Extended calculation of the neoclassical viscosity for TJ-II: Physics of the shear layer formation at stellarators

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The reduction of turbulence by sheared flows is generally accepted as a key ingredient in the formation of transport barriers, but the identification of the physical mechanisms that create these sheared flows is still an open issue. Zonal flows are considered a possible catalyst of the transition. Nevertheless, in non-quasisymmetric stellarators, the radial electric field E_r is basically determined by the neoclassical theory of collisional transport in magnetized plasmas. Therefore, the interaction between neoclassical and turbulent processes, and concretely the extent to which turbulence can overcome the neoclassical viscosity and modify the $E\times B$ rotation through momentum transport, has been the subject of recent works. Here, we contribute to the above programme with the study of the so-called low density confinement transition (see Ref. [1] and refs. therein) in the heliac TJ-II. The formation of the shear layer is described from first principles in the framework of neoclassical theory and the turbulent phenomena that arise in the neighborhood of the transition are shown to be regulated by neoclassical transport.

TJ-II undergoes a spontaneous confinement transition typically at a line-averaged electron density $\overline{n_e}=n_{cr}\approx 0.6\times 10^{19}\,\mathrm{m}^{-3}$. At this empirical critical density n_{cr}, E_r changes from positive to negative and, at the same time, a transport barrier is generated close to the edge. The reversal of E_r starts where the density gradient is maximum and then propagates across the entire region $0.5<\rho<0.9$ ($\rho=r/a$ is the normalized radius) at a speed of the order of several m/s. Whereas similar transport barriers, related to jumps between roots of the ambipolar equation, have been observed in the low density regime of other stellarators, the very detailed study carried out in TJ-II has revealed additional interesting phenomena during the transition. First of all, the level of turbulence and the $E\times B$ flux are seen to increase prior to the transition, associated to long-range-correlated (LRC) electrostatic potential structures that grow when approaching the critical density [2]. The potential relaxation time in biasing turn-off experiments [3] peaks at n_{cr}, and so does the shear-flow susceptibility in electrode-biasing experiments [4].

In this work, we perform a dynamical neoclassical calculation of the formation of the sheared E_r. We simulate the density ramp-up that leads to the transition and describe, from first principles, the formation and evolution of the shear flow in good agreement with the experiment. Furthermore, we show for the first time that the behaviour of the three quantities discussed above (amplitude of low-frequency LRCed potential fluctuations, potential relaxation time and shear-flow susceptibility) appear here as natural consequences of the expected neoclassical bifurcation. We start from the momentum balance equation summed over species:

$$m_i\frac{\partial(n\mathbf{u})}{\partial t}+\nabla\cdot\Pi_i+\nabla\cdot\Pi_e=\mathbf{j}\times\mathbf{B}\,,\qquad \mathbf{u}=2\pi\left(\frac{p_i'(\psi)}{ne}+\phi'(\psi)\right)\mathbf{e}_\theta+\Lambda(\psi)\mathbf{B}\,. \quad (1)$$

Here, $\mathbf{u}$ is the (lowest order incompressible) ion flow tangent to flux surfaces, Π_s is the viscosity tensor, $\Pi_s = m_s\int\mathbf{v}\mathbf{v}f_s(\mathbf{x},\mathbf{v},t)d^3\mathbf{v}$, or momentum flux of species s, f_s its distribution function, and $\mathbf{j}\times\mathbf{B}$ is the Lorentz force. We have assumed a quasineutral plasma consisting of singly charged ions and electrons ($n_e=n_i=n$). Note that we have dropped the inertia since $m_e \ll m_i$. We work in Hamada magnetic coordinates (ψ,θ,ξ). The flux surface label ψ is the toroidal magnetic flux and the prime stands for derivative. The first term on the RHS of Eq. (1) contains the diamagnetic and $E\times B$ perpendicular flows (p_i is the ion pressure, e the elementary charge, ϕ is the electrostatic potential, and $\mathbf{e}_\theta\times\mathbf{B}=(2\pi)^{-1}\nabla\psi$) together with the parallel Pfirsch-Schlüter flow ($\nabla\cdot\mathbf{e}_\theta=0$ and $\langle\mathbf{e}_\theta\cdot\mathbf{B}\rangle=0$ for a currentless stellarator). The term $\Lambda\mathbf{B}$ is the ion bootstrap flow. If we project Eq. (1) along $\mathbf{e}_\theta$ and take flux-surface-average $\langle\cdot\rangle$ we obtain:

$$\frac{\partial E_r}{\partial t}=\frac{1}{n}\frac{\partial}{\partial t}\left(\frac{p_i'(r)}{e}\right)-E_r\frac{1}{n}\frac{\partial n}{\partial t}+\frac{(\psi'(r))^2}{4\pi^2mn\langle\mathbf{e}_\theta\cdot\mathbf{e}_\theta\rangle}\left(e(\Gamma_e-\Gamma_i)+\langle\mathbf{j}\cdot\nabla r\rangle\right), \quad (2)$$

