Vlasov simulation of the parametric instability of ion acoustic waves

T. Chapman[1], R. L. Berger[1], B. I. Cohen[1], E. A. Williams[1], and S. Brunner[2]

[1] *Lawrence Livermore National Laboratory, P.O. Box 808, Livermore, CA 94551, USA*

[2] *Centre de Recherches en Physique des Plasmas, Association EURATOM-Confédération Suisse, Ecole Polytechnique Fédéral de Lausanne, CH-1015 Lausanne, Switzerland*

Abstract

The Ion Acoustic Wave (IAW) decay instability has been shown in theoretical studies to be an important saturation mechanism for Stimulated Brillouin Scatter (SBS) [1, 2, 3], possibly relevant to Inertial Confinement Fusion (ICF) experiments [4]. Previous detailed studies [2, 3] have used Particle-In-Cell (PIC) simulations to explore the decay process. These studies were limited by low resolution in k-space (8 simulated fundamental wavelengths) and the inherently high noise of PIC methods which may seed instabilities at a level such that the nature of the instability is obscured. We present results of a study of the 1D parametric instability of IAWs using the 1D1V Vlasov code SAPRISTI [5].

Utilizing extremely low noise (numerical double precision) and high resolution in $k-$space (64 simulated fundamental wavelengths), we show for the first time the fine structure of the growth of IAWs below the fundamental wave and indeed between each of its harmonics up to the 64^{th} harmonic. Theoretical work [6] has previously suggested that the nature of the instability should be sensitive to the sign of the mismatch in frequency between the pump IAW and its decay waves arising due to the nonlinear dispersion of IAWs and nonlinear frequency shifts (both fluid and kinetic). This sensitivity is investigated by means of varying ZT_e/T_i over a broad range of values.

Results and discussion

The interaction of particles resonant with a plasma wave in a Maxwellian distribution may rapidly deplete the wave field energy via Landau damping. In a one-dimensional (1D) periodic system, IAWs may evolve into an indefinitely-stable Bernstein-Green-Kruskal (BGK)-like mode [7] via trapping of electrons [8] and ions with velocities near the phase velocity of the wave, v_ϕ, in both single [5] and multi-species plasmas [9]. In a 1D periodic system of length L equal to the IAW wavelength λ, this wave is stable to the growth of sidebands via the Trapped Particle Instability (TPI) [10] and decay into longer wavelengths via Two Ion-wave Decay (TID). However, for system lengths greater than λ, the wave may become unstable.

Using results obtained from SAPRISTI [5], this phenomenon is demonstrated in Fig. 1: A prescribed ponderomotive driver is applied to the electrons to excite an IAW in a periodic system. After the desired amplitude is obtained, the driver is switched off and the wave allowed to propagate freely. In Fig. 1, the transition from stability to instability as L is increased is evident. The evolution in time of the field energy of the wave converges quickly as λ is increased. In this figure and unless explicitly stated otherwise, $(k\lambda_{De})^2 = 0.1$ and $ZT_e/T_i = 11$, where λ_{De} is the electron Debye length, Z is the ion charge number, and T_e and T_i are the electron and ion temperatures, respectively. The electric field E is normalised such that $\tilde{E} = e\lambda_{De}E/T_e$ and the potential ϕ such that $\tilde{E} = -\lambda_{De}\nabla\tilde{\phi}$.

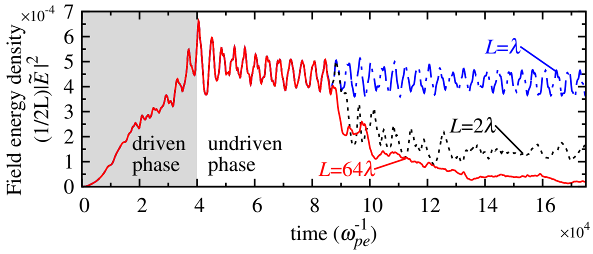

Figure 1: *Sensitivity of the stability of the wave to the system length. The oscillations are related to the bounce frequency of trapped particles.*

Three cases are presented in Fig. 2, differing only in the amplitude to which the IAW was driven before the driver was switched off. In each, $L = 64\lambda$. Analysis of the Fourier k spectrum shows the rapid establishment of the harmonic spectrum in each case, which we label as k_n, $n = 1, 2, \ldots,$ where $n = 1$ is the 1st harmonic (the range in Fig. 2 is restricted to $0 < k\lambda_{De} < 0.66$). A threshold for instability is observed in the potential amplitude $\tilde{\phi}$: below $\tilde{\phi} \sim 0.01$, no subharmonic growth is observed (see Fig. 2a), while above this threshold, the wave is unstable (see Figs. 2b,c). In all cases where subharmonic growth is observed, the fastest growing wave number lies at or close to $k_{n-1/2}$. Growth up to $n = 64$ is observed (spatial resolution $\Delta x = \lambda/128$), and the growth rate is constant to good approximation across all $k_{n-1/2}$ (see Fig. 3a).

