Effects of ion trapping on the nonlinear evolution of drift turbulence

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The aim of this paper is to contribute to the understanding of the effects of trajectory eddying (trapping) in the structure of the stochastic potential on the evolution of drift turbulence. These are the first analytical results on this complex problem that are in agreement with numerical simulations. A Lagrangian approach is developed [1], which extends the type of methods initiated by Dupree [2] to the nonlinear regime characterized by trapping.

Test modes on turbulent plasmas were studied for drift turbulence in constant magnetic field starting from the basic description provided by the drift kinetic equations. Analytical expressions are derived, which approximate the growth rates γ and the frequencies ω of the test modes as functions of the characteristics of the background turbulence. They provide the tendencies in turbulence evolution. A different perspective on important aspects of the physics of drift type turbulence in the strongly non-linear regime (as zonal flow mode generation) is deduced. The main role in these processes is shown to be played by the ion stochastic trapping or eddying.

We consider the (universal) drift instability described by the drift kinetic equations in the collisionless limit. Electron kinetic effects produce the dissipation mechanism to release the energy and, combined with the ion polarization drift, make drift waves unstable. Beside this, the polarization drift has a more complex influence determined by its nonzero divergence, which produces compressibility effects in the background turbulence.

The dispersion relation of the test modes in turbulent plasma is shown to be the same as in quiescent plasma, except for a time dependent function $M(\tau)$, which embeds all the effects of the background turbulence:

$$-i\left(k_y V_{*e} - \omega \rho_s^2 k_\perp^2\right)\int_{-\infty}^{t} d\tau\; M(\tau) \exp\left[-i\omega(\tau - t)\right] = 1 + i\sqrt{\frac{\pi}{2}}\,\frac{\omega - k_y V_{*e}}{|k_z| v_{Te}} \tag{1}$$

$$M(\tau) \equiv \left\langle \exp\left[i\mathbf{k}\cdot\left(\mathbf{x}(\tau) - \mathbf{x}\right) - \int_{\tau}^{t} d\tau' \,\nabla\cdot\mathbf{u}_p\left(\mathbf{x}(\tau')\right)\right]\right\rangle. \tag{2}$$

$V_{*e} = T_e /(eBL_n) = \rho_s c_s / L_n$ is the diamagnetic velocity produced by the density gradient (taken along the x direction), $\rho_s = c_s / \Omega_i$, $c_s = \sqrt{T_e / mi}$, T_e is the electron temperature, m_i is the ion mass, e is the absolute value of electron charge, $\Omega_i = eB / m_i$ is the cyclotron frequency and $k_\perp = \sqrt{k_x^2 + k_y^2}$ is the perpendicular wave number.

The function $M(\tau)$ depends on the background potential through the statistical average on the trajectories denoted by $\langle \; \rangle$, and also through the compressibility term determined by the polarization drift

$$\mathbf{u}_p = \frac{m_i}{eB^2} \partial_t \mathbf{E}_\perp . \tag{3}$$

The trajectories (characteristics of the drift kinetic equation) are solutions of

$$\frac{d\mathbf{x}(\tau)}{d\tau} = -\frac{\nabla \phi(\mathbf{x} - \mathbf{V}_{*e} t) \times \mathbf{e}_z}{B}, \tag{4}$$

calculated backwards in time with the condition at $\tau = t$, $\mathbf{x}(t) = \mathbf{x}$.

In the case of quiescent plasmas $M = 1$.

This function and its evolution is estimated using semi-analytical statistical methods (the decorrelation trajectory method DTM [3] and the nested subensemble approach NSA [4]), which were developed for the study of test particle stochastic advection. Essentially, DTM and NSA reduce the problem of determining the statistics of the stochastic trajectories to the calculation of weighted averages of some smooth, deterministic trajectories determined from the Eulerian correlation (EC) of the stochastic potential. These methods are in agreement with the statistical consequences of the invariance of the potential so that they are able to describe trajectory trapping. We have shown that trapping determines memory effects, anomalous diffusion regimes, quasi-coherent behavior and non-Gaussian distribution. The trapped trajectories form quasi-coherent structures similar to fluid vortices.

The EC of the background potential is initially modelled according to the frequencies and the growth rates of drift modes in quiescent plasma and then it is changed as required by the modification of these quantities in the evolution of the turbulent plasma. The distribution of displacements strongly depends on the ordering of the main characteristic times of the stochastic process: the diamagnetic time $\tau_* = \lambda_y / V_{*e}$ defined by the motion of the potential along y with the diamagnetic velocity, and the time of flight (or eddying time) $\tau_{fl} = \lambda_y / V_y$. The correlation time of the potential τ_c, which is the characteristic time for the change of the shape of the potential, is of the order $\tau_c \cong \gamma^{-1}$, and it is larger than the diamagnetic time ($\tau_c > \tau_*$). The trapping parameter for drift turbulence is $K_* = \tau_* / \tau_{fl} = V_y / V_{*e}$. Trajectory trapping exists when $K_* > 1$. The fraction of trapped trajectories n_{tr} and the maximum size of trajectory structures are increasing functions of K_*

Drift turbulence develops in the initial stage on a wide range of wave numbers. Ion trajectories are not trapped at these small amplitudes of the background turbulence and they have Gaussian distribution. Their diffusion determines the damping of the large $\mathbf{k}$ modes. The correlation lengths ρ_i and the inverse of the dominant wave number k_0 remain during this stage close to ρ_s. Turbulence amplitude β increases (with the largest growth rate) and the shape of the EC is not changed.