where $E_r\equiv-\phi'(r)$ and the minor radius r is a geometric flux label defined in terms of the volume $V(r)\equiv\pi r^2L_{\rm ax}$, where $L_{\rm ax}$ is the length of the magnetic axis. We have obtained the radial particle fluxes from $\Gamma_s=-\frac{2\pi}{q_s\psi'(r)}\langle\mathbf{e}_\theta\cdot\nabla\cdot\Pi_s\rangle$. The viscosity tensor can be split into a neoclassical part, given by the gyrotropic pressure tensor, and an anomalous contribution $\Pi_s=\Pi_s^{NC}+\Pi_s^{an}=p_{s\|}\mathbf{b}\mathbf{b}+p_{s\perp}(\mathbf{I}-\mathbf{b}\mathbf{b})+\Pi_s^{an}$. As mentioned, in non-quasisymmetric confining magnetic topologies, the leading order contribution to Eq. (2) is $\langle\mathbf{e}_\theta\cdot\nabla\cdot\Pi_s^{NC}\rangle$, being much larger than $\langle\mathbf{e}_\theta\cdot\nabla\cdot\Pi_s^{an}\rangle$, which will be thus neglected, together with the shear-flow viscosity.

We use the Drift Kinetic Equation Solver (DKES) to evaluate the pressure anisotropy in the TJ-II magnetic field in the parameter range usually found experimentally in the vicinity of the transition. Details of the calculation and convolution of the monoenergetic coefficients may be found in Ref. [5] and references therein. From Eq. (2), the time-evolution of E_r is fully determined if we know, at every instant of time, the magnetic configuration b_{mn} and the profiles n, T_e, T_i. Since we simulate a pure proton-electron plasma, the effective charge Z_{eff} (which mainly affects collisionality) is set equal to one. We perform a numerical simulation of a density ramp across the critical density for a plasma with profiles $n(\rho,t)$, $T_e(\rho,t)$ and $T_i(\rho,t)$ that mimic the experimental ones, see Fig. 1 and Ref. [1]. We set $\langle\mathbf{j}\cdot\nabla r\rangle=0$ unless otherwise stated. This is implied by quasineutrality ($\nabla\cdot\mathbf{j}=0$), but a net radial plasma current can be induced in plasma biasing experiments. Note that although we assume that the leading non-ambipolar particle fluxes are neoclassical, we make no particular assumption on the total particle or energy fluxes.

Fig. 2 shows the formation and precise evolution of the shear layer in TJ-II plasmas, in good agreement with the experiment [1]. It starts to develop approximately where the gradient is maximum and then propagates inwards and outwards: a speed of the order of 1 m/s may be extracted, determined by the evolution time of the local collisionality. This general behaviour is expected for neoclassical simulations of TJ-II low-density plasmas. In Fig. 3, we show the ambipolar equation at $\rho = 0.7$ for several times. Since we start from low collisionality, E_r is positive ($t = 10$ ms). As n is raised, a negative stable root appears ($t = 50$ ms and $t = 80$ ms), but the two stable roots are separated by an unstable root, so E_r stays positive. Only for $n = n_{cr}$, when the electron root disappears ($t > 90$ ms), the system *jumps* to negative E_r. This picture is only slightly modified by considering the measured turbulent momentum transport.

When the transition is approached from below, the non ambipolar neoclassical fluxes display a weak dependence on E_r around the ambipolar value and large E_r excursions may be caused by turbulent momentum fluxes or external forcing (biasing) as is observed experimentally. To make the argument more precise we define a neoclassical poloidal viscosity as the linear coefficient of the difference between the electron and ion radial fluxes expanded around the ambipolar electric field, $[\Gamma_e - \Gamma_i](E_r) = -\mu_p(E_r - E_r^0) + O((E_r - E_r^0)^2)$. Eq. (2) then approximately yields:

$$\frac{\partial E_r}{\partial t} \approx \frac{e(\psi')^2}{4\pi^2 mn\langle \mathbf{e}_\theta \cdot \mathbf{e}_\theta \rangle}\left[\mu_p(E_r - E_r^0) - \frac{\langle \mathbf{j} \cdot \nabla r\rangle}{e}\right] = -\nu_p(E_r - E_r^0) + \check{j}_r\,. \tag{3}$$

