Nonlinear saturation of the Toroidal Alfvén Eigenmodes computed with the VENUS+δf, HAGIS and KINX codes

M.Yu. Isaev[1], S.Yu. Medvedev[2], S.D. Pinches[3], S.E. Sharapov[4] and JET EFDA Contributors*

[1]National Research Centre "Kurchatov Institute", Moscow, Russia

[2]Keldysh Institute, Russain Academy of Science, Moscow,Russia

[3]ITER Organization, Route de Vinon-sur-Verdon, 13115, St Paul-lez-Durance, France

[4]EURATOM/CCFE Culham Science Centre, Abingdon,Oxfordshire, OX14 3DB, UK

*See the Appendix of F. Romanelli et al., Proc. of the 24th FEC 2012, San Diego, US

1. Introduction

The successfull benchmarking of the linear growth stage of fast particle driven TAE with a broad variation of numerical models by the ITPA Energetic Particle Topical Group for an n = 6 Toroidal Alfvén Eigenmode (TAE) number was described in [1]. In this paper, the second stage in the wave evolution – the non-linear saturated state - is computed for the same n = 6 test case (Section 2) and also for an n = 4 TAE in JET #40214 discharge (Section 3). The fixed spatial structure of TAE modes $\eta_{mn}(s)$ ($s = (r/a)^2$ is in this paper a normalised toroidal (or can be also a poloidal in some codes) flux function, a is a minor plasma radius) is calculated with the KINX [2] and MISHKA codes [3]. The fast particle dynamics, TAE growthrates and wave saturation levels are computed with the HAGIS [4] and VENUS+δf [5, 6] orbit following codes.

Fourier decomposition in Boozer coordinates (s, θ, ζ) of the TAE mode with a poloidal index m and the mode frequency ω has a form

$$\xi = A(t)\, \Sigma\, \eta_{mn}(s) \cos(m\theta - n\zeta - \omega t). \qquad (1)$$

The main equation for the TAE amplitude $A(t)$ evolution due to the fast particle-wave interaction without plasma damping is

$$dA/dt = - \langle \int Z\, e\, \delta f\, \boldsymbol{V}\!\cdot\!\boldsymbol{E}\, d\tau \rangle / (2K \omega^2 A), \qquad (2)$$

here $K = \int \rho_b\, \xi^2\, dV$ – the kinetic energy of the plasma perturbation, $\rho_b = m_b\, n_b$ – mass density of the bulk plasma, volume unit, $d\tau = d^3x\, d^3V$ – phase volume unit, $Z{\cdot}e$ – particle charge, δf – perturbed distribution function of fast particles, $\boldsymbol{V}$ – particle velocity vector, $\boldsymbol{E}$ – wave electric field. The TAE growth rate γ is computed from $\gamma = dA/(Adt)$, and the radial component of the TAE perturbation $\delta\boldsymbol{B}$ = rot$\alpha\boldsymbol{B}$, normalized to the central magnetic field B_0, is defined as

$$\delta B_r/B_0 = \delta\boldsymbol{B}\, \nabla s/[|\nabla s| B_0] = -\mu_0 (I\partial\alpha/\partial\theta + J\partial\alpha/\partial\zeta)/[|\nabla s| g^{1/2} B_0], \qquad (3)$$

where J and I are the toroidal and poloidal current flux functions, $g^{1/2}$ is a Boozer jacobian.

2. Simulation results for the ITPA-EP test case with n = 6 TAE

A circular tokamak with major and minor radii R = 10 m, a = 1 m respectively, safety factor $q(s) = 1.71 + 0.16\, s$, and magnetic field on-axis of 3T was chosen as an ITPA-EP test case because of the restrictions of the 9 participating codes with respect to geometry or numerical properties. A detailed description of this test case is provided in Ref. [1]. The plasma cross-section with the n = 6 TAE perturbation computed with the KINX code, is shown in Fig. 1. The successfull benchmark of the linear TAE growth with a Maxwellian distribution of fast ions (deuterons) with a temperature range of T = 100 - 800 keV has been performed with both zero and finite Larmor radius effects. The maximum linear growth rate was $\gamma_L = 5\times10^4\ s^{-1}$. The typical nonlinear saturation of the TAE mode function $\delta B_r/B$ for T = 400 keV, computed with the VENUS code, is shown on Fig. 2 in green. The evolution of the growth rate is shown in blue. Good agreement between the VENUS nonlinear simulations and a theory scaling $\delta B_r/B \sim (\gamma_L)^2$ [7] is shown in Fig. 3.

