Diamagnetic effects and Landau damping on geodesic acoustic modes

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Introduction: Due to their role in suppression of turbulence in tokamaks [1], geodesic acoustic modes (GAM) [2], also recognized as a high frequency branch of zonal flows (ZF), have been widely investigated in fluid [3, 2] and kinetic [4, 5] models. The kinetic model predict the GAM frequency to be of the form: $\omega_{\text{gam}}^2 = (\Omega_{\text{g0}} + \text{smaller order terms})v_{T_i}/R_0$ where $\Omega_{\text{g0}}^2 = 7/4 + \tau_e$, $\tau_e = T_e/T_i$, R_0 is the major radius of the tokamak and $v_{T_i}^2 = 2T_i/m_i$.

GAM are believed to be part of drift wave (DW) turbulence and for this reason investigation of diamagnetic effects caused by density and temperature gradients may help to understand the turbulence suppression mechanism, which is an essential issue in fusion science. In [6] using a two fluid model it was shown that for $\eta_i = \partial_r \ln T_i/\partial_r \ln n_0 > 3/4$, a non oscillatory instability takes place and the growth rate of this instability is proportional to $\eta_i\omega_{*e}$, where $\omega_{*e} = T_e/eBrL_N$ is the electron drift frequency, $L_N^{-1} = \partial_r \ln n_0$ is the characteristic scale scale length of the density gradient. Corrections of $\mathscr{O}(q^{-2})$ (q is the safety factor) which was not included in [6] is computed in this paper up to first order.

It is also important to determine the characteristic time scale for the GAM existence which can be obtained from the computation of the damping rate. Collisionless damping via Landau mechanism is a fundamental proccess characterised by the wave-particle interaction was investigated in [7, 5] considering GAM without diamagnetic effects.

Here we complement the work of [6] by using a gyrokinetic linearised model to investigate GAM and their related modes in the presence of density and ion temperature gradients (diamagnetic effects) taking into account ion Landau damping. Three modes are found and their respective collisionless damping rate.

The model: We start with the gyrokinetic equation as derived by[8],

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_{g\alpha}\cdot\nabla\right)\tilde{g}_\alpha = e_\alpha J_0(k_\perp v_\perp/\omega_{c_\alpha})\left(\frac{\partial F_{M_\alpha}}{\partial \mathscr{E}_\alpha}\frac{\partial}{\partial t} + \frac{\mathbf{b}\times\nabla F_{M_\alpha}}{m_\alpha\omega_{c_\alpha}}\cdot i\mathbf{k}_\perp\right)\tilde{\Phi} \tag{1}$$

to find the perturbed distribution function, $\tilde{f}_\alpha = e_\alpha\tilde{\Phi}\partial F_{M_\alpha}/\partial\mathscr{E}_\alpha + \tilde{g}_\alpha \exp(i\mathbf{k}_\perp\cdot\boldsymbol{\rho}_\alpha)$, which is an expansion of the equilibrium energy, $\mathscr{E}_\alpha = v^2/2m_\alpha$ ($\Phi_0 = 0$ in non rotating systems). The subscript α labels for ions and electrons of the hydrogen plasma that we consider, e_α, m_α and $\omega_{c_\alpha} = eB/m_\alpha > 0$ stands respectively for the charge, mass and gyrofrequency, F_{M_α} is the

Maxwellian equilibrium distribution, $J_0(x)$ is the zero order Bessel function, $\rho_\alpha = \mathbf{b}\times\mathbf{v}_\perp/\omega_{c_\alpha}$ is the vectorial gyroradius and $\mathbf{k}_\perp \approx \hat{\mathbf{e}}_r \mathrm{k}_r + \hat{\mathbf{e}}_\theta \hat{k}_\theta$ is the perpendicular wave vector. We consider perturbations with intermediate wave length, $r^{-1} \ll \mathrm{k}_r \ll \rho_i^{-1}$. The following methodology is then applied: First, by solving the gyrokinetic equation we obtain the perturbed distribution function as $\tilde{f}_\alpha = \tilde{f}_\alpha(\tilde{\Phi}(r),\theta)$, where $\tilde{\Phi}$ is the perturbed electrostatic potential. Then integration of $\tilde{f}_\alpha$ in velocity space, $\langle ... \rangle = \int_0^{2\pi} d\gamma \int_0^\infty dv_\perp v_\perp \int_{-\infty}^{\infty} dv_\parallel (...)$ must be performed to find the density, $\tilde{n}_\alpha$. In this part it is useful to use the relation $\overline{\exp(i\mathbf{k}_\perp\cdot\rho_\alpha)} = J_0(k_\perp v_\perp/\omega_{c_\alpha})$ [8] where $\overline{(...)}$ denotes the average over the gyro-angle, γ. Since GAM are low frequency modes, by determining $\tilde{n}_\alpha$ we can apply the quasi-neutrality condition, $e(\tilde{n}_i - \tilde{n}_e) \approx 0$, to obtain the desired frequency in the continuum.

