Particle-string formation in complex plasmas with ion flows

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Attractive forces between like-charged particles in a complex (dusty) plasma are one of the most intriguing phenomena in this field. For particles suspended in the sheath region of a low-pressure plasma, an attractive force arises from Coulomb scattering of the flow of positive ions by a negatively charged upstream particle, which leads to positive charge accumulation in the wake of that particle and a resulting attraction of a second particle downstream [1]. Wakefield attraction in supersonic ion flows ($v > v_B = (kT_e/m_i)^{1/2}$) is well understood in terms of linear response theory [2] and particle-in-cell simulation [3].

Recently, we had reported the formation of particle strings for a region of reduced dust density inside the plasma bulk [4]. This experiment was performed under microgravity conditions and the estimated speed of the ambipolar ion flow was subsonic, $v < 0.1 v_B$. Calculations of the restoring forces by subsonic ion flows in terms of a trajectory model [5] and PIC simulations [6] demonstrate that the restoring force by wake charging vanishes below a critical Mach number $M \approx 0.3 - 0.4$. We have reanalyzed this phenomenon by molecular dynamics simulations with 'Yukawa-ions' that are shielded by thermal electrons [7] (see Fig. 1), which are in fair agreement with PIC-simulations [6]. Because all three approaches confirm the vanishing at low Mach numbers of the restoring force by ion drag, alternative mechanisms for the formation of strings will be discussed in the following.

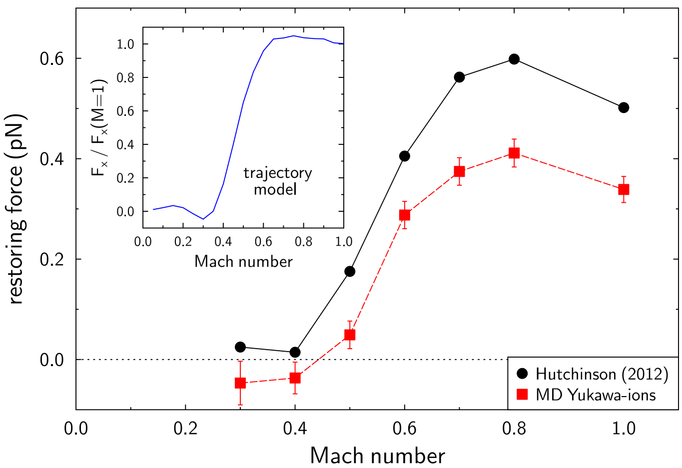

Figure 1: Restoring force transverse to the flow for a pair of particles of 12μm radius as a function of the Mach number from PIC-simulation [6] and molecular dynamics simulation [7]. The inset shows the same effect for a 3μm particle from a trajectory model [5].

First, we give a more detailed description of the experimental situation. String formation is found in a dilute dust region in the midplane of the parallel plate reactor [see Fig. 2(b)]. In panel (c) of that figure, a magnification of this region is shown as a superposition of an initial state (red) and a state 100 ms later (green). The typical string structure becomes evident with a smaller particle spacing inside the string and a larger spacing between strings. Most of the strings are

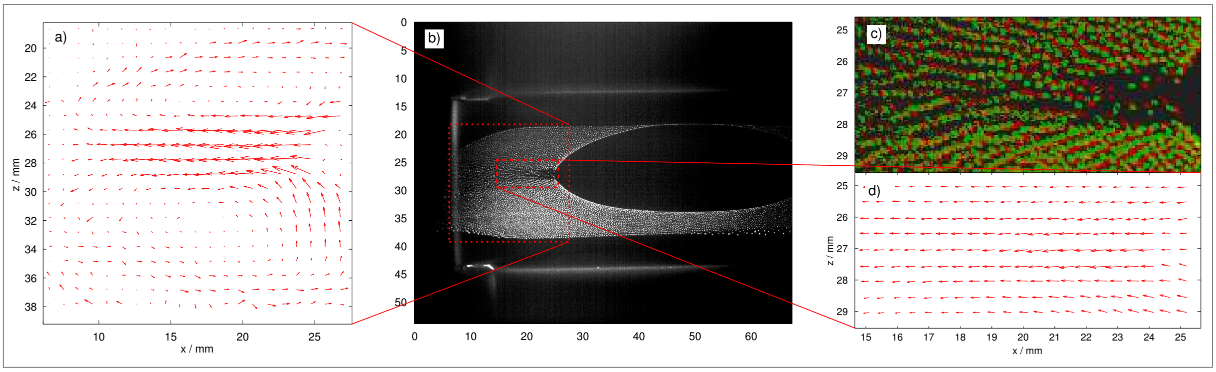

Figure 2: Vertical cross section of the parallel-plate discharge. (a) Formation of a double-vortex in the particle flow under microgravity, (b) the dust cloud with the central void, (c) superimposed snapshots of the particle arangements at 100 ms delay, (d) the flow pattern in the same region.

found tilted w.r.t. the midplane. The strings are slowly displaced to the left, i.e., away from the central void region, at a velocity of $\approx$ 3mm s^{-1}. A PIV[1]-analysis of the time-averaged flow shows that the dust flow in this region has parallel stream lines and moderate shear. The global dust flow has the topology of a double vortex [Fig. 2(a)]. While a balance of ion drag force and electric field forces governs the void boundary, we conjecture that in the dilute dust region, where the electric field is low, the dust motion is primarily driven by a double vortex in the neutral gas flow. Global flow patterns of the neutral gas were reported before for magnetized dusty plasmas [8]. There, the driving force was an ion flow that transfers momentum to the neutral gas by charge-exchange and elastic collisions.

