Resistive wall mode coupling to neoclassical tearing modes in rotating tokamak plasmas

R. McAdams[1,2], H. R. Wilson[1], I. T. Chapman[2]

[1] *York Plasma Institute, Dept of Physics, University of York, Heslington, York, YO10 5DD, UK*

[2] *EURATOM/CCFE Fusion Association, Culham Science Centre, Abingdon, Oxon, OX14 3DB, UK*

Introduction

In order to optimise gain in a future fusion reactor, the plasma parameter, β, should be maximised. The Resistive Wall Mode (RWM) can be destabilised at high beta, reducing thermal pressure and disrupting the plasma. In addition, magnetic reconnection leading to magnetic "islands" at rational surfaces (Neoclassical Tearing Modes, or NTMs), causes a loss in core plasma pressure, but these islands are often unstable at high beta only if seeded above a threshold width. Both instabilities are detrimental to fusion performance-especially, as we will show, when they couple.

System Of Nonlinear Equations

A nonlinear system of equations has been derived to describe a toroidally rotating plasma, unstable to RWMs and metastable to NTMs, using a linear RWM model [1] and a nonlinear NTM model [2]:

$$\Delta_L = \frac{1 - \delta\tau p}{-\varepsilon + \tau p} \tag{1}$$

$$2w(\gamma - i(\omega - \Omega)) = \Delta_L + \frac{\hat{\beta}}{w}\left(1 - \frac{w_c^2}{w^2}\right) \tag{2}$$

$$(\Omega_0 - \Omega(a/(a - r_s)) = Aw^4 Im[\Delta_L] \tag{3}$$

where w is the island half-width normalised to minor radius a; δ, ε are stability parameters; $p(t) = \gamma(t) - i\omega(t)$ the complex growth rate; derivatives are with respect to $\hat{t} = t/\tau_r$ where τ_r is the resistive diffusion time, ω and Ω (the plasma's rotation frequency at the rational surface located at $r = r_s$) are normalised to τ_r also. In addition, $\hat{\beta} = \frac{8\sqrt{\varepsilon}\beta_\theta r_s}{L_p s}$, where s is the magnetic shear, L_p the pressure length scale, ε the inverse aspect ratio, and β_θ is the poloidal beta. Note that $\gamma = 2\dot{w}/w$. We have also introduced a heuristic NTM threshold factor $(1 - w_c^2/w^2)$, which could be attributed to the polarisation current effects in the plasma, for example [3]. Also, $A = s^2\varepsilon^2 a^3 \tau_V \tau_r / 512 r_s^3 q^2 \tau_A^2$, τ_V is the momentum confinement time, q=2 the safety factor at $r = r_s$, τ_A is the Alfvén time. A driving force which maintains $\frac{d\Omega}{dr}|_{r=0} = -\Omega_0/a$ is assumed.

Solutions In Small τ Limit

The system of equations is expanded in the limit where $\tau\omega << 1$, $\tau = \tau_w/\tau_r$ and τ_w is the wall diffusion time, but $\tau\gamma$ is allowed to be of order unity in order to capture the RWM physics. If $\tau = 0$ then the RWM is not a solution (the external kink reflects itself as an infinite growth rate). If τ is finite and small, then the RWM branch is a solution of the equations (as well as the NTM branch), shown in Figure 1. Figure 1a) shows that the NTM branch retains its characteristic threshold-saturation curve, and plasma and mode both lock to the wall. In Figure 1b) it can be seen that for small island widths, the RWM has a positive constant growth rate, and the zero mode frequency indicates it is locked to the wall. The plasma rotation is slowed by the torque exerted by the growing RWM as w approaches w_c. However, after the island crosses the NTM threshold width, the RWM growth increases rapidly above its projected linear growth. The plasma rotation is also increased and the RWM becomes unlocked and
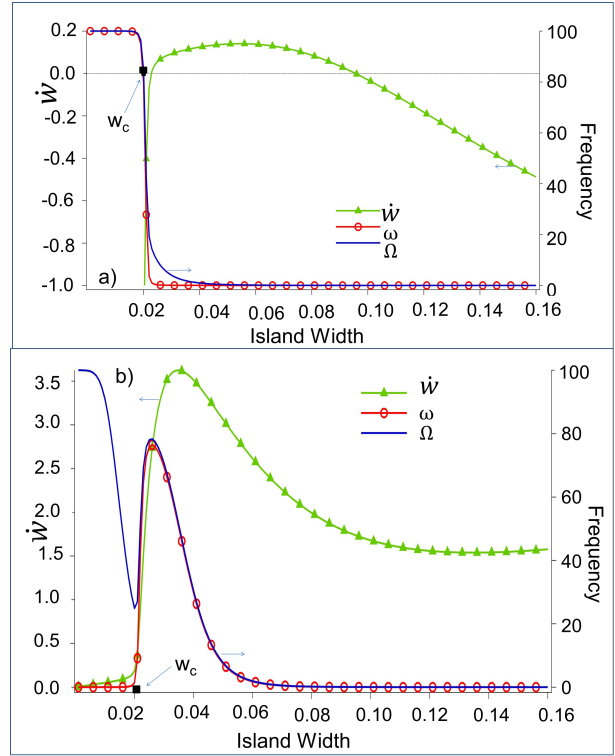

