Simulation of the W7-X 140 GHz Gyrotron Resonator with an explicit 3D Discontinuous Galerkin Method based Particle-In-Cell Scheme

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Numerical simulation of gyrotron resonators is an essential design tool since the 1990's where state-of-the-art fast codes such as SELFT [7] or MAGY [3] have been developed. Also much slower Particle-In-Cell (PIC) codes have been developed and used for the simulation of resonant cavities [5, 2]. While the fast codes procure their rapidity by using basic simplifications, e.g. a pre-selected mode spectrum and assumed invariance of certain physical parameters, PIC codes simulate the entire self-consistent Vlasov-Maxwell system describing the complex particle-field interaction [1]. Therefore PIC codes where too slow to be used in the design process. Growing computational capabilities and new high-order Discontinuous-Galerkin (DG) based PIC methods [6] nowadays allows for highly parallelized PIC codes to simulate resonator cavities in 3D and without significant physical simplifications. First results on a 30 GHz resonator revealed new insights to the physics and demonstrated the relevance of these methods to research and for design issues [8]. In this paper we intend to simulate the 140 GHz 1 MW continuous wave gyrotron aimed for the W7-X Stellerator [4] with a 3D transient high-order DG based PIC method [8] and to compare the results to those achieved with SELFT [7].

Simulation Setup

Due to the high computational effort of the DG-PIC simulations we have to use SELFT in preparatory to get a reasonable setup. Initially we start with a soft excitation (SE) setup in order to excite a $TE_{28,8}$ mode by exceeding its starting current. After a short startup phase we switch to a hard excitation (HE) setup, where the mode's starting current exceeds the

Symbol	[unit]	Soft Excitation	Hard Excitation
E_p	[keV]	81.25	-
α		1.3	-
γ	[-]	1.159	-
B_z	[T]	5.587	5.537 ($\downarrow$)
r_g	[mm]	0.14168	0.14296 ($\uparrow$)
r_b	[mm]	10.1	-
I	[A]	44	40 ($\downarrow$)

Tab. 1: Physical parameters.

actual operating current, in order to raise the efficiency and the power output to a level that is desired in continuous operation mode of the 140 GHz gyrotron. Table 1. summarizes the chosen parameters for the different setups where E_p is the electron's energy, α the ratio of perpendicular to parallel electron velocity, γ the relativistic factor, B_z the static
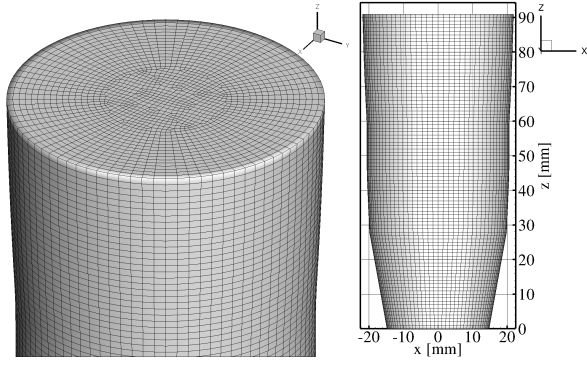
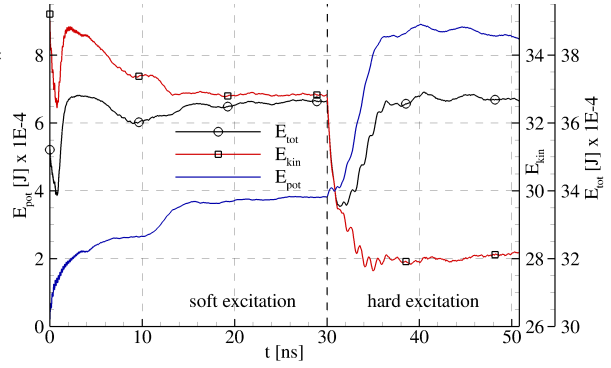Fig. 1: Computational grid with 163436 hexahedral elements.