Preferred decay to $k_{1/2}$ is an established signature feature of TID in experiments, simulations and theoretical treatments, distinct from TPI where k of the fastest growing mode (and its associated growth rate γ_k) is a function of ϕ [10, 11]. In the 3-wave fluid model of TID elaborated

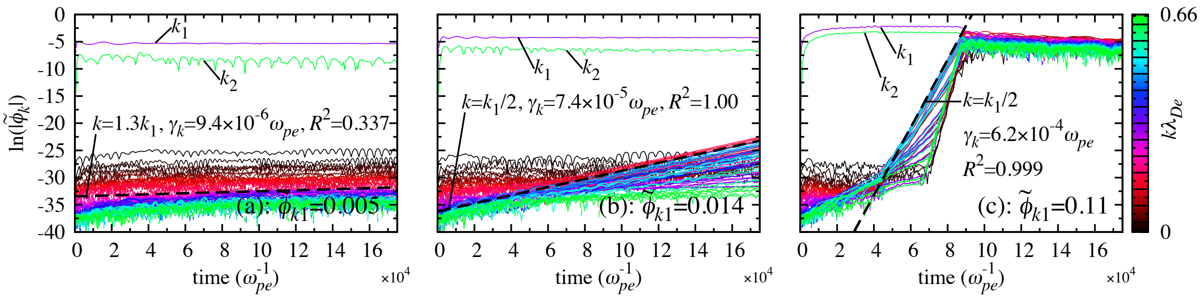

Figure 2: *Evolution of the wave Fourier content (a) just below, (b) just above, and (c) significantly above the potential amplitude threshold for the TID instability.*

by Karttunen *et al.* [1] in which the dispersion of the IAW is linear and nonlinear frequency shifts due to trapping or harmonic generation are ignored (the simplest possible model of TID), a threshold for TID is expected which scales with the square root of the product of the damping rates of the daughter waves.

Assuming applicability of this simple model and using the calculated linear values of Landau damping, one finds a calculated threshold of $\tilde{\phi} \sim 0.025$, significantly above what is observed in Vlasov simulations. Before coupling is established (at which point ω and k of the daughter waves are fixed by conservation of energy and momentum and their phase velocities are identical to that of the first harmonic, v_ϕ), the phase velocities of the daughter waves are determined by the IAW dispersion relation but lie close to v_ϕ, since $(k_1\lambda_{De})^2 = 0.1$ and the dispersion relation is near linear.

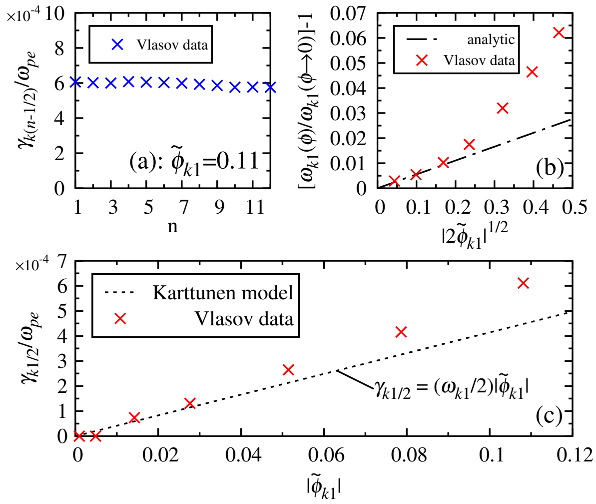

Figure 3: *(a) Growth rate as a function of harmonic index n. (b) Nonlinear frequency and (c) growth rate of the $k_{1/2}$ mode as a function of ϕ_{k1}.*

