

# Isotope Separation with Variable-Frequency Lasers

L. D. Amorim, J. Vieira, F. Peano, L.O. Silva

*GoLP/Instituto de Plasmas e Fusão Nuclear - Laboratório Associado, Instituto Superior  
Técnico, Lisbon, Portugal*

## Abstract

Isotope separation is critical to obtain enriched isotopes samples required for many areas of science and industry. We explore a novel all-optical isotope separation method based on the ponderomotive force produced by two variable frequency, counter-propagating, laser beams. We determined analytically the laser requirements for isotope separation as a function of the different masses and charges of the isotopes for a given desired yield of purified isotopes. Our work suggests that isotope separation with variable frequency lasers provides a robust scheme to obtain highly purified isotope samples.

## INTRODUCTION

The increase of the relative abundance of isotopes in a sample i.e. isotope enrichment has been a world-wide concern for the last 80 years. The enriched isotopes are fundamental, for nuclear fusion and nuclear power plants [3], and are also required for high energy physics experiments using heavy ion beams. They are used for a high number of medical procedures and therapies, which rely on enriched radioisotopes [2].

There are different techniques to separate isotopes [4]. The most common methods include gaseous and diffusive mechanisms, along with electromagnetic cyclotrons. These traditional methods are inefficient and to obtain a final product with high purity and the separation has to occur through a multi-stage system. Complex and large experimental facilities are thus required. Therefore it is vital to find a novel isotope separation schemes with potential to simplify the separation process. Here, we explore a new separation method based on a recently developed all-optical heavy particle acceleration process, [1].

## ACCELERATION PROCESS - CONFIGURATION AND PROPERTIES

Reference [1] describes an ion acceleration method that consists of two counter-propagating laser beams with constant amplitudes and with variable frequencies that propagate in one dimension (x). The interaction of the two beams results in a ponderomotive beat wave with phase velocity and acceleration determined by the change in frequency of the lasers, the chirps. The acceleration of ions is achieved by tuning those chirps and it is given by:

$$\frac{d\hat{p}_x}{dt} = -\frac{\hat{A}_1 \hat{A}_2}{\gamma} \hat{k} \sin(\Phi_1 - \Phi_2) \quad (1)$$

Where  $\hat{p}_x = \frac{p_x}{Mc}$ ,  $\hat{A}_i = \frac{q}{Mc^2} A_i$ ,  $\hat{x} = k_0 x$  and  $\hat{t} = \frac{t}{k_0 c}$ , and where  $q$  and  $M$  are the charge and mass of the accelerated particle.  $p_x$  its longitudinal momentum and  $\gamma$  the Lorentz factor. Furthermore,  $A_i$  is the amplitude of laser  $i = 1, 2$ ;  $\hat{k}$  is the wavenumber of the beat wave and  $c$  is the light speed in vacuum;  $\Phi_i$  is the phase of laser 1 or 2. The argument in the sine is  $\Phi_1 - \Phi_2 = \phi_0 + 2(\hat{x} - \beta_0 \hat{t} - \sigma_+ \hat{x} \hat{t}) + \sigma_- (\hat{x}^2 + \hat{t}^2)$ . Where the initial phase of the beat wave is  $\phi_0$  and  $\beta_0$  is the initial velocity normalized to  $c$ . In addition,  $\sigma_- = \sigma_1 - \sigma_1$  and  $\sigma_+ = \sigma_1 + \sigma_1$ ,  $\sigma_1$  and  $\sigma_2$  referring to the chirp of each laser. The ion's acceleration is proportional to  $\hat{A}_1 \hat{A}_2 \sim \left(\frac{q}{M}\right)^2$ . This scaling is very important. In traditional EM separation schemes the forces acting on the isotopes scale with  $\frac{q}{M}$ . Thus, the method described in this paper has higher sensitivity for subtle  $\frac{q}{M}$  changes than in conventional techniques.

The acceleration of the beat wave,  $\alpha_{\phi 0}$ , varies linearly with  $|\sigma_-|$ . When the beat wave acceleration is lower than the acceleration associated with the ponderomotive force of the particle, the particle is considered trapped in the beat wave structure. A trapped particle will travel with the beat wave and hence it will be accelerated. An un-trapped particle loses the beat wave and will not accelerate. The trapping efficiency,  $\eta$ , studied in [1], varies with  $\sqrt{\frac{q}{M}}$ .

The acceleration combined with the trapping sensitivities to different ratios enable the isotope separation to occur.

## SEPARATION METHOD

To illustrate the ion separation scheme consider a mixture of two species, A and B, to be enriched on isotope B (that is lighter) and with initial concentrations of  $X_A, X_B$ . The species B will have higher trapping efficiency and higher acceleration so a higher portion of type B will get separated and to the final sample, rather than of type A.

The purity of the product obtained may be calculated through the expression  $P = \frac{\eta_B}{\eta_B X_B + \eta_A X_A}$ . The purity is therefore determined by the initial concentrations of the species and by the trapping efficiencies (which are a result of the chirps).