3. Simulation results for the JET#40214 discharge with TAE n = 4

JET DD discharge #40214 at t = 46.38 s has been selected for making more realistic simulations as it has an elongated plasma cross-section, moderate aspect ratio and a core-localised n = 4 TAE mode with a frequency $\omega = 1161100\ s^{-1}$ (see Fig. 4 which shows the plasma cross-section and TAE perturbations computed with the KINX code). The bulk is taken to be a DT plasma (50% of deuterium and 50% of tritium) with a central plasma density $n_0 = 2\times10^{19}\ m^{-3}$ and a flat ion density profile $n = n_0\,(1 - 0.15\ \Psi^2 - 0.85\ \Psi^4)$, where Ψ is the normalised poloidal flux and the ion temperature is constant, T = 15 keV. The fast α-particles are assumed to have an exponential density profile $n_f = n_{f0}\exp(-5.5\ \Psi)$ and a slowing down distribution function in energy, $f(E) = C(1 - \mathrm{erf}(x))/(E^{3/2} + (E_c)^{3/2})$ with $E_c = 4.942\times10^5$ eV, $E_0 = 3.5\times10^6$ eV, $dE = 4.105\times10^5$ eV, $x = (E - E_0)/dE$. The constant C has been defined to provide a central α-particle density of $n_{f0} = 6\times10^{16}\ m^{-3}$.

Fig. 5 shows the nonlinear saturation of the n = 4 TAE mode computed with the VENUS code for the JET #40214 discharge with different initial amplitudes: A(0) = 1.e-9, 1.e-8, 1.e-7, 3.e-7, 5.e-7, 7.e-7. The saturation level $\delta B_r/B \approx 2\times10^{-5}$ does not depend on this initial value, however, the saturation can be achieved much faster (after about 200 wave periods) with a large value of A(0) = 7.e-7. These VENUS computations use about 10^6 particles, time step 5.e-8 s. The saturation level $\delta B_r/B \approx 2\times10^{-5}$ has been achieved with the HAGIS code with 10^6 particles after 800 wave periods (Fig. 6). The linear stage has the normalized growth rate $\gamma_L/\omega \approx 0.15\%$ from both the HAGIS and the VENUS computations, however, for this core-localized TAE case VENUS results depend on the complicated particle orbits near the magnetic axis.

4. Summary and future plans

After the successfull benchmark for the linear stage of the TAE evolution in the frame of ITPA-EP group, we present the simulation results of the nonlinear evolution of the TAE modes with a fixed spatial structure and phase. Nonlinear TAE saturation levels for the ITPA-EP $n = 6$ test case computed with the VENUS+δf code are in an agreement with the theory scaling.

Nonlinear saturation levels for JET #40214 discharge with an $n = 4$ TAE mode, computed with the HAGIS and VENUS+δf codes, are in a good agreement and equal to $\delta B_r/B \approx 2\times10^{-5}$. Further benchmarks will be performed in the frame of ITPA-EP group. VENUS code will explore the wide orbit effects near the plasma edge and near the magnetic axis, where the VMEC code can have a poor equilibrium force balance [8]. Extended JET-DT TAE experimental details, phase evolution equation, damping effects and relaxation effects (sink, source, diffusion or drag) will be considered in the future to explore predictions for TAE behavior in ITER.

Acknowledgements

The authors thank Dr. S.P. Hirshman for providing us the DESCUR and VMEC codes and Prof. W.A. Cooper for providing us the TERPSICHORE code. The work was supported by the RosAtom Contracts H.4A.52.90.13.1027. The computations were performed on Kurchatov cluster (Kurchatov Institute, Russia). The authors thank ITPA-EP colleagues for the collaboration support and fruitful discussions. This work was supported by EURATOM and carried out within the framework of the European Fusion Development Agreement. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

References

[1] A. Könies *et al*, 24th IAEA Fusion Energy Conference, CN-197, ITR/P1-34(2012).

[2] L.Degtyarev *et al*, Comp. Phys. Comm., 103, 10(1997).

[3] A.B.Mikhailovskii *et al,* Plasma Phys. Rep., 23, 844(1997).

[4] S.D.Pinches *et al*, Comp. Phys.Comm., 111, 133(1998).

[5] W.A.Cooper *et al*, Plasma Phys. Contr. Fus. 53, 024001(2011).

[6] M.Yu.Isaev *et al*, Plasma and Fus. Res., 7, 1403077(2012).

[7] H.L.Berk, B.N. Breizman, Phys. Fluids B 2, 2226(1990).

[8] R.Sanchez *et al*, Comp. Phys.Comm., 141, 55(2001).