When the amplitude reaches values that make $K_* > 1$, ion trajectory trapping appears and generates vortical structures of trapped ions. They determine the decrease of the frequencies, which leads to the decrease of γ and to the displacement of the unstable range of wave numbers to small values of the order $1/s$, where s is the average size of trajectory structures. Turbulence amplitude continues to increase in this stage, but with a smaller rate, and large scale correlations are generated.

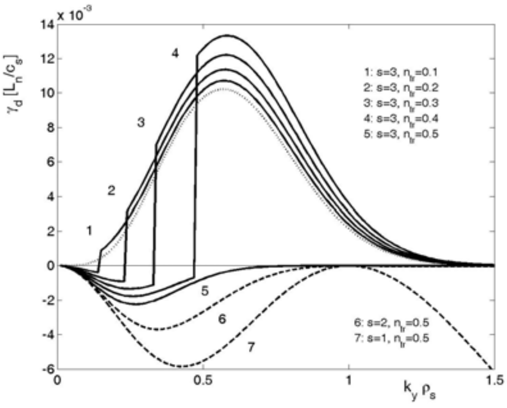

Figure 1. The growth rates of the drift modes in the strongly nonlinear regime characterized by ion flows

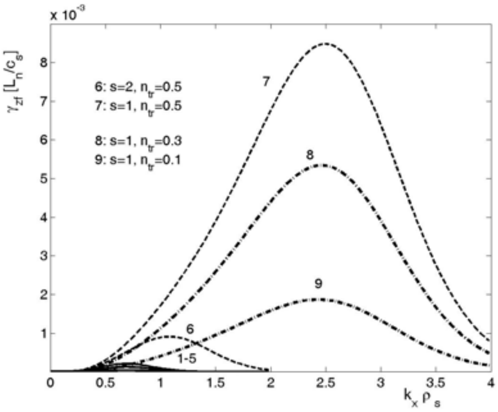

Figure 2. The growth rates of zonal flow modes in the same condition as in Figure 1. and for the same parameters

When the fraction of trapped ions n_{tr} becomes comparable with the fraction of free ions n_f, ion zonal flows are generated by the trapping process. Trapped ions move with the potential, while free ions move in the opposite direction with the velocity $V_f = -V_{*e} n_{tr} / n_f$ such that the total flux is zero. This determines the splitting of the distribution of ion displacement and an essential change of turbulence, which consists of two effects: drift mode attenuation and generation of zonal flow modes. The attenuation of the drift modes is determined by the ion flows. As seen in Figure 1, it begins with the damping of small k modes and it extends to the entire spectrum as n_{tr} increases (the growth rates are negative for the entire range of $\mathbf{k}$ at $n_{tr} = 1/2$.) A new type of modes appear in this strongly nonlinear regime, the zonal flow modes (Fig. 2) with $k_y = 0$ and very small frequencies. They are produced by the combined action of the ion flows and of the compressibility due to the polarization drift in the background turbulence. The damping of the drift modes is not determined by the zonal flows. There is only an indirect contribution through the diffusive damping, which is increased by the zonal flow modes. They modify the correlation of the turbulence by introducing components with $k_y = 0$ in the spectrum, which determine a strong increase of D_y. The decay of the drift turbulence determines the decrease of n_{tr} and of the growth rate of the zonal flow modes, which is proportional to n_{tr}.

The growth and the decay of turbulence are produced on different paths (hysteresis process). Large scales are generated at the increase of turbulence amplitude, when trapping is weak. Later in the nonlinear evolution, when trapping is stronger and produces ion flows, turbulence amplitude continues to increase, but it is accompanied by the decrease of the correlation length and by the generation of zonal flow modes. A closed evolution curve in the (β, λ) space is described by the turbulence, which remains in the nonlinear stage characterized by trapping and oscillates between weak and strong trapping. The characteristic time Δt for turbulence and transport oscillations estimated as the inverse of the growth rates is in agreement with numerical simulations.

The predator-prey paradigm is not sustained by these results, although there is time correlation between the growth of zonal flow modes and the damping of the drift modes.

In conclusion, this first principle semi-analytical approach is able to describe the complex evolution of drift turbulence and yields results in agreement with numerical simulations.

1. Vlad M., Physical Review E 87 053105 (2013).
2. Dupree T. H., Phys. Fluids 15, 334 (1972).
3. Vlad M., Spineanu F., Misguich J.H., Balescu R., Physical Review E 58 (1998) 7359.
4. Vlad M. and Spineanu F., *Phys. Rev. E* **70**, 056304 (2004).