Effect of Impurity Toroidal Viscosity on Offset Toroidal Rotation

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Introduction

One of essential difference between tokamak and helical is the geometrical symmetry [1]. Recently, changes in plasma properties induced by the application of non-axisymmetric field to tokamak attract strong interests. Especially, observation of offset toroidal rotation [2] is important for stabilizing RWM even if it is small. Analytical offset toroidal rotation formula derived in 2011 has been extended to include effect of impurity toroidal viscosity.

Offset Toroidal Rotation due to Bulk Ion Toroidal Viscocity

Ambipolarity is ensured in the axisymmetric tokamak, irrespective of electric field. If the symmetry braking occurs, the ambipolarity of the particle flux is broken. Then the radial electric field (electrostatic potential) is adjusted to satisfy ambipolarity (non-ambipolar flux=0). Assuming impurity toroidal viscosity, which is in Pfirsh Schluter regime, ambipolarity (non-ambipolar flux=0) condition is give by $\langle \boldsymbol{B}_t \cdot \nabla \cdot \boldsymbol{\Pi}_i \rangle = 0$. Shaing [3] gave following expression for this neoclassical toroidal viscosity (NTV) ($' = d/dV$).

$$\langle \boldsymbol{B}_t \cdot \nabla \cdot \boldsymbol{\Pi}_i \rangle = e_i B_t B_p \Gamma_{ri} = -e_i n_i B_t B_p q^2 \tau_{ii} \left(\frac{\delta T_i}{e_i B r} \right)^2 \left(\frac{\varepsilon}{\pi} \right)^{3/2} \left[\lambda_{1i} \left(\frac{p_i'}{p_i} + \frac{e_i \Phi'}{T_i} \right) + \lambda_{2i} \frac{T_i'}{T_i} \right] \quad (1)$$

Therefore non-ambipolar flux=0 condition, $\langle \boldsymbol{B}_t \cdot \nabla \cdot \boldsymbol{\Pi}_i \rangle = 0$ gives $\frac{d\Phi}{d\psi} + \frac{1}{eZ_i n_i}\frac{dP_i}{d\psi} = -\frac{\lambda_2}{eZ_i\lambda_1}\frac{dT_i}{d\psi}$.

To get an analytic expression for the offset toroidal rotation, we use the 0-th order radial force balance equation to see a relation among the offset toroidal rotation, the thermodynamic force, and the residual poloidal flow.

$$\boldsymbol{u}_{i0} \cdot \nabla \zeta = -\left[\frac{d\Phi}{d\psi} + \frac{1}{eZ_i n_i}\frac{dP_i}{d\psi} \right] + q \boldsymbol{u}_{i0} \cdot \nabla \theta \quad (2)$$

, where q is the safety factor. By using an analytic expression for the residual poloidal rotation by Kim [6], $\boldsymbol{u}_{i0} \cdot \nabla \theta = -\frac{K_1 F(\boldsymbol{B}\cdot\nabla\theta)}{eZ_i \langle B^2 \rangle}\frac{dT_i}{d\psi}$, an analytic expression for the offset toroidal rotation of the bulk ion for NTV, $u_{i\zeta 0} = R\boldsymbol{u}_{i0} \cdot \nabla \zeta$ is given by Kikuchi [4], [5] as,

$$u_{i\zeta 0} = R \left[\frac{\lambda_2}{eZ_i \lambda_1} - \frac{qK_1 F(\boldsymbol{B}\cdot\nabla\theta)}{eZ_i\langle B^2 \rangle} \right] \frac{dT_i}{d\psi} \quad (3)$$

In the large aspect ratio cylindrical plasma, offset toroidal rotation is given as follows,

$$u_{i\zeta 0} = \frac{3.54 - K_1}{eZ_i B_\theta}\frac{dT_i}{dr} \quad (4)$$

Since measurement of toroidal rotation is made using impurity toroidal rotation, the expression of the offset toroidal rotation of the impurity is required for comparison to the experiment. We use a formula $u_{I\zeta 0} - u_{i\zeta 0} = -1.5K_2 \frac{dT_i}{dr}$ [1] to obtain following formula.

