Equilibrium topology for plasmas with reversed current density

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Non-inductive drive mechanisms help to sustain the plasma current for longer pulses with small or negative inductive drive where zero current densities and current holes are observed [1, 2] and during slow transitions from positive to negative plasma current [3, 4]. A relevant question arising from this situations is that of the structure of the equilibrium magnetic field when the toroidal current density becomes negative in some region inside the plasma. Different numerical and analytical works [5, 6, 7, 8] have asserted that internal current density reversals lead to non-nested magnetic surfaces.

In the following, we show that, for current density reversals, the nested topology is an unlikely degenerated situation for two-dimensional plasmas. Then we demonstrate that non-nested topologies define several current channels inside the plasma, where the positive current has about twice the magnitude of the negative one. This result is obtained by the introduction of an anisotropy parameter that is related to the geometric properties of the equilibrium. To illustrate such relation we develop a local description of the magnetic field with two geometric parameters. The solution is obtained without specifying further plasma profiles or arbitrary functions and is valid in a region of interest where the toroidal current density is both positive and negative. The obtained configurations agree with several published topologies [5, 6, 7, 8] and allow to identify transitions between different magnetic configurations through changes in the anisotropy.

In usual axisymmetric equilibrium the magnetic field lines remain attached to magnetic surfaces forming nested tori inside the plasma. The total current flowing inside a torus labeled by its poloidal magnetic flux ψ, is obtained from the Ampère's law as

$$\mu_0 I_t(\psi) = \oint_{\Gamma_\psi} \vec{B}\cdot d\vec{l} = \pm \oint_{\Gamma_\psi} |\nabla\psi| \frac{dl}{R}, \tag{1}$$

where it was used that $\nabla\psi\times\nabla\phi$ is the poloidal magnetic field and ϕ is the azimuthal coordinate. In eq. (1), $d\vec{l}$ follows clockwise the *magnetic circuit* Γ_ψ, obtained from the intersection of the torus ψ with any azimuthal plane $\phi = const$. The current is negative when $\nabla\psi$ is inwards and positive otherwise. The presence of both positive and negative current densities suggests that some torus ψ_0 contains a net vanishing current. From (1) this would require $|\nabla\psi| = 0$ in every point of the circuit $\Gamma_0 = \Gamma_{\psi_0}$. As a consequence, for *any* pair of coordinates $\{u, v\}$ in a plane $\phi = const.$, the equation $\partial_u\psi(u, v) = 0$ must lead to the same curve that $\partial_v\psi(u, v) = 0$. In addition, the curve must satisfy $\psi(u, v) = \psi_0$. This degeneracy is possible for one-dimensional problems, but is structurally unstable in two-dimensions [7]. In other words, the usual nested topology is not compatible with current density reversals in two dimensions.

A more feasible configuration requires the existence of saddles in the poloidal flux and a separatrix Γ_s. Each saddle leads to a poloidal field inversion without requiring a degenerated torus. The magnetic topology depends on how the separatrix Γ_s connects two branches of the same hyperbolic point when followed smoothly. If Γ_s connects two non-opposite branches of any saddle the total number of saddles is odd (Fig. 1a); otherwise is even (Fig. 1b).

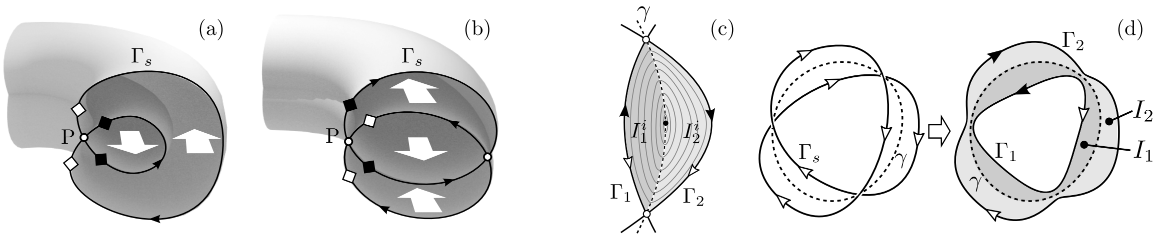

Figure 1: (a-b) Separatrix for odd and even systems of axisymmetric islands respectively. Branches of the saddle P with corresponding diamonds are connected smoothly by the separatrix Γ_s. (c) Splitting of the i'th positive channel into its half-currents by the curve γ. (d) Decomposition of a separatrix into simple circuits and the regions for the currents in (2). Open and filled arrows show the circuit orientation and the direction of $\vec{B}_p$ respectively and white arrows show the direction of the total current for each channel.

