Three dimensional tokamak equilibrium and stability for MAST-like plasmas with external magnetic perturbations applied for ELM control

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Motivation

The Edge-Localized Mode (ELM) is a repetitive instability, thought to be related to peeling-ballooning modes [1], that occurs at the edge of tokamak plasmas in high confinement mode (H-mode). ELMs need to be suppressed or mitigated (frequency of ELMs increased and peak heat flux decreased) in ITER to prevent significant damage to the divertor region [2]. There are several methods for controlling ELMs in tokamaks including pellet pacing, vertical kicks and external, resonant magnetic perturbations (RMPs). We focus on ELM control using RMPs which has been demonstrated on several tokamaks [3], including MAST. ELMs on MAST have been mitigated with RMPs with a toroidal mode number of n=3, 4 and 6 [3] using the 18 in-vessel ELM control coils, six coils in a row above the mid-plane and 12 below [4]. Factors that influence ELM control with RMPs include non-axisymmetric plasma geometry, screening of the applied field, and pedestal evolution. We focus on three dimensional corrugation of the plasma edge [5], and ideal infinite n ballooning stability here.

Equilibrium

We generally assume tokamak equilibria are axisymmetric which allows the derivation of the Grad-Shafranov equation. The presence of the symmetry gives a conserved quantity in the system so we have nested flux surfaces and well defined flux coordinate systems. However, the application of RMPs breaks axisymmetry and there is no equivalent of the Grad-Shafranov equation in general. Non-axisymmetric effects have been seen in tokamak plasmas for a long time in the form of saturated instabilities such as neoclassical tearing modes and more recently the helical core or long lived mode in MAST [6,7].

VMEC [8] is a code which is in very wide use in the stellarator community for calculating non-axisymmetric equilibria. It minimizes the total energy $W = \int\left(\frac{|B^2|}{2\mu_0} + \frac{p}{\gamma - 1}\right)d^3x,$ where B is the magnetic field, p is the plasma pressure and γ is the ratio of specific heats, to find equilibrium states. VMEC makes the assumption that there are nested flux surfaces which means that islands or ergodic regions cannot form in the equilibria it calculates. Other codes do allow islands and ergodic regions, such as PIES, HINT2 and SPEC [9], and these each have different constraints.

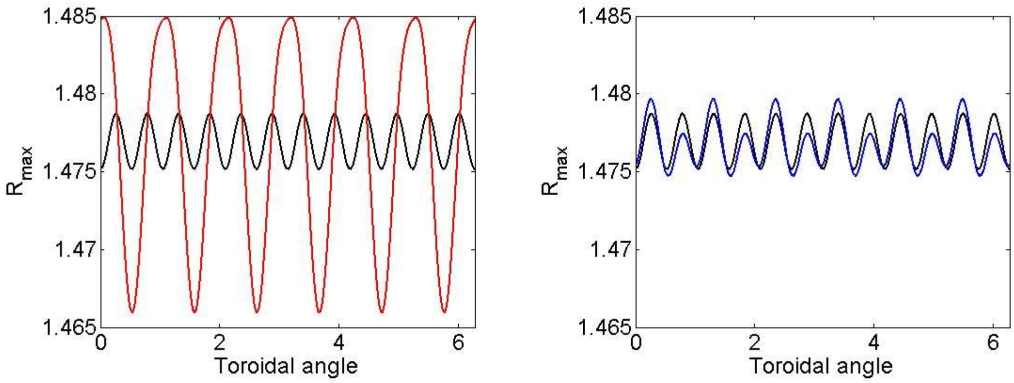

Fig.1: The mid-plane radius with toroidal angle with n=6 odd parity (red left) and even parity (blue right). The black curves show the result with no RMP applied; the n=12 ripple is due to the toroidal field coils.

We consider two MAST-like equilibria here, one connected double null and the other lower single null. The profiles used are based on MAST H-mode plasmas which include a calculated bootstrap current. VMEC was run without the assumption of stellarator symmetry with 97 radial grid points. The results of even and odd parity of the ELM coils with n=6 is shown in Fig. 1. It can be seen that the odd parity is in resonance in this case. Previous work has shown that the phase and size of the corrugations is in agreement with experiment [10].

The current applied for the connected double null case with n=6 odd parity was increased in steps of 1.4kA up to 5.6kA. The mid-plane displacement scaled linearly with this applied current, see Fig. 2. Experimentally, a threshold in current is observed before any displacement is seen at the X-points [11]. This is due to the plasma screening the applied field which is not reproduced by VMEC. In the lower single null case we find that the lower coils have the greatest effect on the plasma due to their proximity to the plasma boundary. The mid-plane displacements are of the order of 1cm at full current which is in broad agreement with experimental evidence.

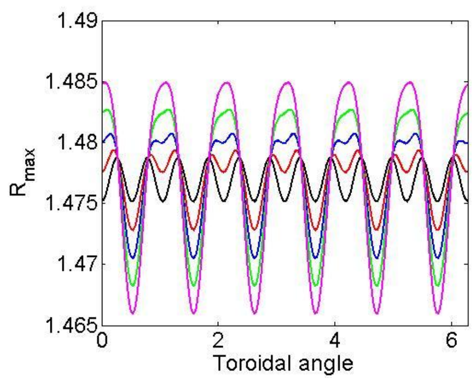

Fig.2: The mid-plane displacement with toroidal angle is shown with increasing current in the ELM coils. Black (no current), red (1.4kA), blue (2.8kA), green (4.2kA) and pink (5.6kA). The displacement scales linearly with the applied current.