$$u_{I\zeta 0} = \frac{3.54 - 1.5K_2 - K_1}{eZ_i B_\theta} \frac{dT_i}{dr} \tag{5}$$

Effect of Impurity Toroidal Viscosity on Offset Toroidal Rotation

In a multi species plasma, the zero non-ambipolar flux condition is given as,

$$\sum_a e_a \Gamma_{na}^a = 0 \tag{6}$$

, where

$$\Gamma_{na}^a = \frac{\langle \boldsymbol{B}_t \cdot \nabla \cdot \boldsymbol{\Pi}_a \rangle}{e_a \psi' \phi'} \tag{7}$$

Here, $\boldsymbol{B} \cdot \nabla\zeta = \phi'$ and $\boldsymbol{B} \cdot \nabla\theta = -\psi'$. Considering the electron viscosity is small, condition for zero non-ambipolar flux $\sum_a e_a \Gamma_{na}^a = 0$ is given as,

$$\langle \boldsymbol{B}_t \cdot \nabla \cdot \boldsymbol{\Pi}_i \rangle + \langle \boldsymbol{B}_t \cdot \nabla \cdot \boldsymbol{\Pi}_I \rangle = 0 \tag{8}$$

Explicit form of the impurity toroidal viscosity is given in the Appendix. Here we use successive approximation to obtain first order correction to offset toroidal rotation. We expand offset toroidal rotation as $u_\zeta = u_{\zeta 0} + u_{\zeta 1} + --$. 0-th and 1-st order equations are given as follows,

$$\langle \boldsymbol{B}_t \cdot \nabla \cdot \boldsymbol{\Pi}_i \rangle_0 = 0 \tag{9}$$

$$\langle \boldsymbol{B}_t \cdot \nabla \cdot \boldsymbol{\Pi}_i \rangle_1 + \langle \boldsymbol{B}_t \cdot \nabla \cdot \boldsymbol{\Pi}_I \rangle_0 = 0 \tag{10}$$

Obviously, the solution for (9) is given by (4). Solution to the 1-st order equation (10) may be given by approximating $u_{\zeta 1} \sim E_{r1}/B_p = -\frac{d\Phi_1}{dV}\frac{dV}{dr}/B_p$ and using the equation (1) to obtain,

$$\langle \boldsymbol{B}_t \cdot \nabla \cdot \boldsymbol{\Pi}_i \rangle_1 = p_i \tau_{ii} \left(\frac{\delta}{R}\right)^2 \left(\frac{\varepsilon}{\pi}\right)^{3/2} r'(V) B_t \lambda_{1i} u_{\zeta 1} \tag{11}$$

Therefore, 1-st order toroidal flow due to impurity NTV, $u_{\zeta 1}$ is given by,

$$u_{\zeta 1} = -\frac{dV}{dr} \frac{\pi^{3/2} R^2 \langle \boldsymbol{B}_t \cdot \nabla \cdot \boldsymbol{\Pi}_I \rangle_0}{p_i \tau_{ii} \varepsilon^{3/2} \delta^2 B_t \lambda_{1i}} \tag{12}$$

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Appendix: Impurity NTV in PS regime

Neoclassical toroidal viscosity (NTV) in Pfirsh-Schluter regime for impurity is given in Hamada coordinates by Shaing[8] as,

$$\langle \boldsymbol{B}_t \cdot \nabla \cdot \boldsymbol{\Pi}_I \rangle = 3\left[\langle \frac{\boldsymbol{B}_t \cdot \nabla B}{B} \frac{\partial B}{B\partial\theta} \rangle (\mu_{I1} \boldsymbol{u} \cdot \nabla\theta + \mu_{I2} \frac{2\boldsymbol{q}}{5p_I} \cdot \nabla\theta) + \langle \frac{\boldsymbol{B}_t \cdot \nabla B}{B} \frac{\partial B}{B\partial\zeta} \rangle (\mu_{I1} \boldsymbol{u} \cdot \nabla\zeta + \mu_{I2} \frac{2\boldsymbol{q}}{5p_I} \cdot \nabla\zeta) \right] \quad (13)$$