The coefficient μ_p can be calculated directly from the data of Fig. 3, and we have absorbed n, m, e, ψ', $\langle \mathbf{e}_\theta \cdot \mathbf{e}_\theta \rangle$, and constants into ν_p and $\check{j}_r$. The dependence of ν_p on n during the transition is shown in Fig. 4: it is smaller before the transition than after it and, more importantly, goes to zero when approaching n_{cr} from $n < n_{cr}$. We now show that the behaviour of the neoclassical viscosity provides a simple, unified explanation of the observed phenomena that accompany the transition. The characteristic relaxation time in biasing turn-off experiments (ν_p^{-1} in Eq. 3) shows a peak around the critical density and decreases for larger density plasmas in the ion root [3]. This is reproduced by the curve shown in Fig. 4. Similarly, when a low frequency external biasing is applied, the response E_r is in phase with the biasing and its amplitude increases close to n_{cr} [4]. This is to be expected from Eq. 3, for in that case $E_r(t) \approx E_r^0 + \nu_p^{-1}\check{j}_r(t)$. Finally, to better discuss the observations of LRCs close to the transition [2] we Fourier transform Eq. (3):

$$i\omega\hat{E}_r(\omega) = -\nu_p\hat{E}_r(\omega) + \hat{j}(\omega) \Rightarrow |\hat{E}_r(\omega)|^2 = \frac{1}{\nu_p^2 + \omega^2}|\hat{j}(\omega)|^2 \equiv A(\omega)|\hat{j}(\omega)|^2\,, \tag{4}$$

for time scales faster than that of the density ramp, i.e., $\omega > \partial_t \log(E_r^0) \sim \partial_t \log(n) \sim 10$ Hz.

Within this simplified framework, Eq. 4 shows that the amplitude of the fluctuations $\hat{E}_r(\omega)$ driven by a given broadband turbulent forcing $\hat{j}(\omega)$ is modulated by the NC viscosity, which damps fluctuations of frequencies lower than ν_p. Below and above the transition (Fig. 4 inset), the fluctuations $\hat{E}_r(\omega)$ with $\omega < 10\,\mathrm{kHz}$ are neoclassically damped. It is only close below n_{cr} that ν_p drops, leaving the low frequency E_r fluctuations (which display higher LRC) undamped.

References

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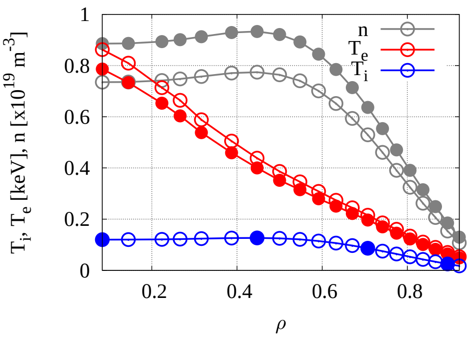

Fig. 1: Plasma profiles: low (high) n in open (closed) circles.

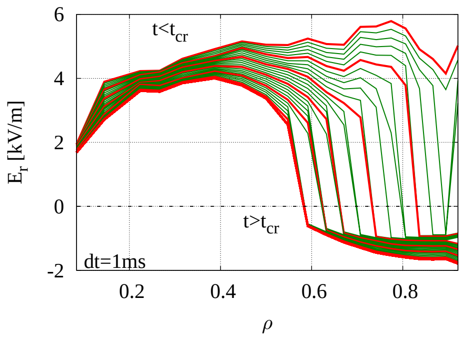

Fig. 2: E_r-profile for representative times.

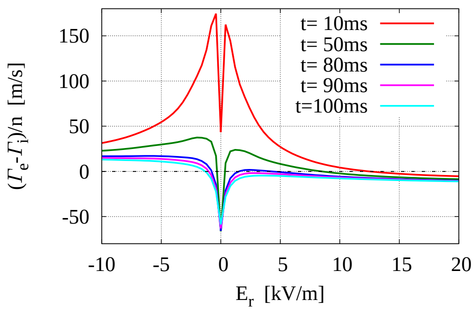

Fig. 3: Ambipolar equation at $\rho = 0.7$.

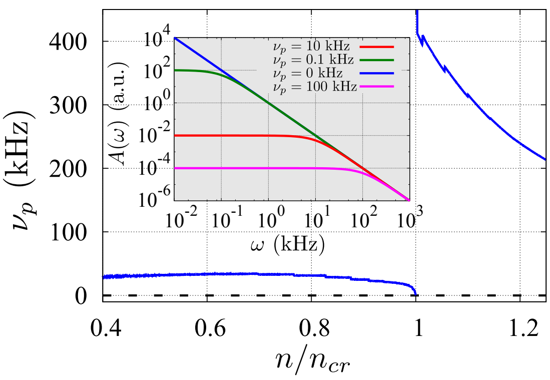

Fig. 4: ν_p vs. n/n_{cr} during the transition (inset: ω-dependence of the NC damping).