Electron Geodesic Acoustic Modes in Electron Temperature Gradient Mode Turbulence

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There has been overwhelming evidence that coherent structures such as vortices, streamers and zonal flows ($m = n = 0$, where m and n are the poloidal and toroidal modenumbers respectively) play a critical role in determining the overall transport in magnetically confined plasmas. [1] Some of these coherent structures, so called streamers, are radially elongated structures that cause intermittent, bursty events, which can mediate significant transport of heat and particles, for instance, imposing a large heat load on container walls. Zonal flows on the other hand may impede transport by shear decorrelation. [1] The Geodesic Acoustic Mode (GAM) [2, 4, 3, 5] is the oscillatory counterpart of the zonal flow ($m = n = 0$ in the potential perturbation, $m = 1$, $n = 0$ in the perturbations in density, temperature and parallel velocity) and thus a much weaker effect on turbulence is expected. Nevertheless experimental studies suggest that GAMs are related to the L-H transition and transport barriers. The GAMs are weakly damped by Landau resonances and moreover this damping effect is weaker at the edge suggesting that GAMs are more prominent in the region where transport barriers are expected. [5] In this work the first demonstration of an electron branch of the geodesic acoustic mode (el-GAM) driven by electron temperature gradient (ETG) modes is presented. The frequency of the el-Gam is higher compared to the ion GAM by the square root of the ion-to-electron mass ratio ($\Omega_q(electron)/\Omega_q(ion) \approx \sqrt{m_i/m_e}$ where $\Omega_q(electron)$ and $\Omega_q(ion)$ are the real frequencies of the electron and ion GAMs, respectively.).

The linear Electron Temperature Gradient Mode

In this section we will describe the preliminaries of the electron-temperature-gradient (ETG) mode which we consider under the following restrictions on real frequency and wave length: $\Omega_i \leq \omega \sim \omega_\star << \Omega_e$, $k_\perp c_i > \omega > k_{||} c_e$. Here Ω_j are the respective cyclotron frequencies, ρ_j the Larmor radii and $c_j = \sqrt{T_j/m_j}$ the thermal velocities. The diamagnetic frequency is $\omega_\star \sim k_\theta \rho_e c_e / L_n$, $k_\perp$ and $k_{||}$ are the perpendicular and the parallel wavevectors. The ETG model consists of a combination of an ion and electron fluid dynamics coupled through the quasineutrality including finite β-effects. [6] The electron dynamics for the toroidal ETG mode are governed by the continuity, parallel momentum and energy equations adapted from the Braginskii's fluid equations. Taking into account the diamagnetic cancellations in the continuity and energy equations we find,

$$
\begin{aligned}
-\frac{\partial \tilde{n}_e}{\partial t} - \nabla_\perp^2 \frac{\partial}{\partial t}\tilde{\phi} - \left(1+(1+\eta_e)\nabla_\perp^2\right)\nabla_\theta \tilde{\phi} - \nabla_{||}\nabla_\perp^2 \tilde{A}_{||} \quad + & \\
\varepsilon_n \left(\cos\theta \frac{1}{r}\frac{\partial}{\partial \theta} + \sin\theta \frac{\partial}{\partial r}\right)(\tilde{\phi} - \tilde{n}_e - \tilde{T}_e) &= [\tilde{\phi}, \nabla_\perp^2 \tilde{\phi}], \quad (1) \\
\left((\beta_e/2 - \nabla_\perp^2)\frac{\partial}{\partial t} + (1+\eta_e)(\beta_e/2)\nabla_\theta\right)\tilde{A}_{||} + \nabla_{||}(\tilde{\phi} - \tilde{n}_e - \tilde{T}_e) &= 0, \quad (2) \\
\frac{\partial}{\partial t}\tilde{T}_e + \frac{5}{3}\varepsilon_n \left(\cos\theta \frac{1}{r}\frac{\partial}{\partial \theta} + \sin\theta \frac{\partial}{\partial r}\right)\frac{1}{r}\frac{\partial}{\partial \theta}\tilde{T}_e + (\eta_e - \frac{2}{3})\frac{1}{r}\frac{\partial}{\partial \theta}\tilde{\phi} - \frac{2}{3}\frac{\partial}{\partial t}\tilde{n}_e &= -[\tilde{\phi}, \tilde{T}_e]. \quad (3)
\end{aligned}
$$

Extended fluid models treating the gyroviscous cancellations by including the higher order moments in the Braginskii's gyroviscous tensor has been presented in [7]. The variables are normalized according to $(\tilde{\phi}, \tilde{n}, \tilde{T}_e) = (L_n/\rho_e)(e\delta\phi/T_{eo}, \delta n_e/n_0, \delta T_e/T_{e0})$, $\tilde{A}_{||} = (2c_e L_n/\beta_e c \rho_e) e A_{||}/T_{e0}$ and $\beta_e = 8\pi n T_e/B_0^2$. Using the Poisson equation in combination with non-adiabatic ions [6] we then find

$$
\tilde{n}_e = -\left(\frac{\tau_i n_i/n_e}{1-\omega^2/k_\perp^2 c_i^2} + \frac{(Z^2 n_I/n_e)\tau_I}{1-\omega^2/(k_\perp^2 c_I^2)} + k_\perp^2 \lambda_{De}^2\right)\tilde{\phi}. \quad (4)
$$

First we will consider the Fourier representation of the linear dynamical equations (1, 2 and 3) combined with (4) in the same manner as in Ref. [6] and we find a linear semi-local dispersion relation.

