The non-marginal Bohm condition in the collisionless plasma diode

S. Kuhn[1], D. D. Tskhakaya (sr)[1,2], L. Kos[3]

[1]*Association EURATOM-ÖAW, Inst. for Theoretical Physics, Univ. of Innsbruck, Austria*

[2]*Andronikashvili Institute of Physics, Georgian Academy of Sciences, Tbilisi, Georgia*

[3]*Association EURATOM-MESCS, Mech. Eng. Faculty, University of Ljubljana, Slovenia*

1. Introduction and summary. In plasma-wall transition (PWT) theory it is common practice to use the following two simplifying approximations [1,2]: (i) The plasma is assumed to be semi-bounded, i.e., bounded on only one side by some electrode ("wall"), in the vicinity of which the PWT layer is investigated, but to extend to infinity on the other; (ii) The PWT layer is split into two sub-layers: the quasineutral "presheath (PS)" and the space-charge-dominated "(Debye) sheath (DS)", separated from each other by the "(Debye-)sheath entrance (SE)". The characteristic length of the PS is λ_{ps} (the relevant collisional or geometrical length), and that of the DS is λ_D (the Debye length at the SE), with the additional approximation $\varepsilon = \lambda_D / \lambda_{ps} \to 0$, called the "asymptotic two-scale (a2s) limit". A good basis for qualitatively discussing the a2s PS is the fairly detailed quasineutrality condition ($n_e = n_i$) presented in Ref. [1], which for planar geometry reads

$$\left[m_i u_i^2 - k\left(T_e^* + \gamma_i T_i\right)\right]\frac{dn_e}{dx} = m_i u_i \left(n_e \nu_{ci} + 2S_i\right), \tag{1}$$

with e the positive elementary charge, k Boltzmann's constant, m_i, $u_i(x)$, T_i, and γ_i the ion mass, fluid velocity, effective temperature and "polytropic coefficient", respectively, ν_{ci} the ion-neutral charge-exchange collision frequency, S_i the ionization rate, $n_e(x)$ the electron density and $T_e^* \equiv en_e/\left(kdn_e/dV\right)$ the "screening temperature". It is well known from DS analysis that at the SE the ions must satisfy the "general Bohm condition (BC)", $u_i \geq c_s$, with $c_s \equiv \sqrt{k\left(T_e^* + \gamma_i T_i\right)/m_i}$ the ion-sound velocity. The BC is called "marginal" or "non-marginal" if the equality or the greater sign applies, respectively. In [3] it was shown that γ_i is actually a polytropic-coefficient *function* (PCF) defined as $\gamma_i(x) = 1 + \left(n_i dT_i/dx\right)/\left(T_i dn_i/dx\right)$. Let us call "collisional" a PS in which charge exchange and/or ionization are important ($\lambda_D << \lambda_{ps} < L$, with L the diode length), and "collisionless" one in which these processes are negligible ($\lambda_D << L < \lambda_{ps}$).

For a *collisional* PS the right-hand side (rhs) of Eq. (1) does not vanish identically, so $|dn_e/dx|$ normally tends to infinity where the bracket in front of it tends to zero. Hence, the vanishing of the bracket, which corresponds to the marginal BC $u_i = c_s$, is normally connected with the "sheath singularity", which indicates the breakdown of quasineutrality and, hence, the SE. Note, however, that in reality the parameter ε may become arbitrarily small but never exactly zero so the so-called "sheath singularity" is actually an arbitrarily narrow spatial region with arbitrarily steep but still finite gradients. As a consequence, the SE "looks" like a singularity if viewed on the PS scale (x/λ_D) but shifts towards infinity if viewed on the DS scale (x/λ_D).

For a *collisionless* PS the rhs of Eq. (1) vanishes identically. If the collisionless PS considered is uniform, dn_e/dx vanishes throughout and the bracket need not vanish for Eq. (1) to be satisfied. This is, to a very good approximation, the case for the monotonically decreasing potential distributions in a sufficiently long plane single-emitter diode (Sec. 2), where the PS ($(n_e - n_i)/n_e \to 0_+$, $dn_e/dx \to 0_-$) and the DS ($n_i > n_e$) extend to the left and right, respectively, of the "plasma point" (the point of exact electric neutrality, $n_e = n_i$), which thus marks the DS entrance, cf. Fig. 1. Here, the ions enter the DS with the non-marginal BC $u_i > c_s$, with the exact values of u_i and c_s depending on the diode parameters as presented in Sec. 4.

In this paper we for the first time investigate the BC in a fully bounded but collisionless plasma and, as argued above, find it to be of the non-marginal type. To be specific, we discuss the problem for the standard plane-diode model of the single-emitter ("single-ended") Q – machine or thermionic converter [4-8], making explicit use of existing results that are of immediate relevance.

