0 hyperbolic. Needless to say, requiring $\kappa_1 = \kappa_2 = 0$ simultaneously imposes a condition over the configuration's parameter set and at least one of these parameters would then be locked to some very particular value, depriving one from the ability to describe arbitrary configurations.

The result above has been derived exclusively from the axisymmetry condition and the two equations in (1), being thus not limited only to GS equilibria which follow aditional conditions (force balance, etc.). Still, a simple but conveying example is given by the analytical equilibrium

$$\frac{1}{c_1 R_0^4}\psi(R,Z) = \frac{1}{8}\left(1-c_2\right)\left[\left(\frac{R}{R_0}\right)^2 - 1\right]^2 - \frac{1}{2}\left[1 - \frac{\mu_0 J_\phi^{(0)}}{c_1 R_0^2} - c_2\left(\frac{R}{R_0}\right)^2\right]\left(\frac{Z}{R_0}\right)^2, \tag{5}$$

which solves the GS equation for a Solovev-type current-density distribution

$$\mu_0 J_\phi(R) = \mu_0 J_\phi^{(0)} + c_1\left(R^2 - R_0^2\right), \tag{6}$$

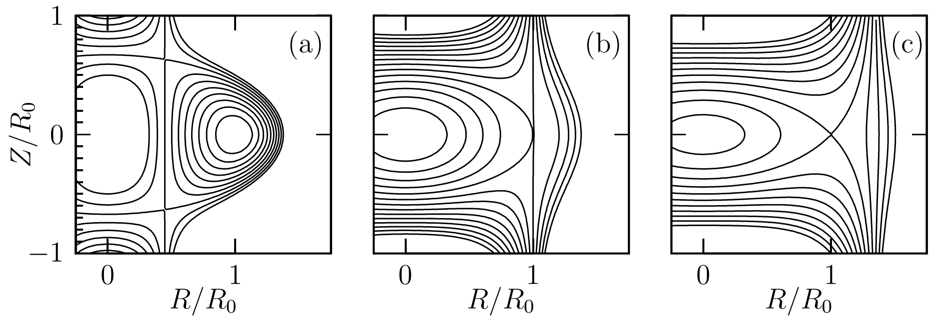

Figure 2: Magnetic surfaces of $c_1^{-1}R_0^{-4}\psi(R,Z)$ from (5) for $c_2 = 0.5$ and $\mu_0 J_\phi^{(0)} = \frac{9}{5}\kappa_1$ (a), κ_1 (b), and $\frac{2}{10}\kappa_1$ (c), with $\kappa_1 = \frac{1}{2}c_1 R_0^2$ positive.

with $J_\phi^{(0)} = J_\phi(z_0)$ and $c_1 = -\mu_0 dP/d\psi$ being the constant flux derivative of the plasma pressure [5]. If the parameter c_2 is chosen in order to make the eigenvalue $\kappa_1 = c_1 R_0^2(1-c_2)$ positive, the condition for an elliptic point at the axis reduces to $\mu_0 J_\phi^{(0)} > \kappa_1$ and the changes in topology induced by decreasing $\mu_0 J_\phi^{(0)} > 0$, at constant c_1 and c_2, are displayed in Figure 2.

Although it succeeds in capturing the essential features of topology transition at positive current density, the solution in (5) is too simple to allow, for instance, any closed magnetic surface which could prevent the plasma from being directed towards the wall when the axis turns hyperbolic. This goal can be accomplished using a more elaborate current-density model,

$$\mu_0 J_\phi(R,\psi) = \mu_0 J_\phi^{(0)}\left(\frac{R}{R_0}\right)^2 - \left(\frac{\sigma}{R_0}\right)^2\left[1-\left(\frac{R}{R_0}\right)^2\right] + \left(\frac{\gamma}{R_0}\right)^2\psi, \tag{7}$$

along with the boundary condition $\psi(R_0,0) = 0$, which results in the equilibrium solution

