Full-f gyrokinetic simulation over a confinement time

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Introduction

In recent years, new generation of gyrokinetic simulations based on the so-called full-f approaches are emerging. In full-f gyrokinetic simulations, both turbulent transport and profile formations are computed under fixed power, momentum, and particle inputs as in the experiment. This approach, in principle, has the capability of dictating plasma profiles, provided that the simulation is performed over a confinement time. In fact, recent Peta-scale supercomputers made such long time scale simulations feasible. However, before proceeding to huge computation, one needs to examine the accuracy of physical and numerical models. In particular, recent work [1] pointed out a serious concern about the accuracy of momentum transport calculations using 1st order gyrokinetics. In order to resolve this critical issue, we have implemented 3rd order gyrokinetics [2] in the Gyrokinetic Toroidal 5D Eulerian code GT5D [3, 4], and the quantitative convergence of turbulent momentum transport with respect to gyrokinetic ordering is examined in the ion temperature gradient driven (ITG) turbulence [5]. After verifying the accuracy issue in basic equations, the convergence of steady temperature and rotation profile is confirmed with respect to time, and mechanisms to sustain the steady profiles are investigated, in particular, focusing on momentum transport.

Convergence study of momentum transport using 3rd order gyrokinetics

GT5D is based on modern gyrokinetic theory, and solves the following gyrokinetic equations,

$$\frac{\partial f}{\partial t} + \{f,H\} = \frac{\partial f}{\partial t} + \dot{\mathbf{R}} \cdot \frac{\partial f}{\partial \mathbf{R}} + \dot{v_\parallel} \frac{\partial f}{\partial v_\parallel} = C(f) + S, \tag{1}$$

$$\frac{en_0}{T_e}\left(\phi - \langle\phi\rangle_f\right) = \delta n_i + n_p, \tag{2}$$

where $\{,\}$ is the Poisson bracket operator, H is the gyro-center Hamiltonian, $C(f)$ is a collision operator, S is a source term. In the quasineutrality condition (2), the lhs denotes the adiabatic electron density, δn_i is the perturbed gyro-center ion density, and n_p is the ion polarization density. In the 3rd order gyrokinetics [2], H and n_p are consistently derived as

$$H = \frac{1}{2} m_i v_\parallel^2 + \mu B + e\langle\phi\rangle_\alpha - \frac{m_i c^2}{2B^2}(\nabla_\perp \phi)^2 + \frac{e}{2}(\nabla_\perp \phi \cdot \nabla_\perp)\left(\frac{\mathbf{P}_E^2}{2}\right), \tag{3}$$

$$n_p = -\nabla_\perp \cdot (n_i \mathbf{P}_E) - \nabla_\perp \cdot \left\{-\frac{n_i}{2}\left[\mathbf{P}_E(\nabla_\perp \cdot \mathbf{P}_E) - (\mathbf{P}_E \cdot \nabla_\perp)\mathbf{P}_E\right]\right\}, \tag{4}$$

where $\mathbf{P}_E = -m_i c^2/eB^2 \nabla_\perp \phi$ is the polarization vector, $\langle\rangle_f$ is flux-surface-average, $\langle\rangle_\alpha$ is gyro-average, and ϕ is the electrostatic potential. Since the above equation system satisfies so-called energetic consistency [6], the toroidal angular momentum is exactly conserved. By multiplying $P_\varphi = -(e/c)\psi + m_i v_\parallel b_\varphi$ and f, Eq.(1) and the Hamilton's equation, $\dot{P}_\varphi = -\partial_\varphi H$, two balance relations are obtained respectively,

$$\left\langle \frac{\partial m_i v_\parallel b_\varphi f}{\partial t} \right\rangle_{gf} + \left\langle \frac{m_i v_\parallel b_\varphi}{J} \frac{\partial}{\partial \mathbf{Z}} \cdot (J\dot{\mathbf{Z}} f) \right\rangle_{gf} - \langle m_i R v_\parallel b_\varphi S \rangle_{gf} = 0, \tag{5}$$

$$\left\langle -\frac{e}{c} f \dot{\mathbf{R}} \cdot \nabla\psi \right\rangle_{gf} + \left\langle \frac{f}{J} \frac{\partial}{\partial \mathbf{Z}} \cdot (J\dot{\mathbf{Z}} m_i v_\parallel b_\varphi) \right\rangle_{gf} + \left\langle f \frac{\partial H}{\partial \varphi} \right\rangle_{gf} = 0, \tag{6}$$

