Effect of a polynomial arbitrary dust size distribution on dust-acoustic double layers in dusty plasmas

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Abstract.

The investigation of the existence of arbitrarily large amplitude electrostatic dust-acoustic double layers[1,2] is conducted in a plasma containing non-thermal ions and massive negatively-charged dust particles that are assumed spheres of different radii distributed according to a polynomial distribution function[3-5]. The influence of an arbitrary dust size-distribution described by a polynomial function on the properties of dust acoustic waves is assessed. An energy-like integral equation involving Sagdeev potential is derived. Conditions of the existence of double layers in terms of the Mach number, concentration of non-thermal ions and dust charge are established.

Introduction.

A dust-acoustic double layer (DADL) is a structure consisting of two space-charge layers of opposite charges. These electrostatic structures have a tremendous role to play in space plasmas as well as laboratory plasmas. Indeed, double layers are considered the appropriate candidate to interpret charged particles acceleration to high energies in plasmas, e.g., in the auroral region of the ionosphere. Double layers may be formed by way of numerous mechanisms, e.g., current driven instabilities, spacecraft ejected electron beams, shock waves in plasmas, laser radiation, injection of non-neutral electrons current into a cold plasma or by electrical discharges.

In this work, we investigate the existence of arbitrarily large amplitude electrostatic dust-acoustic double layers by using the Sagdeev quasi-potential method in dusty plasmas where dust grains are size-distributed according to an arbitrary size distribution. Following Duan *et al.*[6], we assume a polynomial function to describe the arbitrary dust size-distribution that is, for a charged dust grain with radius r_d defined on a given range [r_{d1} , r_{d2}], where r_{d1} and r_{d2} represent the minimum and maximum radii of dust grains respectively, the differential distribution is of the form,

$$n(r_d)dr_d = (a_0 + a_1 r + .).dr_d \qquad (1)$$

where, a_0, a_1, a_2, … are constants satisfying the following condition,

$$N_{tot} = \int_{rd1}^{rd2} n(r_d)dr_d \qquad (2)$$

N_{tot} being the total number density of dust grains. Outside the limits of $r_d < r_{d1}$ and $r_d > r_{d2}$, we use $n(r_d) = 0$.

Basic equations.

Let us consider many components plasma with non-thermal ions and size-distributed negatively charged dust grains, which satisfy an arbitrary dust size-distribution, whereas, the plasma is assumed depleted from electrons, i.e., n_i/n_e>> 1, that is the contribution of electrons on dust acoustic nonlinear structures is ignored.

$$\frac{\partial n_{dj}}{\partial t}+\frac{\partial}{\partial x}\left(u_{dj}n_{dj}\right)=0 \qquad (3), \qquad \frac{\partial u_{dj}}{\partial t}+u_{dj}\frac{\partial u_{dj}}{\partial x}=\left(\frac{Z_{dj}}{m_{dj}}\right)\frac{\partial\phi}{\partial x} \qquad (4)$$

and the closure relationship is nothing but Poisson's equation,

$$\frac{\partial^2\phi}{\partial x^2}=Z_{d0}\sum_j Z_{dj}n_{dj}-n_i, \qquad (5)$$

where the electrostatic potential ϕ is normalized by T_{i0}/e , the dust fluid velocity by the dust acoustic speed $C_d=\sqrt{Z_{d0}T_{i0}/m_{d0}}$, time by $\omega_{pd}^{-1}=\sqrt{m_{d0}/e^2Z_{d0}n_{i0}}$, space by $\lambda=\sqrt{T_{i0}/e^2Z_{d0}n_{i0}}$, and the numerical density of plasma particles by n_{i0} . Furthermore, the dust grain charge Z_{dj} and mass m_{dj} are normalized by the charge and mass of the grain having the most probable radius, viz., $Z_{d0}=Z_d(r_{d0})$ and $m_{d0}=m_d(r_{d0})$. To study the effect of non-thermal ion distribution on the dust acoustic double layer, we choose a more general class of ion distribution which includes the population of non-thermal ion, namely,