Fig. 2: Integral system energies over time. Left scale: E_{pot}=Energy stored in the RF field, right scale: E_{kin}=energy of the electron beam, E_{tot}=total stored energy.

magnetic field, r_g the Larmor radius, r_b the radius of the hollow electron beam and I the beam current. The simulation domain covers the complete resonant cavity including uptaper and downtaper. In total a 91 mm long section around the resonant cavity has been discretized with 163436 hexahedral elements with a 5th-order DG space discretization leading to $\approx$ 16 million degrees of freedom, shown in figure 1. The computations were performed on the CRAY-Hermit cluster of the high-performance computing centre in Stuttgart (HLRS) and on the JUROPA-HPC-FF cluster of the Jülich supercomputing centre (JSC) (both in Germany) with 512 MPI-parallel processes. One nanosecond simulation time required approximately 4.5 hours of computational time leading to a total computation time of 10 days (for 50 ns).

DG-PIC Results

Introductory we discuss the integral energy quantities, shown in figure 2 over a period of 50 ns. At 30 ns the operation parameters are switched from SE to HE. After a short initial phase until 15 ns a TE$_{28,8}$ mode prevails and a steady operation point is reached in the SE setup. After switching to the HE setup at 30 ns the kinetic energy drops drastically while the potential field energy raises in the magnitude. This effect was aimed for the HE
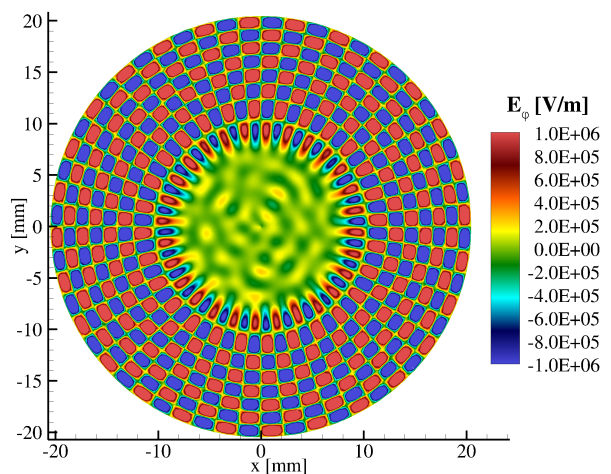

Fig. 3: E_φ in x-y plane at z=51 mm (t=50 ns).

setup. Figure 3 shows a slice in the x-y-plane at z=51 mm with the E_φ-field at t=50 ns. Here, a TE$_{28,8}$ mode pattern is clearly visible. In contrast to experimental measurements we are able to

record the EM field-strength at arbitrary points in the domain to perform a frequency spectrum analysis. We introduce a record point at $(x,y,z) = (11.5, 0, 50)$ mm in the resonator. Figure 4 shows the frequency spectrum by a discrete FFT at the record point for a time window of 20 ns in HE setup. Clearly the 140 GHz frequency peak associated with the TE$_{28,8}$ mode is visible. The second harmonic of this oscillation at 280 GHz can also be found. Some lower peaks at other frequencies indicate that we have parasitic oscillations at lower powers. To compute the output
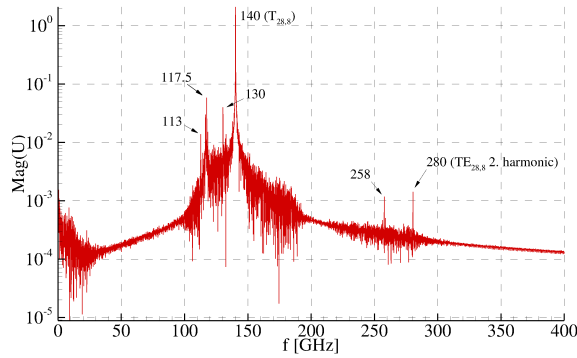

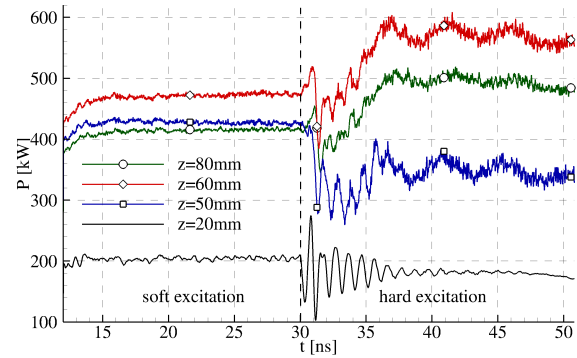Fig. 4: Frequency spectrum in the resonator recorded at (x,y,z)=(11.5,0,50) mm for t=30 − 50 ns.