Consequently, a flattening in the distribution function near v_ϕ due to the comparatively large fundamental wave amplitude should reduce also the Landau damping rate of the daughter waves before and after coupling to the fundamental IAW. The calculated half-widths in velocity of the flattened region of the distribution functions for electrons and ions are given by $\Delta v_j/v_{tj} = 2|q_j\phi_{k1}/T_j|^{1/2}$, where q_j and T_j are the charge and temperature of species j ($j = e$ or i). At $\tilde{\phi} = 0.005$, $\Delta v_e/v_{te} = 0.14$ and $\Delta v_j/v_{te} = 3.2\times10^{-3}$. These values are significantly greater than the offset of $v_\phi^{k1/2}$ from v_ϕ^k when not assuming linear dispersion, for which one finds $\Delta v_\phi = v_\phi^{k1/2} - v_\phi^k = 6.4\times10^{-4}\, v_{te}$. A reduction of the threshold from the value obtained using the model assuming linear Landau damping is therefore expected in this case.

The nonlinear frequency shift of the IAW is shown as a function of ϕ in Fig. 3b. The analytic curve is obtained by calculating the shift due to trapping in the limit of adiabatic electron and ion excitation [12, 5]. In this regime, the positive electron contribution to the nonlinear frequency shift is greater than the negative ion contribution, thus the frequency shift is positive for all ϕ. At higher ϕ, harmonics increase the nonlinearity of the mode both directly, via quadratically coupling harmonics, and indirectly through modifying the shape of the potential that traps particles [5]. The growth rate $\gamma_{k1/2}$ of the $k_{1/2}$ mode (although, as discussed previously, this rate is constant to good approximation across $k_{n-1/2}$) is shown as a function of ϕ in Fig. 3c. The linear scaling of the growth rate with ϕ, in addition to the close agreement with the 3-wave TID model elaborated by Karttunen *et al.* [1], is clear evidence that subharmonic growth in this regime should be attributed to TID (rather than, for example, TPI). The discrepancy between the 3-wave model and the simulation results coincides with the growth of the nonlinear frequency shift in Fig. 3b, but also harmonics. Results across $ZT_e/T_i \lesssim 50$ exhibit remarkably little sensitivity to ZT_e/T_i; the near-linear scaling of γ_k with ϕ is unchanged even at $ZT_e/T_i = 4$, where the ions provide the dominant contribution to an overall *negative* nonlinear frequency shift. However, at $ZT_e/T_i = 50$, the growth rate appears to scale as ϕ^m where $m \sim 2$, consistent with the so-called H2-instability derived by Pesme *et al.* [6], while still maximal very near $k_{n-1/2}$.

The application of a Vlasov code with a high number of simulated wavelengths has allowed for new insights into the nature of the TID instability in 1D and, for the first time, precise measurement of the associated growth rates. This instability has previously been shown in PIC simulations to occur more readily in two-dimensional systems due to more easily achieved matching conditions between the fundamental and decay waves [2]. Further study into the nature of the decay instability in two dimensions using the 2D+2V Vlasove code LOKI will follow.

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References

[1] S. J. Karttunen, J. N. McMullin, and A. A. Offenberger, Phys. Fluids **24**, 447 (1981).

[2] B. I. Cohen *et al.*, Phys. Plasmas **4**, 956 (1997); Phys. Plasmas **16**, 032701 (2009).

[3] C. Riconda *et al.*, Phys. Rev. Lett. **94**, 055003 (2005); Phys. Plasmas **12**, 112308 (2005).

[4] N. B. Meezan *et al.*, Phys. Plasmas **17**, 056304 (2010).

[5] R. L. Berger *et al.*, Phys. Plasmas **20**, 032107 (2013).

[6] D. Pesme, C. Riconda, and V. T. Tikhonchuk, Phys. Plasmas **12**, 092101 (2005).

[7] I. B. Bernstein, J. M. Greene, and M. D. Kruskal, Phys. Rev. **108**, 546 (1957).

[8] T. M. O'Neil, Phys. Fluids **8**, 2255 (1965).

[9] T. Chapman *et al.*, Phys. Rev. Lett. **110**, 195004 (2013).

[10] W. L. Kruer, J. M. Dawson, and R. N. Sudan, Phys. Rev. Lett. **23**, 838 (1969).

[11] R. L. Dewar, W. L. Kruer, and W. M. Manheimer, Phys. Rev. Lett. **28**, 215 (1972).

[12] R. L. Dewar, Phys. Fluids **15**, 712 (1972).