Tokamak Self-Consistent Pressure Profiles Interpretation via a "Thermodynamic" Approach

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Introduction.

Analysis of a wide range of experimental results indicates that the tokamak plasmas are "self-organized", what is most clearly seen in the phenomenon of the pressure profiles self-consistency [1]. This phenomenon can be interpreted in the framework of a "thermodynamic" approach, when a self-consistent solution corresponds to the minimum of the free energy $F = -\theta \cdot S + E$, where entropy S and energy E both depend on the distribution function and θ is some effective temperature [2,3]. The choice of the entropy type defines $S(f)$ unambiguously. Below the Boltzmann and Tsallis statistics are considered. We show that under the assumption that the mechanism of the profiles self-consistency is to be connected directly with the equilibrium diamagnetic currents, one can get a solution which is in a reasonable agreement with experiment. In this case, the energy E is determined by the diamagnetic currents J_d and the potential ψ which is consistent with the space distribution of equilibrium currents:

$$E = (1/2c) \int_0^a (J_d \cdot \psi) r dr \qquad (1)$$

$$\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r\psi)\right] = -(4\pi/c)J_d \qquad ; \qquad J_d = c\nabla p/B_0$$

To find E, the relation between f and pressure gradient is taken to be linear and such that its normalization corresponds to the condition of a fixed total diamagnetic current. Thus the normalized distribution function is defined as

$$f = -(1/r)(\partial p/\partial r) \cdot (a^2/p_0). \qquad (2)$$

Here p_0 is pressure in the centre, $x = r/a$, a is plasma column minor radius.

Self-consistent pressure profiles. Boltzmann statistics case.

The solution $\delta F = 0$ for the Boltzmann-Gibbs entropy $S^{BG} = -\int_0^a (f \ln f) r dr$ gives

$$f_c \propto \exp\left(\frac{p_0}{2B_0 a^2} \cdot \frac{r\psi}{\theta}\right). \qquad (3)$$

Here f_c is the distribution function which provides free energy minimum and corresponds to a self-consistent pressure profile. With account of Eq. (1) and distribution function normalization, one has

$$f_c = -\frac{1}{x}\frac{\partial P_n}{\partial x} = Z_0^{-1} \cdot \exp\left[-\frac{p_0 \beta_0}{4\theta}\int_0^x P_n(x^{'})x^{'}dx^{'}\right] \quad (4)$$

$$Z_0 = \int_0^1 xdx \cdot \exp\left[-\frac{p_0 \beta_0}{4\theta}\int_0^x P_n(x^{'})x^{'}dx^{'}\right] \quad ; \quad P_n = p/p_0 \quad ; \quad \beta_0 = 8\pi p_0/B_0^2$$

The solutions of Eq. (4) are shown in fig.1a for different values of $\gamma = \beta_0 p_0/4\theta$. An increase of γ (that corresponds to a decrease of temperature θ) leads to the pressure profiles sharpening. Interesting is that experimental pressure profiles evolution versus edge safety factor q_L, which is shown in fig.1b, is quite similar with the simulated pressure profiles evolution versus γ, thus connecting these parameters phenomenologically.

Linear relaxation for the Boltzmann entropy case.

Dynamics of the relaxation has to be formulated in a form providing $\partial F/\partial t < 0$ (F tends to a minimum). In addition, in the minimum of free energy, the profile has to be self-consistent. Near the minimum, the linear form, providing the fulfillment of relaxation conditions is similar to Smoluchowski equation [2]

$$\frac{\partial f}{\partial t} = \nabla \cdot \left[\frac{1}{\xi}[\theta \nabla f + f \nabla H]\right] \quad ; \quad H = \frac{\beta_0}{4}\int_0^{x=r/a} p_c(x^{'})x^{'}dx^{'} \quad (5)$$

Here the parameter ξ describes the dissipative processes. In the form of Eq. (5), the condition $\partial F/\partial t < 0$ is strictly satisfied. Because near the minimum of free energy $H = -\theta \ln Z_0 - \theta \ln f_c$ (f_c - distribution function in the minimum), a more compact form is also possible

$$\frac{\partial f}{\partial t} = \nabla \cdot \left[\frac{\theta f}{\xi}\nabla \ln(f/f_c)\right] \quad (6)$$

Taking into account the relation between distribution function and pressure gradient, in given case one has

$$\frac{\partial p}{\partial t} = \frac{\theta}{\xi}\left[\nabla^2 p - \nabla p \cdot \frac{\nabla^2 p_c}{\nabla p_c}\right] \quad ; \quad \nabla^2 = \partial^2/\partial r^2 \quad (7)$$