In the equilibrium the guiding center velocity is governed by the parallel motion and the vertical drift due to the gradient and curvature of the magnetic field (B), i. e.,

$$\mathbf{v}_{g\alpha} = v_\parallel \mathbf{b} + \frac{\mathbf{b}}{\omega_{c_\alpha} R_0} \times \left(\frac{v_\perp^2}{2} \nabla \ln B + v_\parallel^2 \kappa \right), \tag{2}$$

where $\mathbf{b} = \mathbf{B}/B$ and $\kappa = \mathbf{b}\cdot\nabla\mathbf{b}$. For a high aspect ratio tokamak ($r \ll R_0$) the calculation of $\mathbf{v}_{g\alpha}$ in (2) is simplified and from the substitution of (2) in (1) we obtain a tractable equation,

$$\left[1 - \frac{\hat{k}_\parallel v_\parallel}{\omega} + \frac{\omega_{\mathrm{d}\alpha}}{\omega}\left(\frac{v_\perp^2}{2v_{T_\alpha}^2} + \frac{v_\parallel^2}{v_{T_\alpha}^2}\right)\sin\theta\right]\tilde{g}_\alpha = J_0(b_\alpha)\frac{e_\alpha F_{M_\alpha}}{T_\alpha}\left(1 - \frac{\omega_{*\alpha}}{\omega}\nabla \ln F_{M_\alpha}\right)\tilde{\Phi}, \tag{3}$$

where $\hat{k}_\parallel = (\partial_\theta + q\partial_\phi)/qR_0$, $\omega_{\mathrm{d}\alpha} = v_{T_i}\mathrm{k}_r\rho_i/R_0$, $b_\alpha = \mathrm{k}_r v_\perp/\omega_{c_\alpha}$, $\omega_{*\alpha} = T_\alpha/e_\alpha B r L_N$ and $\nabla \ln F_{M_\alpha} = [1 + \eta_\alpha(v_\perp^2/v_{T_\alpha}^2 + v_\parallel^2/v_{T_\alpha}^2 - 3/2)]/L_N$. To solve (3) we consider only the poloidal $m = 0, \pm 1$ and toroidal $n = 0$ harmonics in $\tilde{g}_\alpha = \sum_{m,n} g^\alpha(r)\exp[i(m\theta - n\phi - \omega t)]$.

The quasi-neutrality condition can then be organized as follows:

$$\frac{e^2 n_0}{T_i}\begin{pmatrix} 1 & \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \mathscr{R}_{00} & \mathscr{R}_{0s} & \mathscr{R}_{0c} \\ \mathscr{R}_{s0} & \mathscr{R}_{ss} & \mathscr{R}_{sc} \\ \mathscr{R}_{c0} & \mathscr{R}_{cs} & \mathscr{R}_{cc} \end{pmatrix}\begin{pmatrix} \tilde{\Phi}_0 \\ \tilde{\Phi}_s \\ \tilde{\Phi}_c \end{pmatrix} = 0. \tag{4}$$

where $\mathscr{R}_{ab}$ $(a, b = 0, s, c)$ represent long expressions scaling to

$$\mathscr{R}_{00} \sim \mathrm{k}_r^2\rho_i^2, \quad \mathscr{R}_{s0} \sim \mathscr{R}_{0s} \sim \mathscr{R}_{0c} \sim \mathscr{R}_{c0} \sim \mathrm{k}_r\rho_i, \quad \mathscr{R}_{ss} \sim \mathscr{R}_{cc} \sim \mathscr{R}_{sc} \sim \mathscr{R}_{cs} \sim 1. \tag{5}$$