Here original paper by Shaing [8] includes $p_I \tau_{II}$, which is not necessary. Viscosity coefficients are given by Hirshman-Sigmar [7] as follows,

$$\mu_{a1} = K^a_{11} \, , \, \mu_{a2} = K^a_{12} - \frac{5}{2} K^a_{11} \, , \, \mu_{a3} = K^a_{22} - 5K^a_{12} + \frac{25}{4} K^a_{11} \quad (14)$$

In Pfirsh-Schluter regime, Hirshman[7] gives following expression.

$$K^a_{ij} = p_a \tau_{aa} \iota^a_{ij} \quad (15)$$

$$\iota^a_{11} = \frac{q^{11}_a}{Q_a} \, , \, \iota^a_{12} = \frac{7}{2} \frac{(q^{11}_a + q^{01}_a)}{Q_a} = \iota^a_{21} \, , \, \iota^a_{22} = \frac{49}{4} \frac{(q^{11}_a + q^{00}_a + 2q^{01}_a)}{Q_a} \quad (16)$$

$$Q_a = \frac{2}{5}(q^{00}_a q^{11}_a - q^{01}_a q^{01}_a) \quad (17)$$

$$q^{ij}_a = \sum_b \frac{n_b e_b^2}{n_a e_a^2} (q^{ij}_{ab} - \delta_{ab} r^{ij}_{ab}) \quad (18)$$

$$q^{00}_{ab} = \frac{3 + 5x^2_{ab}}{(1 + x^2_{ab})^{3/2}} \, , \, q^{01}_{ab} = \frac{3}{2} \frac{3 + 7x^2_{ab}}{(1 + x^2_{ab})^{5/2}} \, , \, q^{11}_{ab} = \frac{35x^6_{ab} + \frac{77}{2}x^4_{ab} + \frac{185}{4}x^2_{ab} + \frac{51}{4}}{(1 + x^2_{ab})^{7/2}} \quad (19)$$

$$r^{00}_{aa} = \frac{1}{\sqrt{2}} \, , \, r^{01}_{aa} = \frac{3}{2\sqrt{2}} \, , \, r^{11}_{aa} = \frac{15}{4\sqrt{2}} \quad (20)$$

In case $T_i \approx T_I$, $x_{aa} = 1$, $x^2_{iI} \approx \frac{m_i}{m_I} \ll 1$, $x^2_{Ii} \approx \frac{m_I}{m_i} \gg 1$ when i is deuterium and I is Carbon, since $x_{ab} = v_{Tb}/v_{Ta}$.

$$q^{01}_{II} = \frac{15}{4\sqrt{2}} \, , \, q^{00}_{II} = 2\sqrt{2} \, , \, q^{11}_{II} = \frac{265}{16\sqrt{2}} \quad (21)$$

Since $x^2_{Ii} \gg 1$, we have

$$q^{00}_{Ii} = \frac{5}{x_{Ii}}, q^{01}_{Ii} = \frac{21}{2x^3_{Ii}}, q^{11}_{Ii} = \frac{35}{x_{Ii}} \quad (22)$$

From (18), we have $q^{ij}_I = q^{ij}_{II} - r^{ij}_{II} + q^{ij}_{Ii}/\alpha$, where $\alpha = n_I Z_I^2 / n_i Z_i^2$ is called impurity strength parameter. And,

$$q^{00}_I = \frac{3}{\sqrt{2}} + \frac{5}{\alpha x_{Ii}} \, , \, q^{01}_I = \frac{9}{4\sqrt{2}} + \frac{21}{2\alpha x^3_{Ii}} \, , \, q^{11}_I = \frac{205}{16\sqrt{2}} + \frac{35}{\alpha x_{Ii}} \quad (23)$$

$$Q_I = f(\alpha, x_{Ii}) = \left(\frac{6}{5\sqrt{2}} + \frac{2}{\alpha x_{Ii}} \right) \left(\frac{205}{16\sqrt{2}} + \frac{35}{\alpha x_{Ii}} \right) - \frac{2}{5} \left(\frac{9}{4\sqrt{2}} + \frac{21}{\alpha x^3_{Ii}} \right)^2 \quad (24)$$