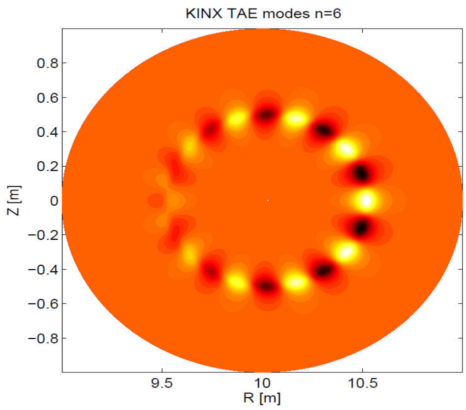

Fig. 1. Plasma cross-section with n = 6 TAE perturbation computed with the KINX code for the ITPA-EP test case.

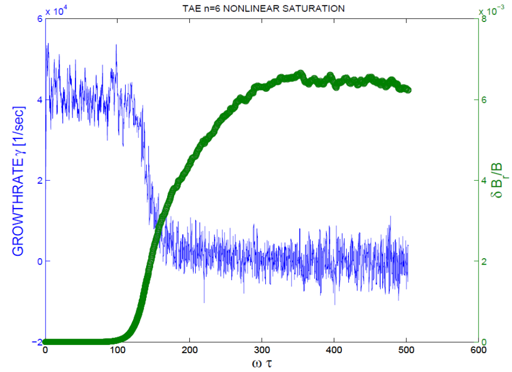

Fig. 2. Nonlinear saturation of n = 6 TAE mode (green), computed with the VENUS code for the ITPA-EP test case. Evolution of the TAE growth rate is shown in blue.

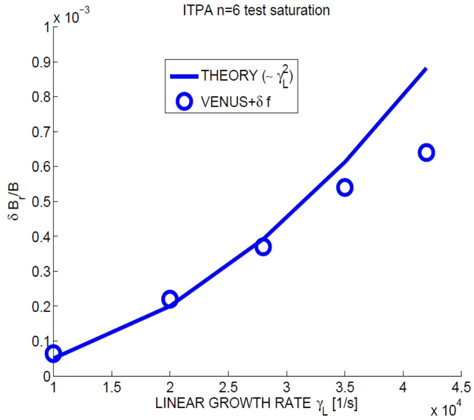

Fig. 3. Saturation level $\delta B_r/B$ of TAE n = 6 mode as a function of the linear growth rate γ_L, computed with the VENUS code and from a theory scaling [7].

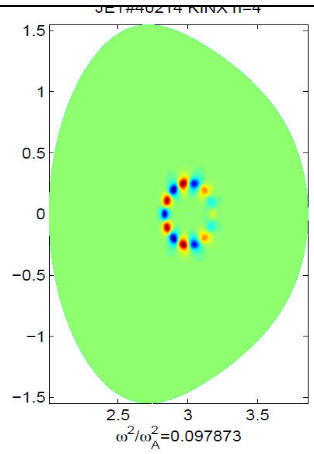

Fig. 4. Plasma cross-section with n = 4 TAE perturbation computed with the KINX code for the JET #40214 discharge.

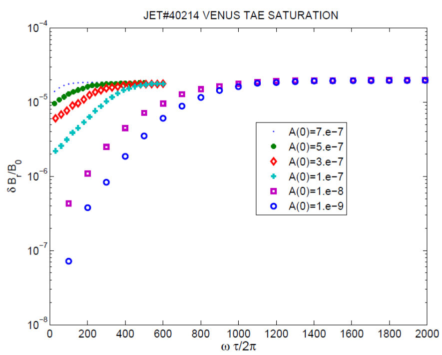

Fig. 5. Nonlinear saturation of n = 4 TAE mode, computed with the VENUS code for the JET #40214 discharge with the different initial amplitudes, A(0).

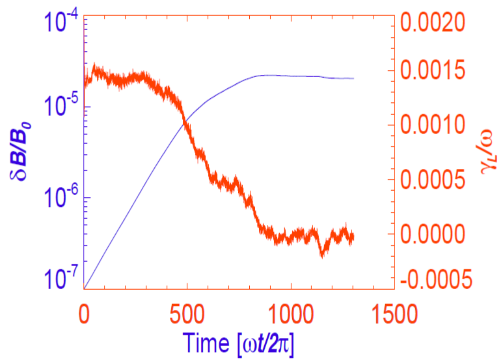

Fig. 6. Nonlinear saturation of n = 4 TAE mode (blue), computed with the HAGIS code for the JET #40214 discharge. Evolution of the TAE growth rate is shown in red.