Figure 1: $\hat{\beta} = 1, \Omega_0 = 200$. *The growth, mode frequency and rotation frequency as a function of island width for finite $\tau = 0.01$. a) The NTM has a threshold width corresponding roughly to $w = w_c = 0.02$, and the plasma and the mode both lock to the wall for $w > w_c$ b)The RWM evolution changes as the island width exceeds the same threshold width.*

starts to rotate with the plasma. As the island grows further, the plasma and RWM both slow rotationally and lock again to the wall; the RWM returns to its linear growth. The RWM evolution for $\beta = 0$ can be seen in Figure 2. In this case, there is no drive for the NTM. The plasma rotation is slowed from its initial value, but the RWM frequency does not match that of the plasma and the mode remains (nearly) locked to the wall as the island width increases. This is the conventional picture of the RWM. The heights of the peaks in $\dot{w}, \omega$ and Ω seen in Figure 1 as w crosses the threshold width w_c increase with β. The above solutions show how the RWM and NTM branches can couple such that there is a positive growth rate for all island widths, which rapidly increases after the island crosses the threshold width. There is in addition a strong dependence upon β, since the deviation from the linear RWM growth rate is proportional to β.

Although strictly the mode shown in Figure 1b) is the RWM branch, it has all the features of a triggerless NTM; in particular, there is no threshold seed island required and the mode can grow from the noise.

Integrated Solutions

Integrating from a seed island of half width w_{seed} at $t = 0$, we obtain the time evolution of w (Figure 3). Choosing the seed island width such that $w_{seed} < w_c$ demonstrates how a mode with the characteristics of a triggerless NTM is observed in the plasma. However, as mentioned above, this is the RWM solution branch. In Figure 3a) we show the NTM branch for $w_{seed} < w_c$. The island has a negative growth rate and the seed island shrinks away. From Figure 3b) for the RWM branch, we see that initially the RWM causes the slow island growth from the seed island (which, in an NTM, would have a negative growth rate). The RWM is however locked to the wall, and the RWM torque drags the plasma rotation frequency toward the stationary wall. When w reaches w_c, the island begins to grow rapidly, and the RWM locks to the plasma-which briefly spins up. This spin up and the subsequent return to decay of the rotation takes place over a time of $\approx 0.2s$ for $T_e = 1keV$, $r_s = 0.5m$, $a = 1.0m$, and $ln\Lambda = 20$. The plasma and RWM then lock to the wall as the island grows to disruption. Note there is no saturation mechanism in the linear RWM model used in the derivation of the equations. The phenomenon of triggerless NTMs near $\beta_N^{no-wall}$, the no-wall β-limit, can be explained by invoking Δ' (which here is equal to Δ_L/r_s). As $\beta_N \to \beta_N^{no-wall}$, $\Delta' \to \infty$ destabilises a classical tearing mode, which in turn destabilises the neoclassical tearing mode when at a sufficiently large amplitude [4]. In our model, this would be equivalent to letting $\tau \to 0$ and $\varepsilon \to 0$, corresponding to the no-wall ideal MHD stability boundary. Here, the RWM is destabilised for $\beta_N > \beta_N^{no-wall}$ but the pole in Δ_L is resolved by its dependence on the wall response. The physics interpretation is therefore different to that provided in [4]. The features of the triggerless NTM appear in this system, although it is the RWM branch which exhibits them while the actual NTM branch retains its threshold characteristics.