These terms are dependent of the plasma dispersion function, $Z(x) = \pi^{-1/2}\int_{-\infty}^{\infty} dy \exp(-y^2)/(y - x)$, coming from the integration of the parallel velocity. Further analytical computation is possible in the limit $|x| \gg 1$, in which, according with [9], $Z(x) \approx 2i\sqrt{\pi}\exp(-x^2) - (1/x + 1/2x^3 + 3/4x^5 + 15/8x^7 + \mathscr{O}(1/x^9))$ if we consider $Im(x) < 0$. The dispersion relation can be conveniently written as

$$\mathscr{D}(\Omega) = \mathscr{F}(\Omega) + \mathscr{K}(\Omega) i\sqrt{\pi} q^5 \exp(-q^2\Omega^2) \tag{6}$$

where

$$\mathscr{F}(\Omega) \approx \Omega^6 - (\Omega_{g0}^2 + 2\Omega_{s0}^2 + \Omega_{*e}^2)\Omega^4 + [(\tau_e - 1/2 - 23/4\tau_e)\Omega_{s0}^2 + (3/4 - \eta_i)\Omega_{*e}^2]\Omega^2 + [(15/2 + 9\tau_e/4)\Omega_{s0}^2 - (\eta_i^2 + 9\eta_i/2 - 17/4)\Omega_{s0}^2]\Omega_{s0}^2/\tau_e, \quad (7)$$

$$\mathscr{K}(\Omega) = \Omega^9 - \Omega^8 + \left[\frac{2}{\tau_e}\left(\Omega_{g0}^2 - \frac{3}{4}\right)\Omega_{s0}^2 + (\eta_i - 1)\Omega_{*e}^2\right]\Omega^7 + \frac{2}{\tau_e}\left(\Omega_{g0}^2 - \frac{3}{4}\right) \times \left(1 - \frac{3}{2}\eta_i\right)\Omega_{s0}^2\Omega_{*e}\Omega^6 - \frac{2}{\tau_e}\left[\left(\frac{7}{2} + \frac{3}{\tau_e} - \frac{1}{\tau_e^2}\right)\Omega_{s0}^4 + \left(1 - \frac{3}{2}\eta_i\right)\eta_i\Omega_{*e}^2\right]\Omega_{s0}^2\Omega^5, \quad (8)$$

stands respectively for the fluid part and the kinetic correction due to Landau damping. The normalised frequencies in (6) – (8) are defined by: $\Omega = \omega R_0/v_{T_i}$, $\Omega_{g0} = (7/4 + \tau_e)^{1/2}$, $\Omega_{*e} = \omega_{*e}R_0/v_{T_i}$ and $\Omega_{s0} = \tau_e/2q^2$. To find the continuum spectrum, first we develop $\mathscr{D}(\Omega)$, where $\Omega = \Omega_R + i\Gamma$, in a series about the point $\Omega = \Omega_R$ (oscillatory part) up to first order in $\Gamma \ll \Omega_R$ (damping part). Then we separate the real and imaginary parts of $\mathscr{D}(\Omega) = 0$, which results in

$$\mathscr{F}(\Omega_R) \approx 0 \quad \text{e} \quad \Gamma = -\frac{\mathscr{K}(\Omega_R)}{\frac{\partial \mathscr{F}}{\partial \Omega}|_{\Omega=\Omega_R}}\sqrt{\pi}q^5 \exp(-q^2\Omega_R^2) \quad (9)$$

which can be solved iteratively in three asymptotic limits, $\Omega \sim \Omega_{g0}$ (GAM branch), $\Omega \sim \Omega_{s0}$ (sound branch) and $\Omega \sim \Omega_{*e}$ (diamagnetic branch) where $\Omega_{g0} \gg \Omega_{s0}, \Omega_{*e}$ and $\Omega_{s0} \gg \Omega_{*e}$ and $\Omega_{*e} \gg \Omega_{s0}$ are considered for the sound and diamagnetic classes of solutions respectively. Also the condition $\Omega_{s0} \gg 1/q$, which is attained when $\tau_e \gg 1$, must be satisfied for the validity of this model. The three solutions and their respective damping rates are then given by:

$$\Omega_G^2 = \Omega_{g0}^2 + (\tau_e + 4 + 23/4\tau_e)(\Omega_{s0}^2/\Omega_{g0}^2) + (\tau_e + 1 + \eta_i)(\Omega_{*e}^2/\Omega_{g0}^2) \quad (10)$$

$$\Gamma_G = -\left\{\Omega_{g0}^4 + \left(\frac{5}{2}\Omega_{g0}^4 - \frac{9}{4}\Omega_{g0}^2 + \frac{29}{32}\right)\frac{1}{q^2} + \eta_i\Omega_{g0}^3\Omega_{*e} + \left[(1 + \eta_i)\Omega_{g0}^2 + \eta_i - \frac{3}{4}\right]\Omega_{*e}^2\right\} \times \frac{q^5}{2}\sqrt{\pi}\exp(-q^2\Omega_G^2), \quad (11)$$

