Development of momentum conserving collisional operator
for Monte Carlo simulation code

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Introduction

The conservation of the momentum during particle collisions is an important issue in studying the electron cyclotron current drive (ECCD), the neoclassical transport and etc. In the present GNET code[1, 2], which solves the drift kinetic equation for energetic particles in 5D phase space, the linear Monte Carlo collision operator is applied. This operator consider the collisional effect between test particle and background particle only as the pitch angle scatter and energy scattering. The change of the background particle distribution and the momentum transferred from the test particle to the background are ignored in this operator. The Ray tracing code shows a large impact of parallel momentum conservation for ECCD simulation [3, 4]. However, in the previous study the finite orbit and radial drift effects are not considered because of a radially local assumption.

In this paper, in order to study ECCD quantitatively, we develop collisional operators conserving momentum for GNET. Two momentum conserving collision operator models are considered applying an iterative method and implemented to GNET code. We apply the developed model to the ECCD in the Heliotron J plasma with the momentum conserving operators.

Momentum conserving model

We assume the particle collision term as $C^{\text{coll}}(\delta f) = C(\delta f, f_{\max}) + C(f_{\max}, \delta f)$, where $C(f_{\max}, \delta f)$ is the field particle operator which represents the collision effect for the background particles. In this study we consider two momentum conserving models. One is very easy to implement but it does not include the information in the velocity space. We named it as the "simple" model. The other includes all information in the velocity space, but it is now being implemented. We named it as "velocity dependent" model.

In the simple model, we assume a high speed limit and $C(f)$ is expressed as

$$C(f_{\max}, \delta f) = p(\mathbf{x}, \mathbf{v}) f_{\max}, \tag{1}$$

where $p(\mathbf{x}, \mathbf{v})$ is a function of the real space coordinate, $\mathbf{x}$, and the velocity, $\mathbf{v}$. $p(\mathbf{x}, \mathbf{v})$ is determined by the conservation laws of energy and momentum. After some calculations, we obtain

$$p(\mathbf{x}, \mathbf{v}) = \mathbf{v} \cdot \mathbf{p}(\mathbf{x}) + \lambda(\mathbf{x}) \left(\frac{v^2}{v_{\text{the}}^2} - \frac{3}{2} \right), \tag{2}$$

$$\mathbf{p}(\mathbf{x}) = -\frac{2}{n_0 v_{\text{the}}^2} \int \mathbf{v} C(\delta f, f_{\max}) d\mathbf{v}, \quad \lambda(\mathbf{x}) = -\frac{2}{3 n_0 v_{\text{the}}^2} \int v^2 C(\delta f, f_{\max}) d\mathbf{v}, \quad (3)$$

where n_0 means the density of background electrons. Once $p(\mathbf{x}, \mathbf{v})$ is obtained from Eq. (2), we can calculate $C(f_{\max}, \delta f)$ which compensates the lost momentum and energy from test particle. Then we can consider $C(f_{\max}, \delta f)$ as a new source–sink term.

In the GNET code, if we iteratively calculate until δf converges, we obtain a final profile of $C(f_{\max}, \delta f)$. We label the steady state solution obtained by using S^{ql} as δf_0 and use $C(f_{\max}, \delta f_0)$ which becomes a new source term. The steady state solution of this source term is δf_1. Obtaining δf_1, we can consider the conservation of momentum when we calculate δf_0. However the test particle lost the momentum due to the collision with background plasma in the process of calculating δf_1. Therefore we iteratively calculate δf_n (n is the natural number) as

$$\frac{\partial \delta f_0}{\partial t} + (\mathbf{v}_d + \mathbf{v}_\parallel) \cdot \frac{\partial \delta f_0}{\partial \mathbf{x}} + \dot{\mathbf{v}} \cdot \frac{\partial \delta f_0}{\partial \mathbf{v}} - C(\delta f_0, f_{\max}) = S^{\text{ql}}(f_{\max}) + L^{\text{orbit}}(\delta f_0),$$

$$\frac{\partial \delta f_1}{\partial t} + (\mathbf{v}_d + \mathbf{v}_\parallel) \cdot \frac{\partial \delta f_1}{\partial \mathbf{x}} + \dot{\mathbf{v}} \cdot \frac{\partial \delta f_1}{\partial \mathbf{v}} - C(\delta f_1, f_{\max}) = C(f_{\max}, \delta f_0) + L^{\text{orbit}}(\delta f_1), \quad (4)$$

$$\vdots,$$

until the lost momentum aproaches 0. At the same time we evaluate the momentum which the test particle lost and calculate the momentum loss rate from them. We stop the iteration when the momentum loss rate becomes small sufficiently. After the iterative method, we obtain the conserving momentum distribution function by calculating $\sum_i^n \delta f_i$.