Modeling Electron Geodesic Acoustic modes

The Geodesic Acoustic Modes are the $m = n = 0$, $k_r \neq 0$ perturbation of the potential field and the $n = 0$, $m = 1$, $k_r \neq 0$ perturbation in the density, temperatures and the magnetic field perturbations. In order to find the relevant equations for the electron GAM dynamics we consider the $m = 1$ component of Eqs (1) - (3) in the low β limit,

$$
\begin{aligned}
-\frac{\partial \tilde{n}_{eG}^{(1)}}{\partial t} + \varepsilon_n \sin\theta \frac{\partial}{\partial r}\tilde{\phi}_G^{(0)} - \nabla_{||}\nabla_\perp^2 \tilde{A}_{||G}^{(1)} &= \langle[\tilde{\phi}_k, \nabla^2 \tilde{\phi}_k]\rangle^{(1)} = 0, \quad (5) \\
\nabla_\perp^2 \frac{\partial}{\partial t}\tilde{A}_{||}^{(1)} - \nabla_{||}(\tilde{n}_{eG}^{(1)} + \tilde{T}_{eG}^{(1)}) &= \langle[\tilde{\phi}_k, \nabla_\perp^2 \tilde{A}_{||k}]\rangle^{(1)} = 0, \quad (6) \\
\frac{\partial}{\partial t}\tilde{T}_{eG}^{(1)} - \frac{2}{3}\frac{\partial}{\partial t}\tilde{n}_{eG}^{(1)} &= -\langle[\tilde{\phi}_k, \tilde{T}_{ek}]\rangle^{(1)} = N_1^{(1)}, \quad (7)
\end{aligned}
$$

where superscript (1) over the fluctuating quantities denotes the $m = 1$ poloidal mode number and $\langle\cdots\rangle$ is the average over the fast time and spatial scale of the ETG turbulence and that non-linear terms associated with parallel dynamics are small since $\frac{1}{q^2} << 1$. Here we have defined the non-linear term on the RHS in Eqs. (1) - (3) as $N_2^{(0)} = \rho_e^3 c_e \hat{z} \times \nabla\tilde{\phi} \cdot \nabla\nabla_\perp^2 \tilde{\phi}$. This can be written $\tilde{T}_e = \frac{2}{3}\tilde{n}_e^{(1)} + N_2^1$, where the $m = 1$ component is determined by an integral of the convective non-linear term as $N_1^1 = -\int dt \rho_s c_s \hat{z} \times \nabla\tilde{\phi}^{(0)} \cdot \nabla\tilde{T}_e^{(1)}$. We now study the $m = 0$ potential perturbations,

$$-\nabla_\perp^2 \frac{\partial}{\partial t}\tilde{\phi}_G^{(0)} - \varepsilon_n \sin\theta \frac{\partial}{\partial r}(\tilde{n}_{eG}^{(1)} + \tilde{T}_{eG}^{(1)}) = \langle[\tilde{\phi}_k, \nabla^2\tilde{\phi}_k]\rangle^{(0)} = N_2^{(0)}. \tag{8}$$

We will use the wave kinetic equation (WKE) [1] to describe the background short scale ETG turbulence for $(\Omega_q, \vec{q}) < (\omega, \vec{k})$, where the action density $N_k = E_k/|\omega_r| \approx \varepsilon_0|\tilde{\phi}_k|^2/\omega_r$. Here $\varepsilon_0|\tilde{\phi}_k|^2$, is the total energy in the ETG mode with mode number k where $\varepsilon_0 = \tau + k_\perp^2 + \frac{\eta_e^2 k_\theta^2}{|\omega|^2}$. In describing the large scale plasma flow dynamics it is assumed that there is a sufficient spectral gap between the small scale ETG turbulent fluctuations and the large scale GAM flow. The electrostatic potential is represented as a sum of fluctuating and mean quantities $\phi(\vec{X}, \vec{x}, T, t) = \Phi(\vec{X}, T) + \tilde{\phi}(\vec{x}, t)$ where $\Phi(\vec{X}, T)$ is the mean flow potential. The coordinates $\left(\vec{X}, T\right)$, $(\vec{x}, t)$ are the spatial and time coordinates for the mean flows and small scale fluctuations, respectively. We will solve the WKE by assuming a small perturbation (δN_k) driven by a slow variation for the GAM compared to the mean (N_{k0}) such that $N_k = N_{k0} + \delta N_k$. The relevant non-linear terms can be approximated in the following form, $\langle[\tilde{\phi}_k, \nabla_\perp^2\tilde{\phi}_k]\rangle \approx q_r^2 \sum_k k_r k_\theta \frac{|\omega_r|}{\varepsilon_0} \delta N_k(\vec{q}, \Omega_q)$ For all GAMs we have $q_r > q_\theta$. The GAM dispersion relation including the non-linear drive from the ETG mode is found to be

$$\Omega_q^2 - \frac{5}{3}q_\parallel^2 - \frac{5}{6}\varepsilon_n^2 = \left(\frac{\Omega_q^2 - \frac{5}{3}q_\parallel^2}{\Omega_q}\right) q_r^2 \sum k_r k_\theta^2 \frac{|\omega_r|}{\varepsilon_0} R \frac{\partial N_k}{\partial k_r}. \tag{9}$$