$$\Omega_S^2 = \left(1 + \frac{7}{4\tau_e^2}\right)\Omega_{s0}^2 + \left[\left(\frac{3}{4} - \eta_i\right)\tau_e + \frac{5}{4} - \eta_i\left(\eta_i + \frac{1}{2}\right)\right]\Omega_{*i}^2 \quad (12)$$

$$\Gamma_S = -\left\{\left(1 - \frac{3\tau_e}{4} + \tau_e^2\right)\frac{1}{q^2} + \sqrt{\frac{2}{\tau_e}}\left[\frac{17}{8}\eta_i - \frac{57}{8} + \left(3 - \frac{5}{4}\eta_i\right)\tau_e + \left(\frac{\eta_i}{2} - 1\right)\tau_e^2\right]\frac{\Omega_{*e}}{q} - \frac{1}{2}\left[4 + (\eta_i - 1)\tau_e + 3\eta_i\left(\eta_i - 2\right)\right]q\Omega_{*e}^2\right\}\frac{q}{4}\sqrt{\pi}\exp(-q^2\Omega_S^2), \quad (13)$$

In the diamagnetic branch for $\Omega_{*i} \gg 1/q$ and $\tau_e \gg 1$, considering $\eta_i \ll 3/4$ or $\eta_i \gg 3/4$ the solution can be approximated by

$$\Omega_D^2 = \left(\frac{3}{4} - \eta_i\right)\frac{\Omega_{*e}^2}{\Omega_{g0}^2} + \frac{4[\Omega_{g0}^2\eta_i^2 + (\Omega_{g0}^4 + \Omega_{g0}^2/2 - 29/16)\eta_i] - (3\Omega_{g0}^4 + 5\Omega_{g0}^2 - 87/16)}{(4\eta_i - 3)\tau_e}\frac{\Omega_{s0}^2}{\Omega_{g0}^2}, \quad (14)$$

The instability occurs at a greater value, $\eta_i > 3/4 + (\tau_e + 11/2 + 19/4\tau_e)(\Omega_{s0}^2/\Omega_{*e}^2)$, then that obtained by [6], in which q is considered to be infinity.

For $\eta \approx 0$, i. e. no temperature gradient, the damping rate in the diamagnetic branch is

$$\Gamma_{\rm D} = -\left(\Omega_{\rm g0}^2 - \frac{3}{4}\right)\frac{\Omega_{*e}^4}{\Omega_{\rm g0}^8}\left[\frac{9}{16}\Omega_{*e}^2 - \frac{3}{4}\left(\Omega_{\rm g0}^4 - \sqrt{3}\Omega_{\rm g0}^3 - \frac{55}{24}\Omega_{\rm g0}^2 - \frac{87}{32}\right)\frac{1}{q^2}\right]\frac{\sqrt{\pi}}{2}\exp\left(-q^2\Omega_{\rm D}^2\right)(15)$$

In figure 1 the three branches of the GAM frequency for $\eta_i \approx 0$ are plotted as a function of q.

Discussion: We observe that in regions of lower values of q (close to the centre of the plasma column) higher ion temperature gradients are necessary to drive the diamagnetic instability. On the other hand in these regions the damping rate are higher than for low values of q, according with figure 1. This effect could be used to explain the difficult in detecting the GAM frequency near the centre. Although higher order terms in q^{-2} are necessary for results closer to experimental data, this model provides a physical comprehension of the Landau mechanism and diamagnetic effects on the GAM.

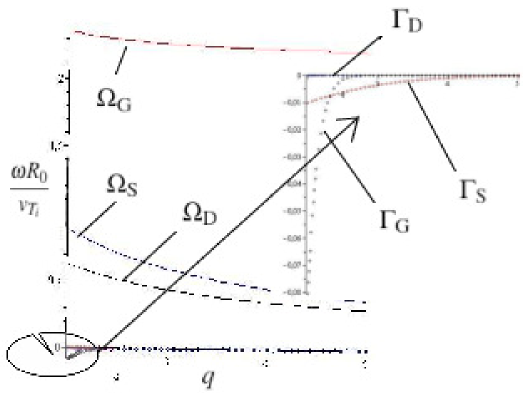

Figure 1: Characteristic GAM frequencies and its damping rates *vs* q.

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