2. The collisionless single-emitter plane-diode model [4-8]. We assume a hot plate at $x = 0$ and a non-emissive, perfectly absorbing cold plate at $x = L$, the intervening space being filled with a collisionless plasma consisting of electrons, singly charged ions, and the related neutrals. The plasma is produced at the hot plate by thermal emission of electrons and surface ionization of neutrals, the desorbing particles being assumed to leave the hot plate with half Maxwellian velocity distribution functions (VDFs). We use the desorbing-electron and desorbing-ion number densities at the hot plate, n_{e0}^+ and n_{i0}^+, as well as the hot-plate temperature T as scaling quantities for the definition of non-dimensional variables as follows:

$$x/\lambda_{De}^{+} \to x,\ n_{e,i}/n_{e0}^{+} \to n_{e,i},\ v_{e,i}/v_{Te} \to v_{e,i},\ u_{e.i}/v_{Te} \to u_{e,i},\ T_{e,i}/T \to T_{e,i},\ \alpha = n_{i0}^{+}/n_{e0}^{+},$$
$$eV(x)/(kT) \to V(x),\quad j_{e,i}/(en_{e0}^{+}v_{Te}) \to j_{e,i},\ f_{e,i}v_{T_e}/n_{e0}^{+},\ \mu = m_e/m_i \qquad (2)$$

for the position coordinate, number densities, single-particle and fluid velocities, effective temperatures, the neutralization parameter α, the electric potential, electric-current densities, particle VDFs and the mass ratio μ, respectively; $v_{Te} = \sqrt{2kT/m_e}$ is the electron thermal velocity and $\lambda_{De}^{+} = \sqrt{\varepsilon_0 kT/(e^2 n_{e0}^{+})}$ is the "electron emission Debye length". From now on we proceed in dimensionless variables. With $\alpha_e = 1$ and $\alpha_i = \alpha$ we have for the fluid quantities

$$n_{e,i} = \alpha_{e,i}\int_{-\infty}^{+\infty} dv f_{e,i},\ u_{e,i} = \frac{\alpha_{e,i}}{n_{e,i}}\int_{-\infty}^{+\infty} dv v f_{e,i},\ T_e = \frac{2}{n_e}\int_{-\infty}^{+\infty} dv (v-u_e)^2 f_e,\ T_i = \frac{2\alpha}{\mu n_i}\int_{-\infty}^{+\infty} dv (v-u_i)^2 f_i. \qquad (3)$$

Poisson's equation for $V(x)$ and the Vlasov equations for the VDFs $f_{e,i}$ read

$$\frac{d^2V}{dx^2} = n_e - n_i,\ (4a) \qquad v\frac{\partial f_e}{\partial x} - E\frac{\partial f_e}{\partial v} = 0,\ (4b) \qquad v\frac{\partial F_i}{\partial x} + \mu E\frac{\partial f_i}{\partial v} = 0,\ (4c). \qquad (4)$$

3. Monotonically decreasing potentials. We specifically consider potential profiles decreasing monotonically from $V = 0$ at $x = 0$ to some negative value $V = U$ at $x = L$, for which we can solve Eqs. (4b, c) with the appropriate boundary conditions to obtain the VDFs

$$f_e(V,v;U) = \frac{2}{\sqrt{\pi}}\exp(V - v^2)H\left(v + \sqrt{V-U}\right),\quad f_i(V,v;\mu) = \frac{2}{\sqrt{\pi\mu}}\exp\left(-V - v^2/\mu\right)H\left(v - \sqrt{-\mu V}\right), \qquad (5)$$

where $H(s)$ is the Heaviside step function. Hence by virtue of (3) we obtain the densities

$$n_e(V;U) = e^V\left(1 + erf\sqrt{V-U}\right),\quad n_i(V;\alpha) = \alpha\,\overline{n}_i(V),\quad \overline{n}_i(V) = e^{-V}\left(1 - erf\sqrt{-V}\right), \qquad (6)$$

where $erf\,\xi = \left(2/\sqrt{\pi}\right)\int_o^{\xi} dt e^{-t^2}$. Inserting (6) into Poisson's equation (4a) and integrating once we arrive at the following differential equation, which is starting point of our discussions:

$$\frac{dV}{dx} = -2\sqrt{E_0^2 - S(V)},\quad \text{with}\quad S(V;U,\alpha) = S_e(V;U) - \alpha S_i(V), \qquad (7)$$