$$\begin{aligned}\psi(R,Z) = \mu_0 J_\phi^{(0)}\left(\frac{R}{\gamma}\right)^2 + \left(\frac{\sigma}{\gamma}\right)^2\left[1-\left(\frac{R}{R_0}\right)^2\right] + c_1\frac{R}{R_0}J_1\left(\frac{\gamma R}{R_0}\right) + c_2\frac{R}{R_0}Y_1\left(\frac{\gamma R}{R_0}\right) + \\ c_3\sin\left(\frac{\gamma}{R_0}\sqrt{R^2+Z^2}\right) + c_4\cos\left(\frac{\gamma}{R_0}\sqrt{R^2+Z^2}\right) + \left[c_5 + c_6\left(\frac{R}{R_0}\right)^2\right]\cos\left(\gamma\frac{Z}{R_0}\right),\end{aligned} \tag{8}$$

where $J_n(x)$ and $Y_n(x)$ are Bessel functions of the first and second kind [6]. Keeping all other parameters $(c_1,\ldots,c_6,\gamma,\sigma,R_0)$ fixed, decreasing the on-axis toroidal current density below its critical value (at which the on-axis curvature vanishes) is seen to turn the initial elliptic axis into an hyperbolic one (Figure 3).

In summary, it was found that, in general, it is not possible to continuously decrease the toroidal current density flowing in the magnetic axis towards zero without first breaking the nested arrangement of the magnetic surfaces. This result highlights the need to handle non-nested magnetic configurations when modelling scenarios with extreme magnetic-shear reversal, even in those cases for which the current density is not assumed to reverse its sign.

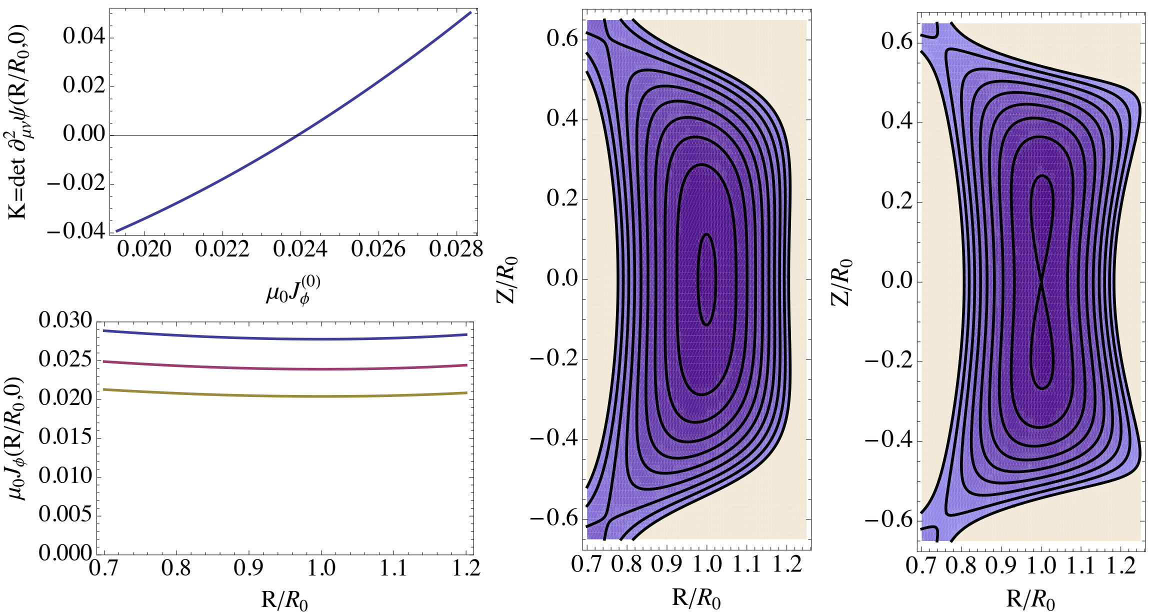

Figure 3: On-axis curvature versus on-axis current density (left panel, top), current-density along the midplane (left panel, bottom) for $\mu_0 J_\phi^{(0)} = 0.028$ (blue line, $K > 0$), 0.021 (brown line, $K < 0$), and the critical value 0.024 (purple line, $K = 0$), and magnetic surfaces corresponding to the positive and negative curvature cases (central and right panel, respectively). Equilibrium parameters are $\gamma = \sigma = 1$ and $R_0 = 6.5$m, while $c_1, \ldots, c_6$ are taken from the boundary shape.

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