References

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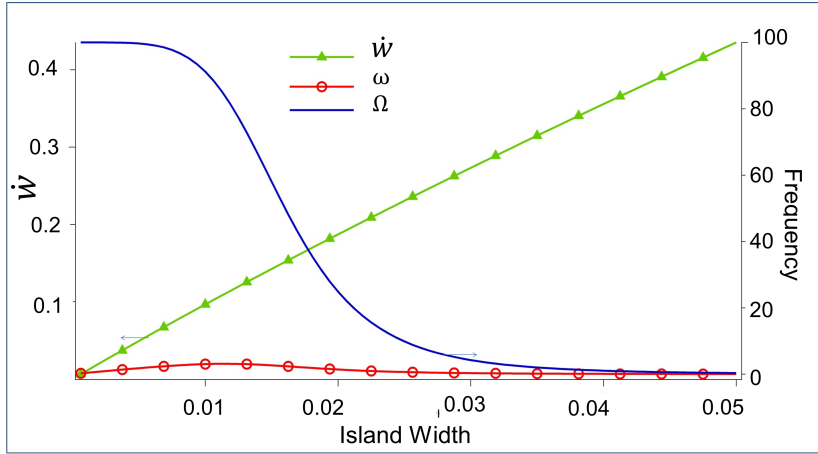[4] D. P. Brennan et al., Phys. Plasmas *10*, 1643, 2003.

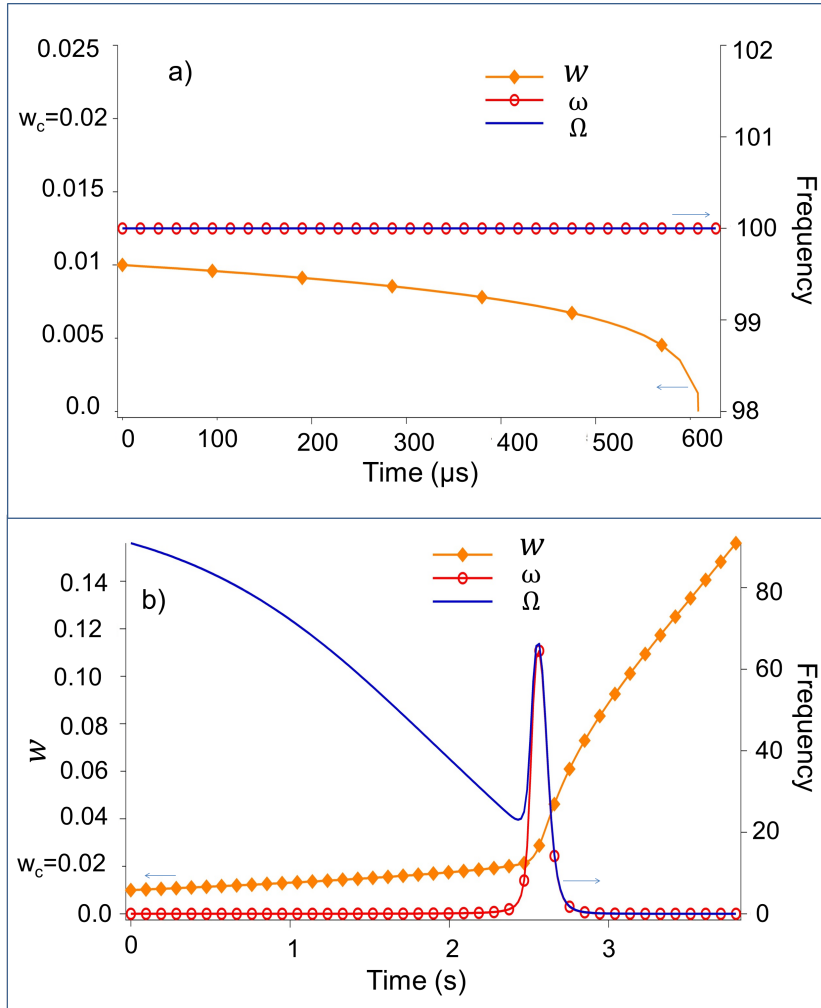Figure 2: *RWM evolution as a function of island half width for* $\hat{\beta} = 0, \Omega_0 = 200.$

Figure 3: $\hat{\beta} = 1, w_{seed} = 0.01, \Omega_0 = 200, a = 1.0m, r_s = 0.5m, T_e = 1keV, ln\Lambda = 20.$ *a) The seed island decays away rapidly in the NTM branch b) The RWM branch shows that the island is growing even below the NTM threshold width.*