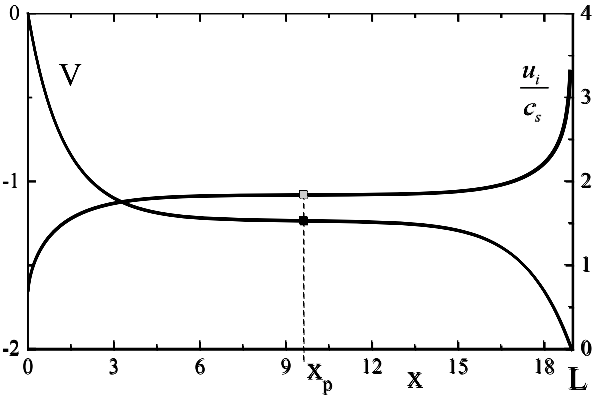
$$S_i(V) = \frac{1}{2}\left\{e^{-V}\left(1 - erf\sqrt{-V}\right) - 1 + \frac{2}{\sqrt{\pi}}\sqrt{-V}\right\}, \qquad (8)$$

$$S_e(V;U) = \frac{1}{2}\left\{1 + erf\sqrt{-U} - e^V\left(1 + erf\sqrt{V-U}\right) - \frac{2}{\sqrt{\pi}}e^U\left(\sqrt{-U} - \sqrt{V-U}\right)\right\}. \qquad (9)$$

Here E_0 is the electric field at $x = 0$. On differentiating Eq. (7) we obtain $d^2V/dx^2 = -2\,dS/dV$, which means that the curvature properties of $V(x)$ follow the slope properties of the function $S(V)$. The inflection points of $V(x)$ occur at those values of V where $S(V)$ assumes extreme values, i.e., $\nu(V;U) \equiv \frac{dS_e}{dV}\Big/\frac{dS_i}{dV} \equiv \frac{e^{2V}\left(1 + erf\sqrt{V-U}\right)}{1 - erf\sqrt{-V}} = \alpha$.

The function $\nu(V;U)$ has just one maximum and satisfies $\nu(U;U) < \nu(0;U)$. Hence, we have only one real solution, i.e, only one inflection point ($V = V_p$) if $\nu(U;U) \le \alpha \le \nu(0;U)$. At this inflection point the quasineutrality condition, $n_e(V_p;U) = n_i(V_p)$, is fulfilled, which is why we call it the "plasma point" and consider it as marking the DS entrance.

4. Numerical example demonstrating the non-marginal BC



From (3) we also find $u_i = \sqrt{\mu/\pi}/\overline{n}_i$. Hence the ratio u_i/c_s is given by

$$\frac{u_i}{c_s} = \left(1 + \frac{\pi}{2}\overline{n}_i^2 \left[\frac{n_e}{dn_e/dV} - \frac{\overline{n}_i}{d\overline{n}_i/dV}\right]\right)^{-\frac{1}{2}}. (10)$$

In Fig. 1 the potential V and the ratio u_i/c_s are plotted versus x for the diode parameters $U = -2$, $\alpha = 1.30$ and $L = 19$. The square points on the curves indicate the inflection ("plasma") point, for which from Eqs. (6) and (10) we calculate $V_p = -1.24$ and $u_i/c_s = 1.85$, respectively. This means that, as expected from the discussion of Sec. 1, the BC is fulfilled in non-marginal form. In a next step towards physical reality we will consider a diode containing *collisional* plasma.

Acknowledgements .This work was supported (i) by the European Commission under the Contract of Association between EURATOM and the Austrian Academy of Sciences and (ii) by the Austrian Science Fund (FWF) under project P223450-N16. It was carried out within the framework of the European Fusion Development Agreement. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

1. K.-U. Riemann, J. Tech. Phys. **41**, 89 (2000).
2. K.-U. Riemann, J. Seebacher, D.D. Tskhakaya (sr.) and S. Kuhn, Plasma Phys. Contr. Fusion **47**, 1949 (2005).
3. S. Kuhn, K.-U. Riemann, N. Jelić, D.D. Tskhakaya (sr.), D.D. Tskhakaya (jr.) and M. Stanojevic, Phy. Plasmas **13**, 013503 (2006).
4. R.G. McIntyre, J. Appl. Phys. **33**, 2485 (1962).
5. W. Ott, Z. Naturf. **22A**, 1057 (1967).
6. S. Kuhn, G. Auer and E. Mravlag, Report UNICP-FSP78/1, Institute for Theoretical Physics, University of Innsbruck, A-6020 Innsbruck, Austria (1978).
7. S. Kuhn, J. Plasma. Phys. **23**, 881(1981).
8. J. K. Buzzi, H.J. Doucet, and D. Grésillon, Phys. Fluids **13**, 3041 (1970).