Though the simple model conserves the momentum, it does not include the exact information in the velocity space. Therefore it is necessary to implement the more exact model. The velocity dependent model is derived from the Fokker–Planck collisional term directly, so it includes exact information more than the simple one.

The field particle operator can be expressed using Legendre polynominals $P_n(\cos \theta)$ as

$$C(f_{\max}, \delta f) = \sum_{n=0}^{\infty} C_n(f_{\max}, \delta f^{(n)}(v)) P_n(\cos \theta), \quad (5)$$

where v is the total velocity of an electron and θ represents the pitch angle. Introducing the Trubnikov-Rosenbluth potential and define $u = \cos \theta$ to simplfy [5, 6], we can describe field particle term $C_n(f_{\max}, \delta f^{(n)}(v))$ as

$$C_n(f_{\max}, \delta f^{(n)}(v_{\text{e}})) = \Lambda^{e/e} f_{\max} \sum_{i=0}^{\infty} P_i(v_{\text{e}}) \left[\delta f^{(n)}(v_{\text{e}}) \right. \quad (6)$$

$$+ 2 \int_0^{v_{\text{e}}} u^2 \delta f^{(n)}(u) \left\{ \left(n_+ \frac{u^{i+2}}{v_{\text{e}}^{i+1}} - n_- \frac{u^i}{v_{\text{e}}^{i-1}} \right) - \frac{1}{2i+1} \frac{u^i}{v_{\text{e}}^{i+1}} \right\} du$$

$$+ 2 \int_{v_{\text{e}}}^{\infty} u^2 \delta f^{(n)}(u) \left\{ \left(n_+ \frac{v_{\text{e}}^{i+2}}{u^{i+1}} - n_- \frac{v_{\text{e}}^i}{u^{i-1}} \right) - \frac{1}{2i+1} \frac{v_{\text{e}}^i}{u^{i+1}} \right\} du \left. \right],$$

where $n_+ = (i+1)(i+2)/(2i+1)(2i+3)$, $n_- = (i-1)i/(2i-1)(2i+1)$, $v_{\text{e}} = v/v_{\text{the}}$. $\Lambda^{e/e}$ represents the amplitude of field particle term and in this paper it is assumed as $\Lambda_{\text{c}} e^4/m_{\text{e}}^2 \varepsilon_0^2$, where Λ_{c} is coulomb logarithm, e is charge, m_{e} is mass of an electron and ε_0 is permittivity in vacuume. In order to obtain the field particle term $C_n(f_{\text{max}}, \delta f^{(n)}(v))$ which is determined by the obtained pertubed distribution function $\delta f^{(n)}(v)$. We can iteratively calculate $\delta f^{(n)}(v)$ in the same way with the simple model case. After the iterative method, we calculate the complete collision operator according to Eq. (5).

Simulation result

In this study we apply the momentum conserving mode assuming the same magnetic config-uration, heating and plasma parameters as the previous paper[2]. We consider the Heliotron J configuration with $\varepsilon_{\text{b}} = 0.01$ at the magnetic flux surface $\rho = 0.67$. The radial heating point is set to $(\rho_0, \phi_0, \theta_0) = (0.1, 45°, 0°)$. We also set the parameters describing the EC resonance con-dition as follows: EC wave frequency is 70 GHz, $2\omega_{\text{ce}}/\omega = 0.98$, $n_\parallel = 0.44$ and $\Delta = 1.0 \times 10^{-3}$. ECRF heating power is deposited at the top of the ripple in this configuration.