Here p_c is the self-consistent pressure profile. Equation (7) for pressure perturbation dynamics can be written also for the separated density and temperature variables, involving the equation of state: $p = nT, T = T_e + T_i$.

$$\frac{\partial n}{\partial t} = -n\left[\frac{1}{T}\frac{\partial T}{\partial t}\right] + \frac{\theta}{\xi}\left[n \cdot A + \nabla n \cdot B + \nabla^2 n\right] \quad (8)$$

$$A = \frac{\nabla^2 T}{T} - \frac{\nabla T}{T} \cdot \frac{\nabla^2 p_c}{\nabla p_c} \quad ; \quad B = 2\frac{\nabla T}{T} - \frac{\nabla^2 p_c}{\nabla p_c}$$

It is known that with an auxiliary heating, the density profile evolution provides the stiffness of normalized pressure profiles. Figure 2 shows simultaneous evolution of electron temperature (main contribution to plasma pressure), density and normalized pressure profiles in the discharge with the central ECRH .

The Tsallis entropy case.

The Tsallis entropy $S^T = -1/(\mu - 1)\int_0^a (f^\mu - f) r dr$ is intensively used for the analyses of complex non-equilibrium systems. The solution $\delta F = 0$ for the Tsallis entropy gives:

$$f_c \propto [1 + \gamma \cdot \frac{(\mu - 1)}{\mu}(\frac{r}{a}) \cdot \psi^*]_+^{1/(\mu-1)} \; ; \; \psi^* \cdot (\frac{4\pi p_0}{B_0} a) = \psi \; ; \; [y]_+ = \begin{cases} y & y \geq 0 \\ 0 & y < 0 \end{cases} \qquad (9)$$

Here μ is the so called non-extensivity parameter. As μ tends to 1, distribution (9) becomes the same as (3). For the Tsallis entropy, taking into account the distribution function normalization, the equation for the normalized self-consistent pressure profile is as follows

$$\frac{\partial P_n}{\partial x} = -Z_0^{-1} \cdot x \cdot \left[1 - \frac{p_0 \beta_0}{4\theta} \frac{(\mu - 1)}{\mu} \int_0^x P_n x' dx'\right]_+^{1/(\mu-1)} \qquad (10)$$

$$Z_0 = \int_0^1 x dx \cdot \left[1 - \frac{p_0 \beta_0}{4\theta} \frac{(\mu - 1)}{\mu} \int_0^x P_n x' dx'\right]_+^{1/(\mu-1)}$$

Figure 3a,b shows the solution of Eq. (10) for various μ and two values of $\gamma = \beta_0 p_0 / 4\theta$=30 and γ=100. For the Tsallis entropy, the form satisfying the relaxation conditions is

$$\frac{\partial f}{\partial t} = \nabla \cdot \left[\frac{1}{\xi}[\theta \nabla f^\mu + f \nabla H]\right] \qquad (11)$$

For such a form, the condition $\partial F / \partial t < 0$ is strictly satisfied. In the vicinity of the free energy minimum, an equation can be formulated for the pressure profiles perturbations similarly to the case of Boltzmann entropy. But in the case of μ not equal to unity, this equation becomes nonlinear and needs careful studies.

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References

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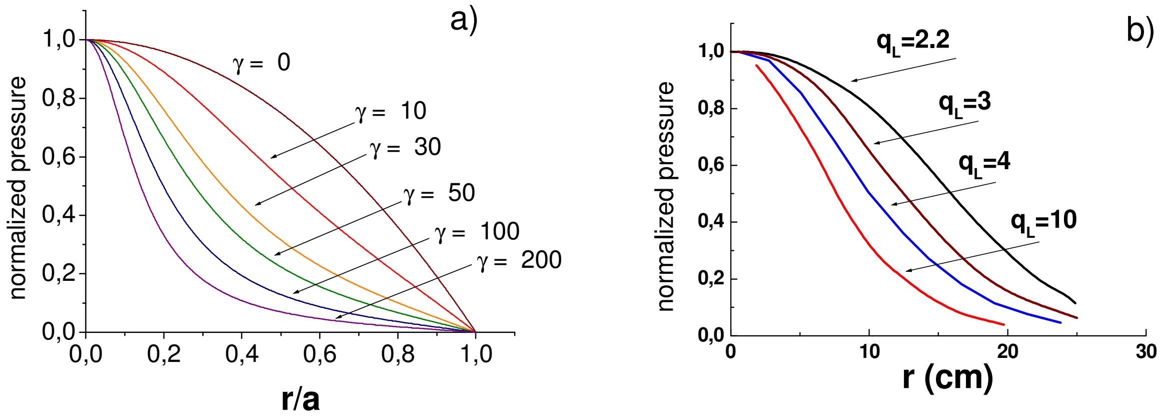