Eq. (9) is the sought dispersion relation for the electron GAM and we solve it perturbatively by assuming $\Omega_q = \Omega_0 + \Omega_1$ where Ω_0 is the solution to the linear part, $\Omega_0^2 = \frac{5}{3}\frac{c_e^2}{R^2}\left(2 + \frac{1}{q^2}\right)$. Now we find the perturbation $\Omega_1 = i\gamma_q$ which will determine the growth rate of the GAM as,

$$\frac{\gamma_q}{c_e/L_n} = -i\frac{5}{12}\frac{\varepsilon_n^2}{\Omega_0^2} q_r^2 \rho_e^2 \sum k_r k_\theta^2 \rho_e^3 \frac{|\omega_r|}{\varepsilon_0}\frac{1}{i\gamma}\left|\frac{\partial N_0}{\partial k_r}\right| \approx \frac{1}{2}\frac{q_r^2\rho_e^2 k_\theta \rho_e}{\sqrt{\varepsilon_n(\eta_e)}}\frac{1}{1 + 1/2q^2}\left|\tilde{\phi}_k \frac{L_n}{\rho_e}\right|^2. \tag{10}$$

Here the main contribution to the non-linear generation of GAMs originates from the Reynolds stress term. The non-linearly driven electron GAM is unstable with a growth rate depending on the saturation level $\left|\tilde{\phi}_k^2\right|$ of the ETG mode turbulence.

Saturation mechanism

We will estimate a new saturation level for the ETG turbulent electrostatic potential $(\tilde{\phi}_k)$ by balancing the Landau damping in competition with the non-linear growth rate of the GAM in a constant background of ETG mode turbulence, according to the well known predator-prey models used, in Ref. ([3]). The Landau damping rate $\left(\gamma_L = \frac{4\sqrt{2}}{3\sqrt{\pi}}\frac{c_e}{qR}\right)$ is assumed to be balanced by GAM growth rate Eq. (10) modified by the neoclassical damping in stationary state $\left(\frac{\partial}{\partial t} \to 0\right)$. In steady state find the saturation level for the ETG turbulent intensity as ($\gamma_q = \gamma_L + \nu^\star$),

$$\left|\frac{e\phi_k}{T_e}\frac{L_n}{\rho_e}\right|^2 \approx \frac{2L_n}{qR}\left(1+\frac{1}{2q^2}\right)\sqrt{\varepsilon_n\eta_e}\left(\frac{4}{3}\sqrt{\frac{2}{\pi}}+\nu^\star\right)\left(\frac{k_\theta}{q_r}\right)^2\left(\frac{1}{k_\theta\rho_e}\right)^3. \quad (11)$$

Here, the saturation level is modified by the neoclassical damping $\nu^\star = \nu_e \frac{qR}{v_{eth}}$ and the $\frac{k_\theta}{q_r}$ factor arises due to the spatial extension of the GAM and we obtain, $\left|\frac{e\phi_k}{T_e}\frac{L_n}{\rho_e}\right| \sim 30-40$. Note that the result found using a mixing length estimate with $\left|\frac{e\phi}{T_e}\frac{L_n}{\rho_e}\right| \sim 1$ is significantly smaller. Here in this estimation we have used $L_n = 0.05$, $q = 3.0$, $R = 4$, $\varepsilon_n = 0.025$, $q_r\rho_e = k_\theta\rho_e = 0.3$ and $\eta_e \sim 1$.

Conclusion

In this paper we have presented the first derivation of an electron branch of the Geodesic Acoustic Mode (el-GAM). The linear dispersion relation of the el-GAM showed that the new branch is purely oscillatory with a frequency $\Omega_q \sim \frac{c_e}{R}$. To estimate the GAM growth rate, a non-linear treatment based on the wave-kinetic approach was applied. The resulting non-linear dispersion relation showed that the el-GAM is excited in the presence of ETG modes with a growth rate depending on the fluctuation level of the ETG mode turbulence. To estimate the ETG mode fluctuation level and GAM growth, a predator-prey model was used to describe the coupling between the GAMs and small scale ETG turbulence. The stationary point of the coupled system implies that the ETG turbulent saturation level $\tilde{\phi}_k$ can be drastically enhanced by a new saturation mechanism, stemming from a balance between the Landau damping and the GAM growth rate. This may result in highly elevated particle and electron heat transport, relevant for the edge pedestal region of H-mode plasmas.

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