Fig. 5: Power over time by Poynting vector integral over xy-plane slices at different z-axis points.

power in the EM field, we use a surface integral of the Poynting vector over a defined slice C in the x-y-plane at a certain z-coordinate, i.e. $P(z,t) = \int_C \mathbf{n} \cdot \mathbf{S} \, da = \frac{1}{\mu_0} \int_C (E_x B_y - E_y B_x) \, da$. Figure 5 shows the temporal evolution of the power in the EM field at four different slices in the x-y-plane versus time. A similar qualitative behavior as for the integral energies shown in figure 2 is visible. At the output port (z=80 mm) a power of approximately 500 kW can be calculated.
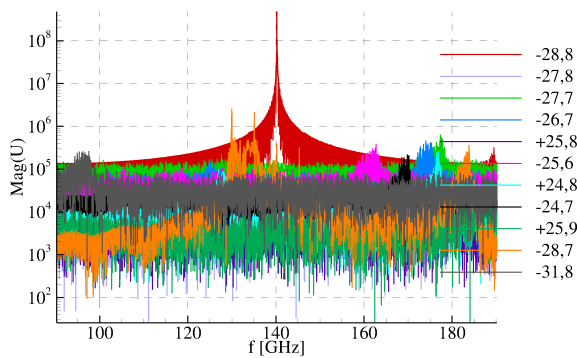

Fig. 6: SELFT power spectrum at t=90 ns of the relevant TE modes included in the simulation.

Fig. 7: Power in selected modes vs. time calculated with SELFT. U, I and B_z are the beam parameters.

SELFT-Results

We computed the 140 GHz resonator with the same parameter setup with the SELFT code as well. SELFT is a fast simulation code for gyrotron interaction processes, based on a slow
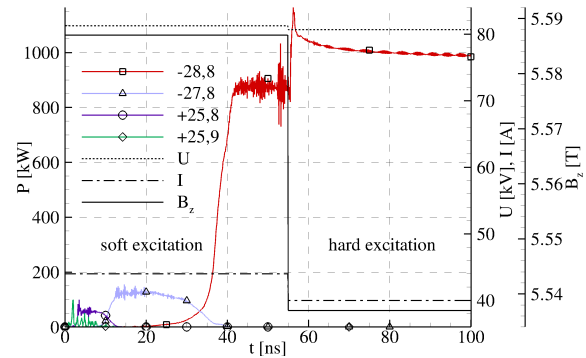
variables formalism [7]. It calculates the electron trajectories and the field profiles of selected relevant modes self-consistently over time, and is thus capable of simulating complete startup sequences of gyrotrons. In contrast to the full wave solver implemented in DG-PIC, SELFT uses a preselected mode spectrum of 25 different TE-modes. Figure 7 shows the output power of the most relevant modes over time. Here, the switch from HE to SE is shifted from 30 ns to 55 ns because the simulation reaches stationarity only after 30 ns. The $TE_{28,8}$ mode prevails much later as dominant frequency compared to the DG-PIC approach. This is probably caused by a difference in the output boundary condition: While SELFT imposes a continuous waveguide boundary condition, HALO implements an irradiating open end, which actually introduces a non-zero output reflection factor. This additional reflection usually enhances and accelerates the oscillation processes. In the startup phase for a short period a $TE_{27,8}$ mode is dominant and vanishes later. The output port has a power of 1000 kW. The presented DG-PIC method shows a good agreement in the predicted operating parameters for the soft and hard excitation region and for the dominating line in the predicted oscillation frequency spectrum (shown in figure 6 for SELFT), while some spurious oscillations at 113 and 117.5 GHz may again be enhanced in the DG-PIC simulation by the artificial output reflections.

Conclusions

Through first simulations, the feasibility of simulating highly oversized gyrotron resonators over physically relevant time frames with a high-order DG-PIC code was demonstrated. While special questions like the appropriate implementation of dedicated boundary conditions still have to be solved, the way towards a simulation tool free of the otherwise necessary approximations and model limitations is shown.

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