The separatrix defines several families of nested magnetic surfaces, each working as a *current channel* inside the plasma. In Fig. 1a-b the poloidal field direction reveals the direction of the toroidal current inside each channel. It is clear from (1) that a magnetic surface just enclosing all the channels has a finite positive current, meaning that the total current in the positive channels must exceed the current of the negative one. To evidence this we build a curve γ orthogonal to the magnetic field (Fig. 1c) passing through every hyperbolic and elliptic point that defines the positive current channels (Fig. 1d). The curve γ divides in two the positive channels and since $\oint_\gamma \vec{B}_p \cdot d\vec{l} = 0$ it contains a vanishing current. By decomposing the separatrix Γ_s into a pair of simple circuits $\{\Gamma_1, \Gamma_2\}$ (Fig. 1d) it can be shown that

$$-\oint_{\Gamma_1} B_p dl = \sum_i I_1^i \equiv I_1 \ , \ \oint_{\Gamma_2} B_p dl = \sum_i I_2^i \equiv I_2, \tag{2}$$

where $I_{1,2}^i$ is the current flowing through the region limited by γ and $\Gamma_{1,2}$ in the i'th positive channel (Fig. 1c). The i'th positive current is $I^i = I_1^i + I_2^i$ and the relative difference between its components $\eta_i = (I_2^i - I_1^i)/I^i$, measures its anisotropy. The current inside all the positive channels is $I_+ = I_1 + I_2$ and the current in the central channel is $I_- = -I_1$. Introducing the mean anisotropy $\eta = \sum_i \eta_i I_i / \sum_i I_i$ and using (2) we obtain

$$I_+ = \frac{2}{1-\eta} |I_-|. \tag{3}$$

For monotonic variation of the current density we expect $0 \leq \eta_i < 1$, leading to $I_+ > 2|I_-|$. This reveals that the combined channels $I_+ + I_-$ carry a net positive current larger that $|I_-|$.

To study the size and number of magnetic islands in terms of the anisotropy, we need to put η in terms of other parameters of the equilibrium. For this, we develop a *local* solution that accounts for the non-nested topology while keeping a simplified physical picture of the equilibrium. From (1) and $\nabla \times \vec{B} = \mu_0 \vec{j}$ it can be verified that

$$R\nabla \cdot (R^{-2}\nabla\psi) = -\mu_0 j_\phi. \tag{4}$$

For a single-fluid MHD equilibrium [9] the current density may be put in terms of two surface functions, leading to the Grad-Shafranov equation [10, 11]. Since the expected equilibrium processes more than one magnetic family, establishing a single form of the pair of surface functions may not be appropriated, however, there are results in this direction exhibiting current inversions [5, 6]. Formally, a broad range of surface functions may lead to current reversals and non-nested surfaces, but the underlying description of the equilibrium topology is not restricted to the particular model. We can cover a wide range of arbitrary choices by imposing existence of a small negative minimum of the current density.

For a hollow-current profile the negative current density, if any, should be small. Assuming up-down symmetry about the minimum, we can write the current density as $j_\phi = j_0 + \iota(r^2 - \kappa r^2 \cos^2\theta)$, with $j_0 < 0$ and (r,θ) local polar coordinates. The parameters ι and κ define the curvature and ellipticity of $j(\vec{r})$ about the minimum. We define a region of interest with radius $a = \sqrt{-2j_0/\iota}$, where the poloidal field reverses in the cylindrical case. Provided that a is small we can solve (4) by a *local* scheme analogous to the successive approximations method [9]. In our case the inverse aspect ratio $\varepsilon = a/R_0$, is a reliably small parameter defined from the region of interest instead of the plasma radius. Defining $R = R_0 + ax$ and $z = ay$ we can write (4) about the current minimum in dimensionless form

