Envelope solitons in a weakly magnetized electron-positron plasma with relativistic temperature

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Introduction

The self-modulation of a relativistic nonlinear circularly (and linearly) polarized electromagnetic wave in an electron-positron plasma was first investigated by Chian and Kennel [1] in the cold unmagnetized case. A large number of papers have improved the original model by Chian and Kennel [1], including the existence of Alfvén vortices in the presence of a strong magnetic field, the influence of a strong magnetic field and ions, and the effects of relativistic temperatures and phonon damping.

In this work we follow this research subject, by studying the self-modulation of a nonlinear circularly polarized electromagnetic wave in a weakly magnetized relativistic hot electron-positron plasma. [2]

Relativistic plasmas with a finite temperature

It is possible to formulate a theory for hot relativistic plasmas in a covariant form [3] where the fluid and the electromagnetic fields are unified into a single field. The plasma dynamics is given by Maxwell's equations, the continuity and the motion equations for each fluid:

$$\frac{\partial}{\partial t}\left(n_j\gamma_j\right)+\vec{\nabla}\cdot\left(n_j\gamma_j\vec{v}_j\right)=0\ , \tag{1}$$

$$\left(\frac{\partial}{\partial t}+\vec{v}_j\cdot\vec{\nabla}\right)\left(f_j\gamma_j\vec{v}_j\right)=\frac{q_j}{m}\left(\vec{E}+\frac{\vec{v}_j}{c}\times\vec{B}\right)-\frac{1}{n_j m\gamma_j}\vec{\nabla}p_j\ , \tag{2}$$

where $\vec{v}_j$,γ_j, q_j, p_j and n_j are the velocity, relativistic Lorentz factor, charge, pressure and rest-frame number density of each fluid, respectively, m is the electron mass, c is the speed of light, and $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields.

The $f_j \equiv f(T_j)$ function is given by $f(T) = K_3(mc^2/k_BT)/K_2(mc^2/k_BT)$, where T_j is the temperature, $K_3(x)$ and $K_2(x)$ are modified Bessel functions of order 3 and 2, respectively, and k_B is the Boltzmann constant. We use the subindex $j = e$ for the electron fluid and $j = p$ for the positron fluid.

Circularly polarized electromagnetic wave

A circularly polarized electromagnetic wave is an exact solution of the fluid equations for constant, relativistic temperatures. [4, 5] We consider a circularly polarized electromagnetic wave which propagates in a relativistic electron-positron plasma, along an ambient uniform magnetic field $B_0\hat{z}$, that is $\vec{A} = a_0[\cos(kz-\omega t)\hat{x} + \sin(kz-\omega t)\hat{y}]$, where ω is the frequency and k is the wave number.

This wave satisfies the dispersion relation [4, 5]

$$\frac{\omega^2 - c^2k^2}{\omega\,\omega_p^2} = \frac{\gamma_e}{f_e\gamma_e\omega + \Omega_c} + \frac{\gamma_p}{f_p\gamma_p\omega - \Omega_c}\,, \tag{3}$$

where $\omega_p = (4\pi e^2 n_0/m)^{1/2}$ and $\Omega_c = eB_0/(mc)$ are the plasma and cyclotron frequencies.

NLS equation for wave amplitude

We now seek a solution in the weakly magnetized relativistic plasma limit ($\omega \gg \Omega_c$). At the order Ω_c^2/ω^2, for a plasma in thermal equilibrium ($f_e = f_p = f$), it can be shown that the vector potential satisfies the wave equation:

$$\frac{\partial^2\vec{A}}{\partial t^2} - c^2\frac{\partial^2\vec{A}}{\partial z^2} + \frac{2\omega_p^2\vec{A}}{f}\left[1 + \frac{\Omega_c^2 f^2}{\omega^2\left(f^2 + \lambda A^2\right)^2}\right] = 0\,, \tag{4}$$

where $\lambda = e^2/(m^2c^4)$. This equation represents the nonlinear propagation of the circularly polarized electromagnetic wave packet.

In order to study the modulational instability, we now consider a space-dependent wave amplitude, $\vec{A} = a(z,t)[\cos(\omega t)\hat{x} + \sin(\omega t)\hat{y}]$. Then, for a slowly time-varying modulation $\partial_t^2 a \ll \omega^2 a$ in Eq. (4), the small-amplitude limit $f^2 \gg \lambda|a|^2$, and with the further change of variables $a \to a\exp(iR(\omega,|a_0|^2)t)$, the following nonlinear Schrödinger equation is obtained:

$$i\frac{\partial a}{\partial t} + P(\omega)\frac{\partial^2 a}{\partial z^2} + Q(\omega)(|a|^2 - |a_0|^2)a = 0\,, \tag{5}$$

with

$$P(\omega) = \frac{c^2}{2\omega}\,, \quad Q(\omega) = \frac{2\lambda\,\omega_p^2\Omega_c^2}{\omega^3 f^5}\,, \quad R(\omega,|a_0|^2) = \frac{\omega}{2} - \frac{\omega_p^2}{\omega f} - \frac{\omega_p^2\Omega_c^2}{\omega^3 f^3} + \frac{2\lambda\,\omega_p^2\Omega_c^2}{\omega^3 f^5}|a_0|^2\,. \tag{6}$$

This nonlinear Schrödinger equation for the wave amplitude includes thermal effects introduced through the f_j factor, thus improving on previous results [1]. Also notice that $P(\omega)Q(\omega) > 0$, thus the wave is subject to the modulational instability.