$$n_i=\left(1+\beta\Phi+\beta\Phi^2\right)e^{-\Phi} \qquad (6)$$

where β is a constant representing the proportion of non-thermal ions. We look now for stationary solutions of Eqs. (3)-(5), assuming the physical quantities to depend only on, $\xi=x-Mt$ where M is the Mach number. We solve the system of equations (3-5) taking into account the following boundary conditions, $\phi\to\phi_\infty$, $u_{dj}\to 0$, and $n_{dj}\to n_{dj0}$ at $\xi\to\infty$, to obtain the dust density, $n_{dj}=\frac{Mn_{djo}}{\sqrt{M^2+2Z_{dj}(\phi-\phi_\infty)/m_{dj}}}$ (7). Substituting for the particle number densities from Eqs.(6) and (7) into Eq.(5) then integrating the resulting equation, we obtain, $\frac{1}{2}\left(\frac{d\Phi}{d\xi}\right)^2+V(\Phi,M)=0$ (8), where the Sagdeev potential is given by ,

$$V(\Phi,M)=1+3\beta-\left(1+3\beta+3\beta\Phi+\beta\Phi^2\right)e^{-\Phi}-M\,I(\Phi,M) \qquad (9)$$

with,

$$I(\Phi,M)=\sum_j m_{dj}n_{dj0}\left[\sqrt{M^2+\frac{2Z_{dj}}{m_{dj}}\Phi}-M\right] \qquad (10)$$

For the continuous case, Eqs.(1) and (2) are recast as follows,

$$N_{tot} = \sum_{j=1}^{N} n_{dj0} = \int_1^C n(r)dr = \int_1^C [a_0 + a_1 r + ...]\, dr \tag{11}$$

where, $r = (r_d/r_{d1})$ and $C = (r_{d2}/r_{d1})$ is ratio of maximum to minimum grain radius, which describes the extent of the distribution. Furthermore, the discrete summation in Eq. (10) is replaced by integration, $I(\Phi, M) = \int_{r_{d1}}^{r_{d2}} r_d^3 n_0^0 \left[\sqrt{M^2 + \frac{2\Phi}{r_d^2}} - M\right] n(r_d) dr_d$ (12)

so this relation may be written as follows,

$$I(\Phi,M) = Ma_0 \left\{ \left(\frac{\tilde{a}_1 C^2}{5} - \frac{4\tilde{a}_1 \Phi}{15M^2} + \frac{C}{4}\right)\left(C^2 + \frac{2\Phi}{M^2}\right)^{3/2} - \left(\frac{\tilde{a}_1}{5} - \frac{4\tilde{a}_1 \Phi}{15M^2} + \frac{1}{4}\right)\left(1 + \frac{2\Phi}{M^2}\right)^{3/2} - \frac{\Phi C\sqrt{C^2 + \frac{2\Phi}{M^2}}}{4M^2} \right.$$
$$\left. + \frac{\Phi\sqrt{1 + \frac{2\Phi}{M^2}}}{4M^2} - \frac{1}{2}\frac{\Phi^2 \ln\left(C + \sqrt{C^2 + \frac{2\Phi}{M^2}}\right)}{M^4} + \frac{1}{2}\frac{\Phi^2 \ln\left(1 + \sqrt{1 + \frac{2\Phi}{M^2}}\right)}{M^4} - \frac{\tilde{a}_1(C^5 - 1)}{5} - \frac{(C^4 - 1)}{4} \right\} \tag{13}$$

where $\Phi = (\phi - \phi_\infty)$, $a_0 = n_{tot}\left((C-1)(1 + \frac{1}{2}\tilde{a}_1(C+1))\right)^{-1}$ and $\tilde{a}_1 = (a_1 / a_0)$.

It is to be recalled that the conditions under which double layers are allowed, are, i) $V(\Phi,M) = \frac{dV(\Phi,M)}{d\Phi} = 0$ for $\Phi = 0$ and $\Phi = \Phi_m \neq 0$, ii) $V(\Phi,M) < 0$ for $0 < |\Phi| < |\Phi_m|$.

Results.

The Sagdeev potential is plotted versus the electric potential Φ in figures (1) and (2), it is shown that for $\beta = 0{,}5$ and C =10; 15, positive double layers structures are obtained for the value of Mach number M_{DL} = 1,219; 1,107. respectively.

