

Predator-prey modeling of the coupling of co-propagating CAE to kinks

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Co-propagating Compressional Alfvén eigenmodes (CAE) with shorter wavelength and

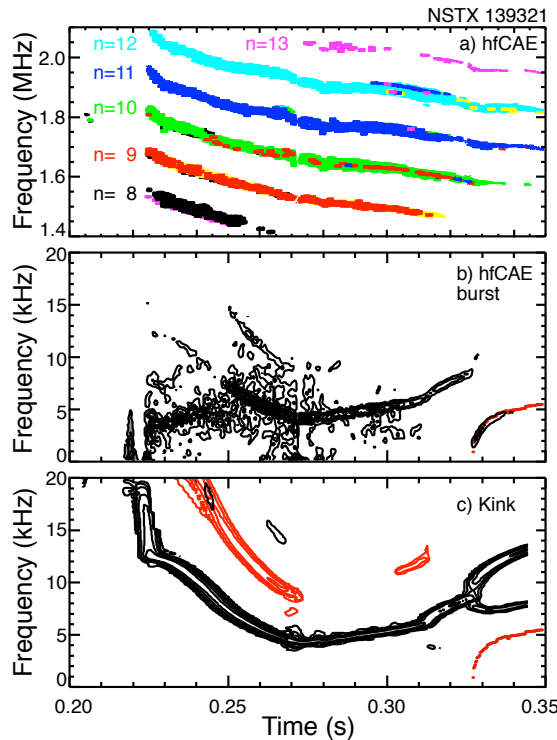


Fig. 1. a) Spectrogram showing hfCAE, b) spectrogram of rms(hfCAE), c) spectrogram showing kink evolution.

higher frequency than the counter-propagating CAE and Global Alfvén eigenmodes (GAE) are commonly seen in NSTX H-mode plasmas [1]. Whereas the lower frequency CAE and GAE are excited through a Doppler-shifted cyclotron resonance, the high frequency CAE (hfCAE) are driven through a simple parallel resonance. The presence of the hfCAE is strongly correlated with a low frequency $n=1$ kink. An example of multiple hfCAE with toroidal mode numbers from $n=8$ up to $n=13$ is shown in Fig. 1a. The hfCAE appear concurrently with an $n=1$ kink-like mode (Fig. 1c), which is probably similar to the long-lived mode seen on MAST [2].

Typically the hfCAE frequencies lie between ≈ 1.2 MHz and ≈ 2.3 MHz ($f_{ci} \approx 2.5$ MHz). The magnetic fluctuations show a strong compressional polarization, as measured with Mirnov coils [1]. The modes are identified as CAE by comparison with the eigenvalues predicted from a 2-D reduced dispersion relation. The frequencies predicted by the dispersion relation are in good agreement with measured frequencies over a range of shots, as shown in Fig. 2.

The focus of this paper is on a curious coupling of the hfCAE to the kink mode [1]. The hfCAE are typically bursting, with a burst frequency on the order of a few kHz. The $n=1$ Doppler-shifted kink frequency is usually

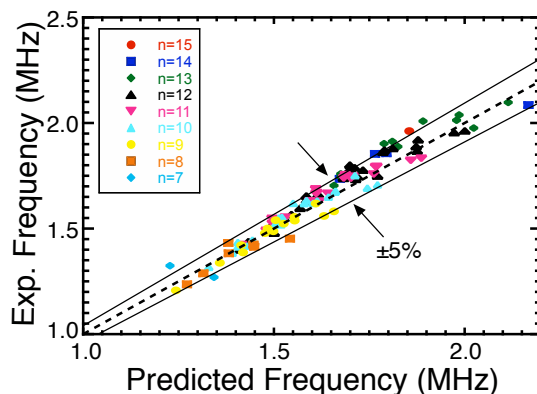


Fig. 2. The measured mode frequency, corrected for the Doppler shift due to the toroidal plasma rotation are plotted vs. the predicted frequency for toroidal mode numbers from $n=7$ to $n=15$.

higher than this, but when the kink frequency does drop towards the natural burst frequency of the hfCAE, the hfCAE burst frequency will become locked to the kink frequency. This is seen in Figs. 1b and 1c which show spectrograms of, respectively, the hfCAE burst amplitude and the $n=1$ kink-like mode. Starting after about 0.25s a strong modulation of the burst rate at the kink frequency is seen. The bursts aren't 'entrained' in the kink, as was reported previously for the coupling of TAE to the kink [3], rather the bursts are axi-symmetric (Fig. 3). A case where the kink slows and locks