$$\left(\partial_x^2 + \partial_y^2 - \frac{\varepsilon}{1+\varepsilon x}\partial_x\right)\psi = (1+\varepsilon x)(1 - 2r^2 + 2\kappa x^2), \tag{5}$$

with r measured in units of a and ψ in units of $\mu_0|j_0|a^2R_0$. The solutions of (5) will depend on κ and ε, allowing a detailed study of the bifurcations that change the topology of the magnetic field. Writing the nondimensional flux as $\psi(r,\theta) = \psi_0(r) + \varepsilon\psi_1(r,x) + O(\varepsilon^2)$ and excluding the ellipticity in the zero-order calculations we can derive the poloidal flux function to the first order in ε.

$$\psi(r,x) = \left(1 - \frac{1}{2}r^2\right)\frac{r^2}{4} + \varepsilon x\left(1 - \frac{5}{9}r^2\right)\frac{3r^2}{16} + \frac{\kappa}{6}x^4. \tag{6}$$

In Fig. 2 we plot the level sets of ψ obtained from the local solution (6) for different ellipticities. The condition for off-axis critical points $|x_c| < r_c$ leads to $|\kappa| \gtrsim \varepsilon/8$, from which we obtain the bifurcation values depicted in Fig. 2-right. As the ellipticity and toroidicity are equilibrium parameters, they are related in a continuous way to the anisotropy η, defined before (4). In Fig.3-left is clear that increasing the elongation leads to a growth of the anisotropy and the toroidicity gives an implicit anisotropy to the system. Consequently, η only vanishes in the cylindrical case and never becomes negative, so it verifies $I_+ > 2I_-$. During the bifurcation at

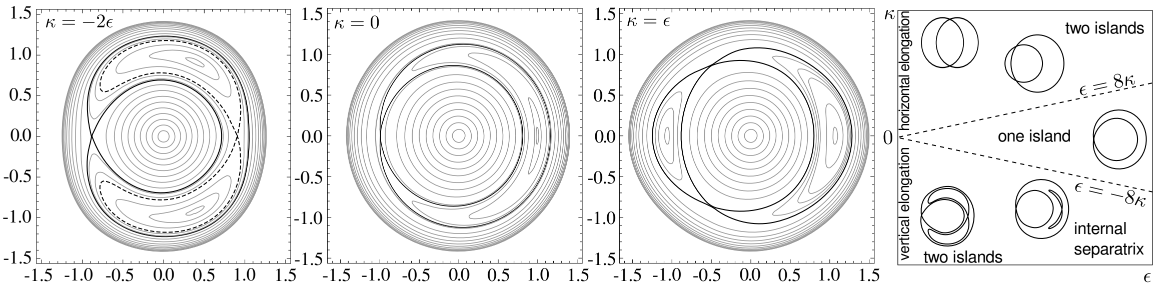

Figure 2: Levels sets of ψ from (6) for $\varepsilon = 0.1$ and different ellipticities κ. In the rightmost frame we illustrate the change of the equilibrium topology in the parameter space.

$\kappa = \varepsilon/8$ a zero-current island is created leaving η stationary (Fig. 3).

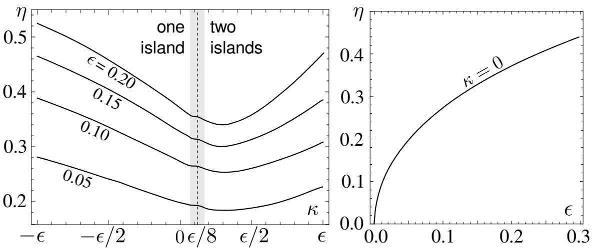

Figure 3: Change in the anisotropy η as a function of the ellipticity κ for different values of ε (left). Intrinsic anisotropy due to the toroidicity with zero ellipticity (right). The gray region is about the bifurcation value $\kappa = \varepsilon/8$ where η is stationary.

In summary, the magnetic topology related to current density reversals defines several current channels within the plasma. The ratio between the current in the positive channels and the central negative current depends on a topological parameter measuring the anisotropy of the positive channels. In general terms the positive current is about twice the size of the central negative current causing the screening of this channel and forming a structure with net positive current. The anisotropy was shown to be related to the geometrical properties of the equilibrium for a local solution of the equilibrium.

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