where J is the Jacobian and $\langle A \rangle_{gf} = \left\langle \int A(\mathbf{Z}) \delta(\mathbf{R} + \rho - \mathbf{x}) d^6 Z \right\rangle_f$. Eqs. (5) and (6) yield the transport equation of the toroidal angular momentum [6]. In Eq.(5), all the ψ terms vanish because of the particle conservation, and the collision term disappears due to its particle and momentum conservation. On the other hand, Eq.(6) is trivially satisfied, when the system keeps the toroidal symmetry or follows the Hamilton's equation. In GT5D, numerical schemes are carefully chosen, so that the discrete system keeps these conservation and symmetry properties, and therefore, the above relations are accurately satisfied in Figs.1(a) and 1(b), where a bipolar flow generation is balanced with the convective terms and the toroidal field term. In Fig. 1(c), the convergence of the parallel momentum flux between 1st and 3rd order gyrokinetics is confirmed. The result shows that unlike the claim in [1], turbulent fluxes are well converged even with 1st order gyrokinetics. It is found that in the ITG turbulence with $k_\perp \rho_i < 1$, higher order corrections to the ion polarization density are negligibly small, and that non-diffusive momentum transport due to residual stress and related intrinsic rotation are an order of magnitude larger than $\sim v_{ti}\rho^*$, which is assumed in the so-called low flow ordering. These corrections to the conventional ordering arguments lead to the convergence of momentum transport.

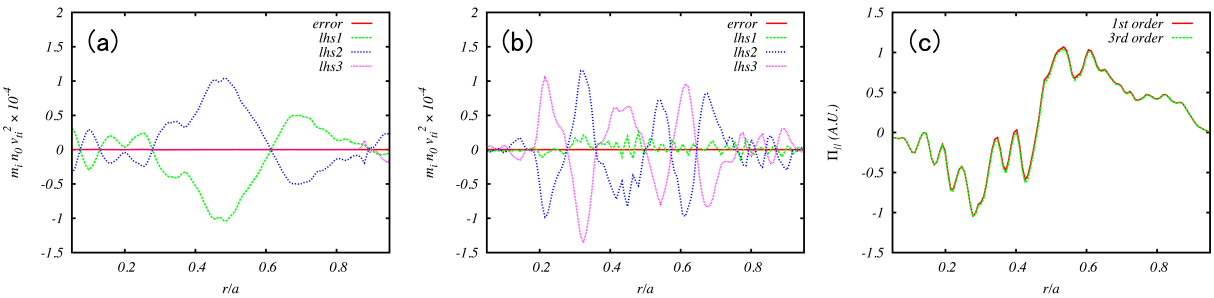

Figure 1: *Toroidal angular momentum conservation in the ITG turbulence with Cyclone parameters. (a) and (b) show balance relations, Eqs. (5) and (6), respectively. (c) shows the convergence of the parallel momentum flux between 1st and 3rd order gyrokinetics. Time average data over* $\sim 400R/v_{ti}$ *is shown.*

Long time ITG turbulence simulations over a confinement time

After verifying the accuracy issue, long time ITG turbulence simulations are performed for a normal shear tokamak with Cyclone like parameters, $\rho^{*-1} \sim 100$, on-axis heating ($P_{in} = 2MW$), and no momentum input. A fixed edge temperature and no slip boundary are imposed by a Krook type sink operator near the plasma surface. In the present parameter, a core stored energy (excluding a part given by the boundary temperature) is $\sim$ 44kJ, and an energy confinement time is $\tau_E \sim$ 22ms, which is an order of magnitude longer than a collision time $\tau_{ii} \sim$ 3ms. In Fig.2(a), the stored energy is almost saturated after $\sim$ 10ms. In Figs.2(b) and 2(c), temperature and (intrinsic) rotation profiles are converged in the quasi-steady phase, while profiles after τ_{ii} are rather close to converged ones. During the whole simulation period, the ITG turbulence show intermittent bursts of avalanche like non-local transport. In the quasi-steady phase, a power balance is sustained on average, while transient drop of the temperature gradient occurs due to the bursty transport. Since the ITG turbulence is driven by the temperature gradient, the power balance dictates the steady temperature profile, where the turbulent ion heat transport $\chi_i \sim 0.3\chi_{GB}$ is an order of magnitude larger than the neoclassical transport $\chi_{nc} \sim 0.05$.

On the other hand, the momentum balance in intrinsic rotation profiles is rather complicated. Intrinsic rotation is often discussed as a null momentum transport state where residual stress and momentum diffusion are balanced. In Fig.3(a), the bursty transport shows non-diffusive momentum fluxes or residual stress. Theoretically, residual stress can be induced by various symmetry breaking mechanisms such as $E \times B$ shear, profile shear, and turbulent intensity gradient. Although quasi-linear calculations in [7] identified profile shear stress, in the presence of mean $E \times B$ shear with $|\gamma_E| \sim \gamma_L$, mode asymmetry and resulting sign of residual stress are dictated mainly by $E \times B$ shear, where γ_E is the shearing rate and γ_L is the linear growth rate. The similar sign relation and mode asymmetry is observed also for the bursty transport.