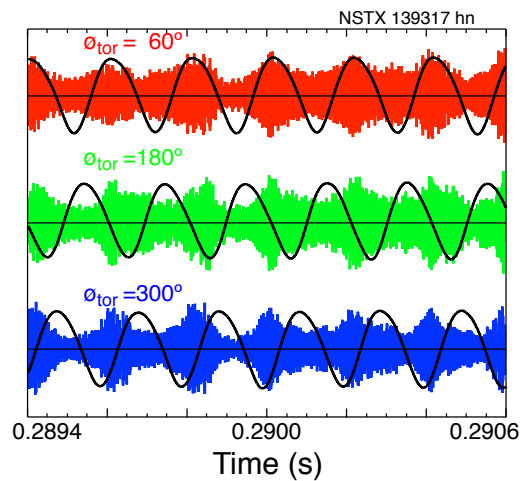


Fig. 3. Filtered signals from a toroidal array of Mirnov coils showing the hfCAE bursts (red, blue and green) and the kink mode (black). The kink mode propagates, the hfCAE bursts are concurrent at all toroidal positions.

to the wall is shown in Fig. 4. The relative phase between the kink and the hfCAE bursts shifts by $\approx 115^\circ$ and the amplitude of the hfCAE bursts remains large as the burst frequency drops to ≈ 1 kHz.

Some of the characteristics of this coupling between the kink and the hfCAE bursting can

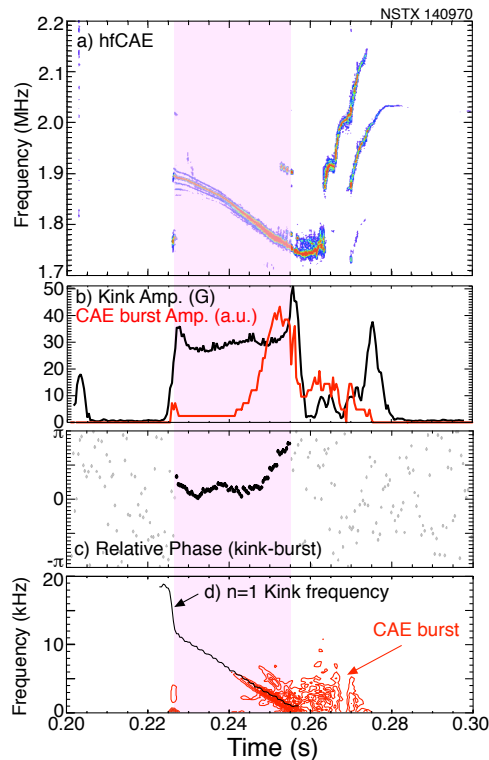


Fig. 4. a) spectrogram showing single $n=12$ hfCAE, b) rms kink amplitude evolution (black) and hfCAE burst amplitude, c) phase between kink and hfCAE bursts, d) spectrogram showing hfCAE burst frequency.

be seen in a simple predator-prey model, with the added assumption that the rotation of the kink modulates in some way the growth or damping rate of the hfCAE. Predator-prey simulations find that with the growth or damping rate modulated by as little as a few percent, at a frequency near the natural burst frequency, the burst frequency becomes locked to the modulation frequency.

The modulation of the damping or drive requires a symmetry-breaking interaction of the kink with some asymmetric feature, such as an error field, the non-axisymmetric vacuum vessel or possibly the asymmetric beam injection. For example, as the kink rotates, the kink interaction with the error field either reinforces or partially cancels the error field. This modification of the error field by the kink could modulate the part of the fast ion distribution driving the hfCAE. Alternatively, the modulation of the plasma shape,

interacting with the non-symmetric vacuum vessel, might directly affect the hfCAE stability. Or the displacement of the plasma by the kink will modulate in a small way the deposition profile of the neutral beams. Any of these effects might weakly affect the mode stability.

Predator-prey systems may be modeled with a coupled set of non-linear equations such as

$$\begin{aligned} A_{n+1} &= A_n + \gamma_d A_n (B_n - T(t_n)) \delta t & \text{or} & \quad \partial A / \partial t = \gamma_d A (B - T(t)) \\ B_{n+1} &= B_n + (S - B_n A_n) \delta t & \text{or} & \quad \partial B / \partial t = S - BA \end{aligned}$$

shown as difference and differential equations. Here the first equation for the parameter A describes the mode amplitude evolution. The second equation for B describes the fast ion beta (mode drive), normalized such that $B=1$ represents marginal stability. The threshold for mode growth may be modulated through the parameter $T=1+\epsilon\sin(\omega t)$. Losses are assumed proportional to mode amplitude and γ_d is the mode growth rate.

For the simple, unmodulated, case with $T=1$, the equations have a stationary solution with $B=1$ and $A=S/B$. The system tends to relax to that state quickly. However, intrinsic to the model is an inherent resonance at some natural bursting frequency. It is well known that the addition of 'noise' to the predator-prey mathematical models can stimulate periodic behavior [2]. Similarly, introducing a coherent perturbation near the intrinsic resonant frequency can also stimulate periodic behavior.

Noise can be introduced in the difference equation realization of the model through the use of a coarse time-step, but in the integral formulation, noise must be explicitly introduced (or just the coherent perturbation to be amplified).