Modulational instability

Writing $a = \sqrt{\rho(z,t)}\exp[i\sigma(z,t)]$ in (5), yields

$$\frac{\partial\rho}{\partial t} + 2P\frac{\partial}{\partial z}\left(\rho\frac{\partial\sigma}{\partial z}\right) = 0\,, \tag{7}$$

$$\frac{\partial\sigma}{\partial t} + P\left[\frac{1}{4\rho^2}\left(\frac{\partial\rho}{\partial z}\right)^2 - \frac{1}{2\rho}\frac{\partial^2\rho}{\partial z^2} + \left(\frac{\partial\sigma}{\partial z}\right)^2\right] = Q(\rho-\rho_0)\,, \tag{8}$$

where $\rho_0 = |a_0|^2$.

Linearizing with respect to the uniform solution,

$$\rho = \rho_0 + \rho_1 e^{ik_L z - i\omega_L t}\,, \quad \sigma = \sigma_1 e^{ik_L z - i\omega_L t}\,,$$

where $\rho_1 \ll \rho_0$ and $\sigma_1 \ll \sigma_0$, we obtain the dispersion relation for the low-frequency modulation. Solving for ω_L shows that the maximum growth rate is $\Gamma = Q\rho_0$, and that the instability is suppressed at large temperatures, $f \approx 4k_B T/mc^2$, $\Gamma \simeq T^{-5}$.

When the modulation grows, the instability evolves into a nonlinear stationary state balancing the dispersion with the nonlinearity. To study these nonlinear states, we focus on Eq. (5) for $a(z,t)$.

Depending on the initial conditions, the solution can be periodic wave trains (Fig. 2), or localized solutions (Fig. 3):

$$\frac{a(z,t)}{a_0} = \Psi\left(\sqrt{\frac{Qa_0^2}{2P}}\,\xi\right)e^{i\eta}\,, \quad \text{where } \xi = z - Vt\,, \quad \eta = \frac{V}{2P}z - \left(\frac{V^2}{4P} + \frac{Qa_0^2}{2}\,\tau^2\right)t\,. \tag{9}$$

For $\tau = 1$ and $\Psi'(0) = 0$, we obtain a localized solitary wave solution $\Psi = \mathrm{sech}\,(\xi)$, so that the solution of (9) for a is

$$a(z,t) = a_0\,\mathrm{sech}\left(\sqrt{\frac{Qa_0^2}{2P}}\,\xi\right)e^{i\eta}\,. \tag{10}$$

The soliton solution gets wider in ξ_n as we increase the temperature, reducing the effect of the nonlinear corrections. Eventually the localized solution becomes effectively a light mode with uniform amplitude for very large temperatures. Similarly, the solitary wave also becomes effectively a uniform solution for large wavenumbers.

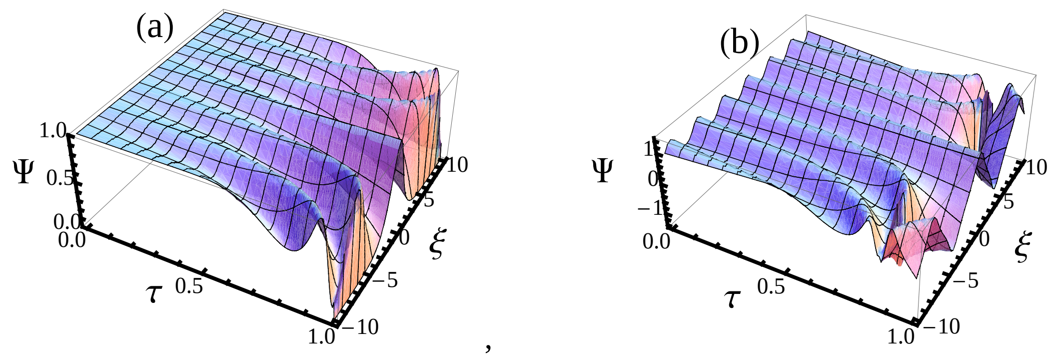

Fig. 2. (a) Solutions for $\Psi(0) = 1$ and $\Psi'(0) = 0$. The constant solution is for $\tau = 0$, whereas a solitary wave solution exists for $\tau = 1$. For values $0 < \tau < 1$ we find periodic wave trains. (b) Solutions for $\Psi(0) = 1$ and $\Psi'(0) = 0.5$. For these values there are only periodic wave trains whose amplitude becomes bigger as the value of τ increases.

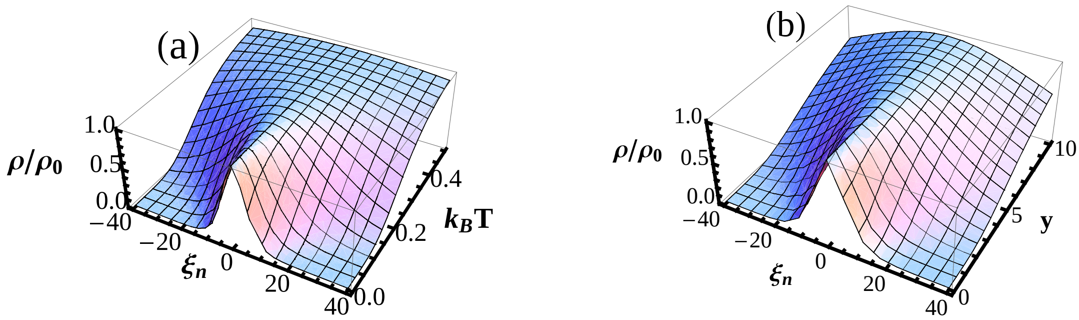

Fig. 3. (a) The soliton solution ρ in terms of $-40 \le \xi_n \le 40$ and $0 \le k_B T \le 0.5$ MeV for $y = 1$. (b) Its behavior with respect to $-40 \le \xi_n \le 40$ and $0 \le y \le 10$ for temperature $k_B T = 0.05$ MeV. We have taken $\alpha = 0.1$.

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