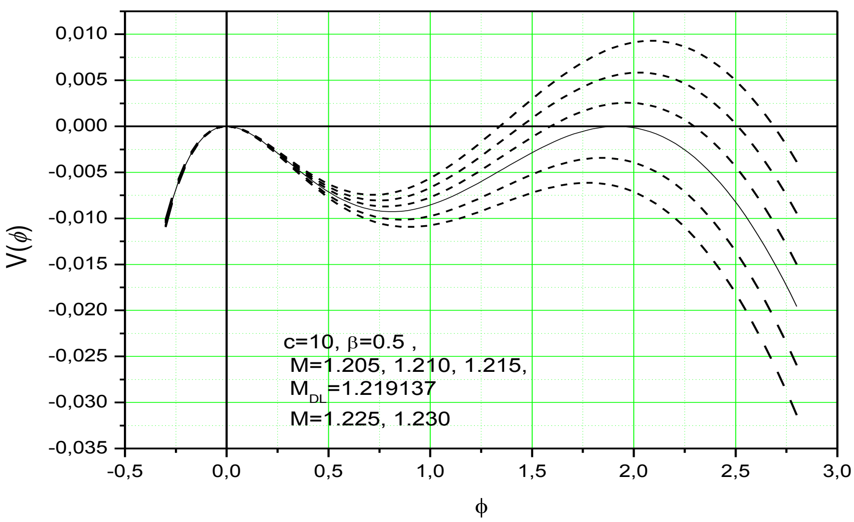

Figure 1: Sagdeev potential $V(\Phi, M)$ vs Φ for C= 10 and $\beta = 0{,}5$.

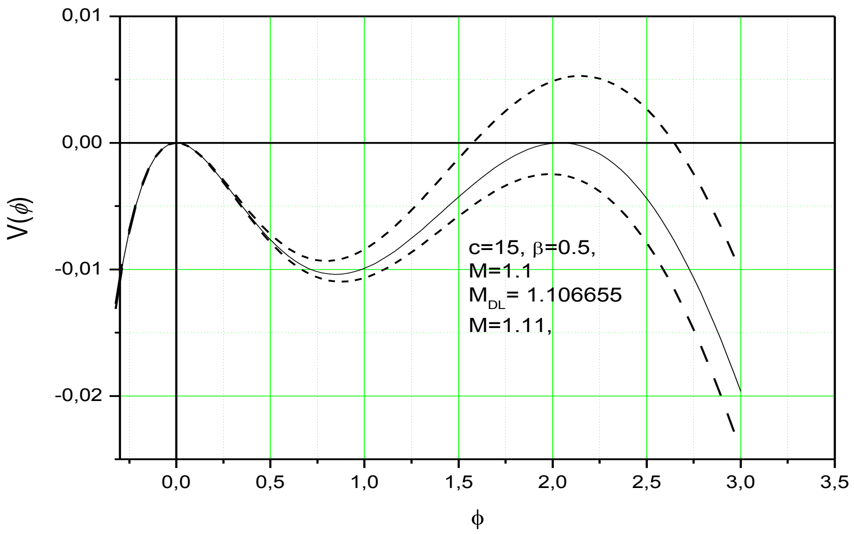

Figure 2: Sagdeev potential $V(\Phi, M)$ vs Φ for C= 15 and $\beta = 0{,}5$.

Conclusion.

In this work, we present a study of the existence of arbitrary amplitude dust acoustic double layers in an unmagnetized dusty plasma with a non-thermal ion distribution and a cold fluid of dust grains of different sizes described by a continuous polynomial arbitrary dust size-distribution. Most of the background electrons are collected by the negatively-charged dust grains. Such plasmas may exist in both laboratory and space environments. The results of this paper confirm that only positive double layers are possible, bay way of varying the Mach number M for a fixed value of β, we could confirm that double layers exist only for Mach number M = 1,219 for C = 10 and M = 1,107 for C = 15.

references.

[1] R.Bharuthram and P.K.Shukla, Planet. Space Sci. **40**, 465 (1992).

[2] M. Ishak -Boushaki, R. Annou and R. Bharuthram, Phys. Plasmas, **19,** 033707(2012).

[3] M. Ishak-Boushaki, D. Djellout and R. Annou, Phys. Plasmas, **19,** 073707(2012).

[4] J.H. Chen and W.S. Duan, Phys. Plasmas, **14** 083702(2007).

[5] J.H. Chen, Chinese Physics B, Vol **18**, No 6, June (2009).

[6] W. S. Duan, G. X. Wan, X. Y. Wang, and M. M. Lin, Phys. Plasmas 11, 4408 (2004).