Figure 5 shows a spectrogram of the burst frequency from the predator-prey model where the stability threshold is modulated by 2%. The modulation frequency starts at 15 kHz, above the 'natural' burst frequency of ≈ 7 kHz, and is swept down to 2 kHz. The stability threshold is also modulated with broadband fluctuations of 4%. Bursting at the natural burst frequency of ≈ 7 kHz can be seen initially, and towards the end of the spectrogram. As the modulation frequency

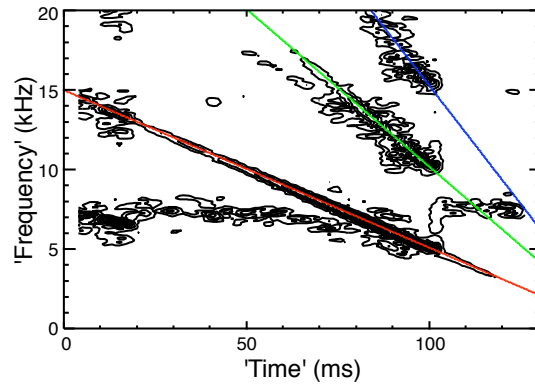


Fig. 5. Spectrogram of predator-prey model of bursting modes with threshold modulation of 2% and broadband noise with rms amplitude of 4% added to the threshold.

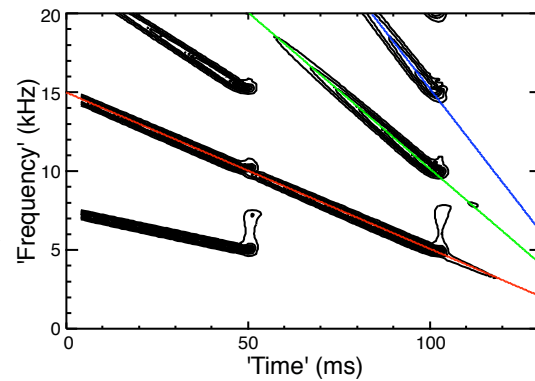


Fig. 6. Spectrogram of predator-prey model of bursting modes with 2% threshold modulation.

sweeps down through the resonant frequency, the bursting becomes modulated by the applied frequency. A similar simulation is shown in Fig 6, but now without the 'noise' added to the stability threshold. Modulation at harmonics, and subharmonics of the modulation frequency are seen.

The simulations demonstrate that modulation of the damping rate or fast ion source can lock the burst frequency to the modulation frequency if the modulation frequency is near enough to the natural burst frequency. For the parameters used here, that means within $\approx \pm 40\%$ of the natural burst frequency. The natural burst frequency in the experiment, however, is not so well known, and may be evolving. In Fig. 1 the natural burst frequency starts at about 3 kHz at 0.22s and increases to ≈ 5 kHz at 0.25s. The kink frequency is 7 kHz at this time, and the burst frequency jumps up to match it. However, the evolution of the natural burst frequency is unknown after 0.25s.

The mode amplitude and fast ion beta evolution from the simulation shown in Fig. 5 are graphed in Fig. 7 to show the cyclical bursting behavior. As the applied perturbation frequency sweeps downwards towards the natural resonance frequency, the mode amplitude modulation increases (black curves). As the modulation frequency

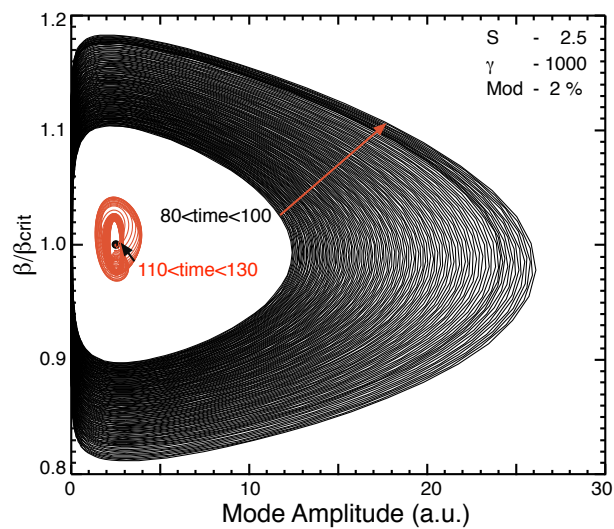


Fig. 7. Evolution of attractor as perturbation frequency sweeps towards natural resonant frequency (black) and below resonant frequency (red). (Arrows indicate direction of time.)

sweeps through the natural burst frequency, the relative phase between the modulation and the mode bursts shift by $\approx 180^\circ$. For frequencies below the natural burst frequency the mode amplitude is only modulated, without clear bursting. This seemingly conflicts with the experimental observations shown in Fig. 4 where strong bursting is seen down to ≈ 1 kHz.

A clear coupling of a low frequency kink mode with the bursting frequency of hfCAE is seen in NSTX. Some features of this coupling can be produced with a predator-prey model for the bursting, where the hfCAE stability threshold is modulated by the rotation of the kink. This would imply an interaction of the kink with some non-axisymmetric feature of the machine, *e.g.*, error fields, vacuum vessel shape or neutral beam injection.

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