So, $Q_I = \frac{267}{40} + O(1/\alpha x_{Ii})$. Taking the leading order for Q_I, we obtain,

$$\iota^I_{11} = \frac{40}{267} \times \frac{205}{16\sqrt{2}} = 1.357 \tag{25}$$

$$\iota^I_{12} = \frac{7}{2} \times \frac{40}{267}\left(\frac{205}{16\sqrt{2}} + \frac{9}{4\sqrt{2}}\right) = 5.586 \tag{26}$$

$$\iota^I_{22} = \frac{49}{4} \times \frac{40}{267}\left(\frac{205}{16\sqrt{2}} + \frac{3}{\sqrt{2}} + \frac{2\times 9}{4\sqrt{2}}\right) = 26.26 \tag{27}$$

If we define normalized viscosity $\hat{\mu}_{Ij} = \mu_{Ij}/p_I\tau_{II}$, we obtain,

$$\hat{\mu}_{I1} = 1.357\ ,\ \hat{\mu}_{I2} = 2.193\ ,\ \hat{\mu}_{I3} = 6.81 \tag{28}$$

If the magnetic field variation in poloidal and toroidal directions is given as,

$$B = B_0[1 - \varepsilon cos\theta - \delta cos(m\theta - n\zeta)] \tag{29}$$

, we have $(\partial B/\partial\theta)/B = \varepsilon sin\theta + m\delta sin(m\theta - n\zeta)$. Using the relation $\boldsymbol{B}_t = \phi'\nabla V \times \nabla\theta$ in Hamada coordinates, we obtain,

$$\langle \frac{\boldsymbol{B}_t\cdot\nabla B}{B}\frac{\partial B}{B\partial\theta}\rangle = -\frac{\phi'}{2}nm\delta^2 \tag{30}$$

$$\langle \frac{\boldsymbol{B}_t\cdot\nabla B}{B}\frac{\partial B}{B\partial\zeta}\rangle = \frac{\phi'}{2}n^2\delta^2 \tag{31}$$

Here, $\phi' = d\phi/dV = \boldsymbol{B}\cdot\nabla\zeta \sim B_t/R \sim B/R$. Inserting these expression, NTV in PS regimes becomes,

$$\langle \boldsymbol{B}_t \cdot \nabla \cdot \boldsymbol{\Pi}_I\rangle = \frac{3}{2}p_I\tau_{II}\phi' n\delta^2\left[\hat{\mu}_{I1}\boldsymbol{u}_I + \hat{\mu}_{I2}\frac{2\boldsymbol{q}_I}{5p_I}\right]\cdot[n\nabla\zeta - m\nabla\theta] \tag{32}$$

References

[1] M. Kikuchi, *Frontier in Fusion Research - Physics and Fusion*, (Springer, London, 2011)

[2] A. M. Garofalo et al., Phys.Rev. Lett., **101**,195005(2008).

[3] K.C. Shaing, Phys. Plasmas**10**,1443(2003).

[4] M. Kikuchi, Proc. 38th European Conf. on Contr. Fusion and Plasma Physics, edited by A. Becoulet, T. Hoang, and U. Stroth (EPS, Strasbourg, France), Vol. 35G, p. 4.115.

[5] M. Kikuchi, M. Azumi, Rev. Mod. Physics **84**,1807(2012).

[6] Y.B. Kim, P. Diamond, R.J. Groebner, Phys. Fluids **B3**,2050(1991).

[7] S.P. Hirshman, D.J. Sigmar, NF **21**, 1079(1981).

[8] K.C. Shaing, et al., PPCF **51**, 075015(2009).