Fig.1. a) The simulated normalized pressure profiles versus parameter $\gamma = \beta_0 p_0 / 4\theta$. Boltzmann statistics case. b) The experimental normalized pressure profiles, measured on the T-10 tokamak in the regimes with the different values of the safety factor at the limiter, q_L.

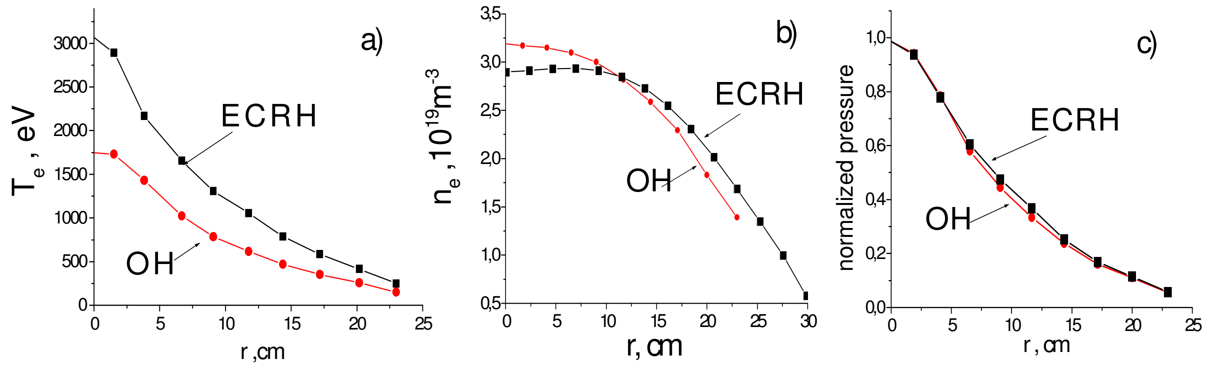

Fig.2. The simultaneous evolution of electron temperature (a), density (b) and normalized pressure profiles (c) in the discharge with the central ECRH in the T-10 tokamak. The normalized pressure profile is conserved.

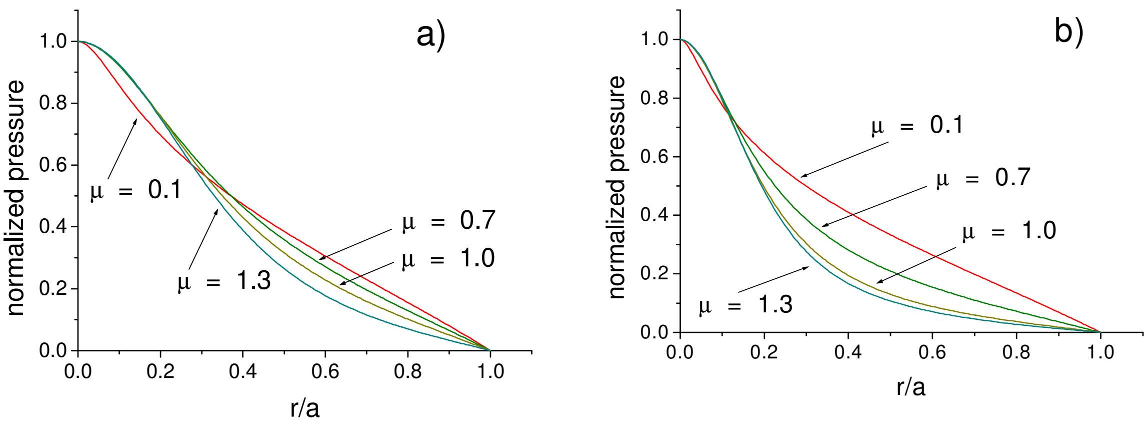

Fig.3. The simulated normalized pressure profiles versus parameter μ. Tsallis statistics case. The parameter $\gamma = \beta_0 p_0 / 4\theta$ is fixed. a) γ=30 , b) γ=100.