One advantage of this method is that it provides the ability to achieve a completely pure sample (containing only the required isotope) as the product of the separation. This occurs when the tuning and chirping of the lasers allows for no trapping of A type isotopes. And corresponds to the condition for the beat wave acceleration:  $\alpha_0 = \hat{A}_1 \hat{A}_2$  with  $\hat{A}_i = \frac{Z_A}{A} \frac{q}{M_p c^2} A$ , being  $Z, A$  the atomic and mass number of isotope of type A. When the final sample is not completely pure more than one stage would then be required. The number of stages required are thus related to

the conditions of the initial sample (feed) and the lasers properties.

Assuming that the chirps for no trapping of species A are used. A perfectly pure sample can always be obtained in a one stage process. The separation efficiency,  $\eta_B$ , is the percentage of the original amount of isotope B that gets to the resulting sample and can be written as:

$$\eta_{\text{sep}} = 1 - \frac{2}{\pi} \arcsin \left[ \left( \frac{q_A M_B}{q_B M_A} \right)^2 \right] \quad (2)$$

As can be seen in Fig. 1 this efficiency is highly sensitive to the ratio between the masses of the two isotopes. In fact, for a 0.99% ratio the efficiency is of about 14%. Hence this scheme may be used to obtain highly purified isotope samples.

The separation factor is the ratio between the concentration of isotope type B in the product and wasted, considering  $P = 1$  we get the expression for this factor:  $S = 1 + \frac{X_A}{X_B \eta_B (1 - \eta_B)}$ . Different separation factors can be achieved based on the initial feed conditions and the laser chirps. An enrichment method is considered to have higher efficiency when the separation factor is higher (less waste). In our method the  $S$  factor goes up to infinity if the trapping efficiency of type B reaches the value 1 which implies no separation and minimal purity. Therefore, a trade-off must be determined between the desired purity and  $S$  factor.

## LASER PULSE REQUIREMENTS

We can determine the minimum time for isotope separation to occur by estimating the maximum time required for an un-trapped isotopes to loose the ponderomotive beat wave. Defining the phase difference between the particle and the beat wave,  $\psi = 2(\hat{x} - \hat{x}_{\phi 0})$  it is possible to show the isotope separation time is [5]:

$$\tau \approx \sqrt{\frac{\Delta\psi}{\hat{A}_1 \hat{A}_2 - \alpha_{\phi 0}}} \quad (3)$$

where  $\Delta\psi$  is the trapping region and so the length the isotope must go through to loose the beat wave. The minimum pulse duration needed is also given by  $\tau$ . From this consideration we can retrieve the required laser parameters.

The laser pulse duration is thus given by:

$$\Delta\tau[\text{ps}] \approx 0.286 \sqrt{\frac{\eta}{1 - \hat{\sigma}_-}} A Z^{-1} (I[10^{20} \text{W/cm}^2])^{-1/2} \quad (4)$$

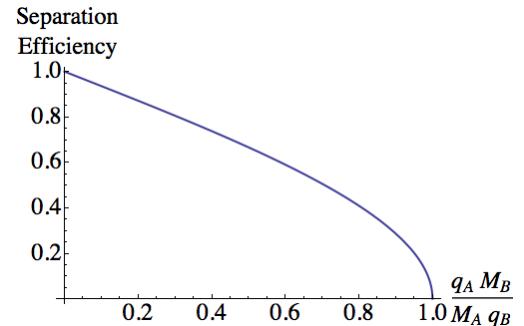


Figure 1: Efficiency of separation, as a function of the mass ratios.

To achieve better results, the laser intensities should be constant all through the separation process. Therefore, the Rayleigh length and beam waist should be of the order of:

$$Z_R[\mu\text{m}] \approx 85.8 \sqrt{\frac{\eta}{1 - \hat{\sigma}_-}} \frac{A}{Z} (I[10^{20}\text{W/cm}^2])^{-1/2} \quad (5)$$

$$W_0[\mu\text{m}] \approx 5.22 \left( \frac{\eta}{1 - \hat{\sigma}_-} \right)^{1/4} \sqrt{\frac{A}{Z}} \left( \frac{\lambda[\mu\text{m}]}{I[10^{20}\text{W/cm}^2]} \right)^{1/4} \quad (6)$$

where we normalized the chirp to  $\hat{\sigma}_- = \frac{|\sigma_-|}{\hat{A}_1 \hat{A}_2}$ .

By solving eq. 1 numerically we simulated the separation of a mixture of  $^{14}\text{C}$  and  $^{20}\text{O}$ , which is a typical separation scenario needed to isolate the radioactive waste of the carbon isotope. We used a laser beam with  $I = 1.3 \times 10^{20}\text{W/cm}^2$ ,  $\lambda = \frac{2\pi}{k} = 0.8\ \mu\text{m}$  and the chirps required for a 100% pure final sample. The estimated and simulated results were of 67%, 65% for the separation efficiency and 19, 18.6 for the separation factor.

## CONCLUSIONS

A new method of isotope enrichment was discussed. We showed that chirped counter-propagating laser beams can lead to a pure sample on the needed isotope through a one stage application. This may allow for more compact isotope separation facilities. Furthermore, the separation factors that can be obtained are unlimited but there is an inverse relation between the factor and the purity achieved. The efficiency of the separation can be of 100%.

## References

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