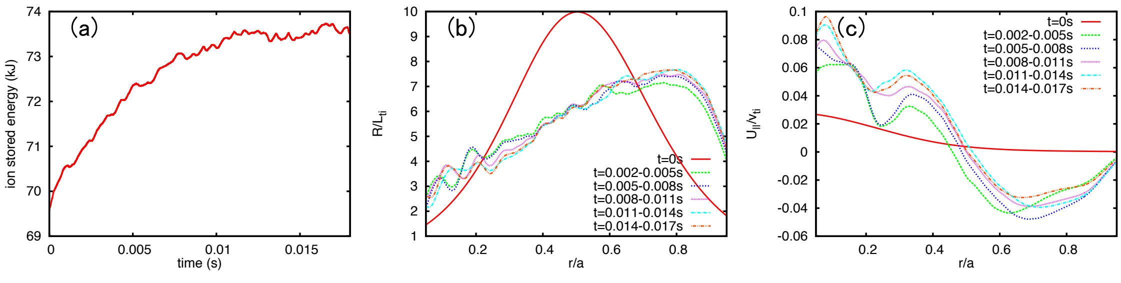

Figure 2: *Long time ITG turbulence simulation. Time evolutions of (a) plasma stored energy, (b) temperature gradient profiles, and (c) parallel flow profiles. After* $\sim$ 10ms*, plasma profiles show quantitative convergence, while profiles after a collision time (*$\sim$ 3ms*) are qualitatively similar to converged ones.*

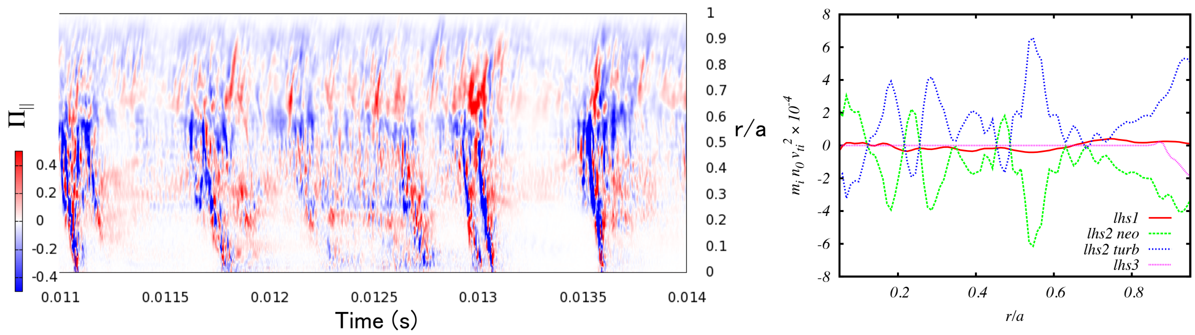

Figure 3: *(a) Spatio-temporal evolution of turbulent parallel momentum flux* $\Pi_{||}$*, and (b) the momentum balance relation, Eq.(5), in the quasi-steady phase, where non-diffusive turbulent momentum transport* ($E \times B$ *drift) is cancelled by neoclassical one (magnetic drift).*

The remaining question is the mechanism to cancel the residual stress. The inter-burst phase often shows diffusive momentum fluxes (see Fig.3(a)). However, their net torque is not enough for canceling the residual torque from the bursty transport. To answer this question, the momentum balance, Eq. (5) is examined again in the quasi-steady phase. It is noted that another balance relation, Eq. (6) is automatically satisfied, and terms in Eq.(6) do not produce a net torque. In Fig.3 (b), the turbulent ($E \times B$ drift) torque is cancelled by the neoclassical (magnetic drift) torque, which also shows similar bursty behaviors as in Fig.3(a) with opposite signs.

Summary

The accuracy issue of momentum transport in full-f gyrokinetic simulations is verified from the convergence of simulation results with respect to the gyrokinetic ordering. Steady temperature and intrinsic rotation profiles are obtained in ITG turbulence simulations over a confinement time. The temperature profile is determined by a balance between fixed power input and turbulent heat transport characterized by intermittent bursts of avalanche-like transport. On the other hand, the rotation profile is sustained by a balance between turbulent residual stress and neoclassical momentum transport, which is induced by non-axisymmetric turbulent perturbations.

References

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