We run the GNET iteratively and obtain the steady state solution, δf. Fig. 1 (a) shows the firstly obtained distribution function. The distribution becomes asymmetric in $v_\parallel$ at the high energy region. This is because many ECRH accelerated electrons hardly become trapped and the collisional relaxtion of the electron deficit in low energy region is faster than that of the accelerated electrons. As a result, the excess of electrons with positive $v_\parallel$ occured and it is found that the negative toroidal current is driven by the Fisch-Boozer effect. Fig. 1 (b) shows the source–sink term to conserve the momentum using the steady state solution δf_0. Then the steady state solution δf_1 is evaluated using the this source–sink term (Fig. 1 (c)) and again the next source–sink term is evaluated (Fig. 1 (d)). In the two source–sink terms we can see the larger distribution in the positive $v_\parallel$ region and this means the lost momentum have large effect in this region. Fig. 2 shows the momentum loss rate at the each iterative calculation in the simple model. We evaluate the momentum loss at each calculation, and define the momentum loss rate as $p_{\text{loss}} = (p_0 - p_n)/(p_0)$, where p_0 is the momentum lost by test particle at first calculation and p_n represents one at the n th iterative caluculation. Fig. 2 shows the momentum loss decreases as the iterative calculation advanced and dropped less than 5% of lost momentum than initial simulation. The calculated ECCD current of the simple model is -20.1kA and the non-conserving one is -18.4kA. We can see the calculated ECCD current is larger by 9.2% than that of non-conserving one.
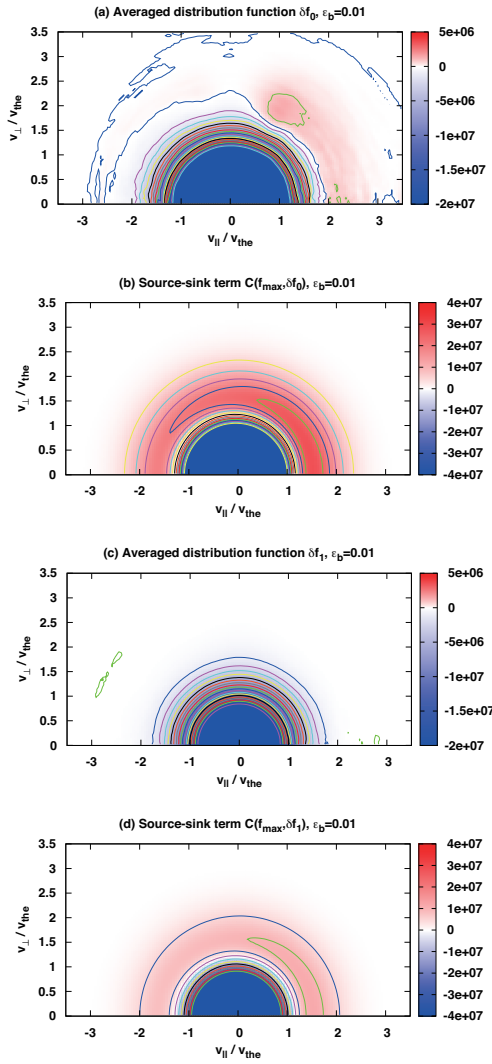
Figure 1: *Flux averaged distribution function of (a) δf_0, (b) source–sink term $C(f_{max}, \delta f_0)$, (c) δf_1 and (d) source–sink term $C(f_{max}, \delta f_1)$.*
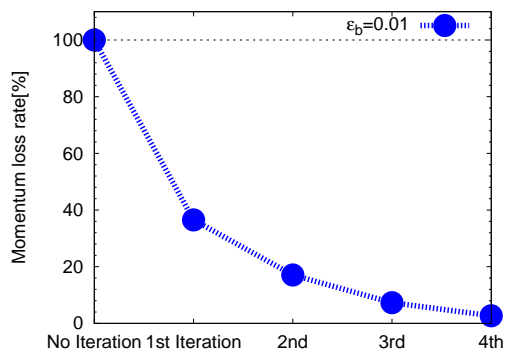

Figure 2: *The momentum loss rate at each iterative calculation.*

Conclusion

In order to study the ECCD physics on helical plasmas, we have simulated the current drive of ECH plasmas in toroidal plasmas by GNET. To evaluate the EC current quantitatively correct, we have improved the collision operator of GNET to conserve the parallel momentum. We have implemented two models; the simple and velocity dependent models. It is easy to implement the simple model and it showed a good conservation of parallel momentum. However it does not include the exact information in the velocity space. Therefore we are implementing the velocity dependent model which is expected to include the exact information in the velocity space.

References

[1] S. Murakami *et al.*, Nucl. Fusion **40**, 693 (2000).

[2] Y. Moriya *et al.*, Plasma Fusion Res. **6**, 2403139 (2011).

[3] H. Maaßberg *et al.*, Phys. Plasmas **19**, 102501 (2012).

[4] N. B. Marushchenko *et al.*, Phys. Plasmas **18**, 032501 (2011).

[5] M. N. Rosenbluth *et al.*, Physical Review **107**, 1 (1957).

[6] MARCO BRAMBILLA, *Kinetic Theory of Plasma Waves Homogeneous Plasmas* (CLARENDON PRESS, 1998).