The VMEC equilibria here assume nested flux surfaces and no islands or ergodic regions. However, if RMPs have been applied axisymmetry is broken and this is unlikely to be true. In particular, helical Pfirsch-Schlüter currents will appear on rational surfaces. MHD force balance implies that $\nabla \cdot J_{\perp} = (B \times \nabla p) \cdot \nabla\left(\frac{1}{B^2}\right) \neq 0$, where J is the current density. If we allow non-axisymmetric fields to penetrate the plasma we can Fourier decompose the magnetic field so that $\frac{1}{B^2} = \sum_{m,n} h_{mn}(\psi) e^{i(m\theta - n\varphi)}$, where m is the poloidal mode number and θ, φ are the poloidal and toroidal angles respectively. This results in a singular parallel current at the rational surface, $J_{\parallel} \sim \frac{h_{mn}}{\psi - \psi_{mn}} p'$, where ψ_{mn} is the location of the rational surface. This singularity indicates a breakdown in the model. Boozer gave the hypothesis that there would be enhanced transport at the rational surface meaning that the pressure would be flattened thus removing the singularity [12]. We have a SPEC [9] equilibrium for the connected double null case with n=6 perturbations applied. Fig.3 shows the Poincare plot for this run. This shows nested flux surfaces in the core of the plasma but island structures and ergodic regions closer to the edge.

Stability

The infinite n ideal ballooning stability of the equilibria with and without the RMPs applied has been calculated using COBRA [13]. When RMP coils are applied the most unstable ballooning mode growth rate increased, see Fig. 4. However, with RMPs applied the ballooning mode growth rate has structure in the toroidal direction which it did not have without RMPs applied. This is due to the change in curvature of the magnetic field lines caused by the RMPs.

Bird and Hegna used a local equilibrium model to investigate the effect of helical perturbations on the stability of the plasma. In their analysis they found the Pfirsch-Schlüter currents near to rational surfaces [14] and they found that these currents made the most unstable infinite n ballooning modes more unstable.

Conclusions

RMPs produce mid-plane displacements and Pfirsch-Schlüter currents at rational surfaces. Both of these effects result in the

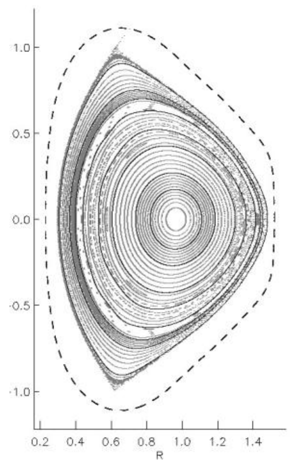

Fig. 3: Poincare plot of the magnetic field calculated by SPEC [9].

most unstable ideal infinite *n* ballooning modes becoming more unstable. Given that the ballooning mode growth rate does not increase more around the rational surfaces, shown in Fig. 4, we conclude that the change in the curvature of the magnetic field lines is the key factor changing the ballooning mode growth rate here.

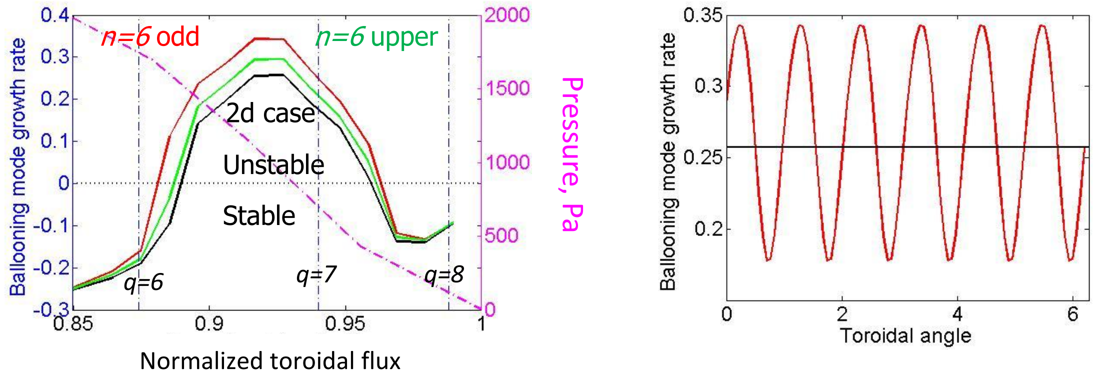

Fig. 4 Ballooning mode growth rate of the most unstable mode calculated by COBRA for the edge plasma is shown (left plot) for the axisymmetric case (black), n=6 upper coils (green) and n=6 odd (red). The edge pressure profile is shown (pink dashed) with values on the right axis. Ballooning mode growth rate is shown against toroidal angle for normalized toroidal flux of 0.92 (right plot).

ELMs are related to peeling-ballooning stability and this indicates these modes will become more unstable with RMPs applied. This may explain ELM mitigation. Infinite *n* ballooning modes may also be used as a proxy for kinetic ballooning modes which are believed to limit the pressure gradient in the pedestal. This mechanism may lower the pedestal pressure gradient. The RMPs are assumed to have fully penetrated in VMEC; there is no screening.

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