Here, the ion flow is the ambipolar plasma flow out of the void region. An order-of-magnitude estimate for the neutral speed v_n in the midplane can be obtained by replacing the double vortex by a shear flow in plane geometry . We assume that the driving force is deposited in an input volume V that is bounded by the box that represents panel (c) of Fig. 2. In this volume, ions transfer momentum to neutrals by collisions, which gives a driving force $F_{\rm gain} = n_{\rm i} m_{\rm i} v_{\rm i} \nu_{\rm in} V$. Here, $n_{\rm i} \approx 10^{15}\,{\rm m}^{-3}$ is the ion density, $m_{\rm i} = 6.64 \times 10^{-26}$ kg is the ion mass, $\nu_{\rm in} = 4.13 \times 10^{6}\,{\rm s}^{-1}$ is the ion-neutral collision rate at $p = 30$ Pa, $v_{\rm i} \approx 100\,{\rm m\,s}^{-1}$ is the typical ion speed estimated in Ref. [4]. This force is balanced by viscosity in the sheared neutral flow, which is given by the viscous force $F_{\rm loss} = \eta A {\rm d}v/{\rm d}z$, that depends on the velocity gradient ${\rm d}v/{\rm d}z \approx v_{\rm n}/r$, which has a scale length comparable to the vortex radius $r = 4$ mm. The viscosity of argon at room temperature is $\eta = 22 \times 10^{-6}$ Pa s. Because the volume V drives a double vortex, the loss surface A has to be taken as twice the contact surface with the electrodes, which has a width of 10 mm and yields a ratio $V/A \approx 2.5$ mm. The predicted neutral velocity then is $v_{\rm n} = 6.2\,{\rm mm\,s}^{-1}$, which is larger than the observed speed. We further conjecture that the dilute dust region may be a consequence of the inhomogeneous flow speed in the dust vortex flow.

A quite different approach to the alignment of particles in a string can be based on considering the assembly of strings as a kind of porous matter with aligned pores. Then the question of

[1]particle image velocimetry

string formation transforms into the problem of string stability. This basic idea is explored by assuming that a string of particles is embedded in a cylindrical cavity of radius R inside a homogeneous arrangement of particles with repulsive Yukawa force

$$F_{\mathrm{Y}}(r) = \frac{q_d^2}{4\pi\varepsilon_0 r_{ij}^2}\left(1+\frac{r_{ij}}{\lambda}\right)\exp\left(-\frac{r_{ij}}{\lambda}\right) . \qquad (1)$$

The radial restoring force on a test particle at a radial distance ρ from the cylinder axis is then obtained by integration of this force over the entire ambient particle cloud. With an interparticle distance Δ between the particles in the dust cloud and normalizing the force $F_{\mathrm{r}}' = F_{\mathrm{r}}/F_{\mathrm{C}}$ by the Coulomb force $F_{\mathrm{C}} = q_{\mathrm{d}}^2/(4\pi\varepsilon_0\Delta^2)$, normalizing the coordinates r, z and the considered position ρ by the cavity radius R, and defining the shielding strength $\kappa = R/\lambda$, we obtain

$$F_{\mathrm{r}}'(\rho') = -2(n_{\mathrm{d}}\Delta^3)\frac{R}{\Delta}\int_0^{2\pi} \mathrm{d}\varphi \int_1^{\infty} r'\mathrm{d}r' \int_0^{\infty} \mathrm{d}z'(1+\kappa d')\frac{\exp(-\kappa d')}{d'^3}(r'\cos\varphi - \rho') , \qquad (2)$$

in which the distance of the test particle from the considered volume element is $d'(\rho'; r', z', \varphi) = [r'^2 - 2r'\rho'\cos\varphi + \rho'^2 + z'^2]^{1/2}$. The last term in the integrand denotes the projection of the force in the direction of the displacement ρ. The interparticle distance is related to the Wigner-Seitz radius $a_{\mathrm{WS}} = [3/(4\pi n_{\mathrm{d}})]^{1/3}$ by $\Delta \approx 1.8a_{\mathrm{WS}}$, which gives $n_{\mathrm{d}}\Delta^3 = 1.4$. The normalized restoring force depends only on the ratio R/Δ, the normalized position ρ' and the shielding strength κ. The integral is numerically evaluated for $R/\Delta = 1.5$, which is very close to the experimental value $R/\Delta = 1.44$. In this way, the ambient particles around the cylindrical cavity generate a net restoring force that pushes a string towards the cylinder axis, see Fig. 3(a). Moreover, for small displacements ρ', the confinement is harmonic $F_r'(\rho') = -m\omega_0^2\rho'$.

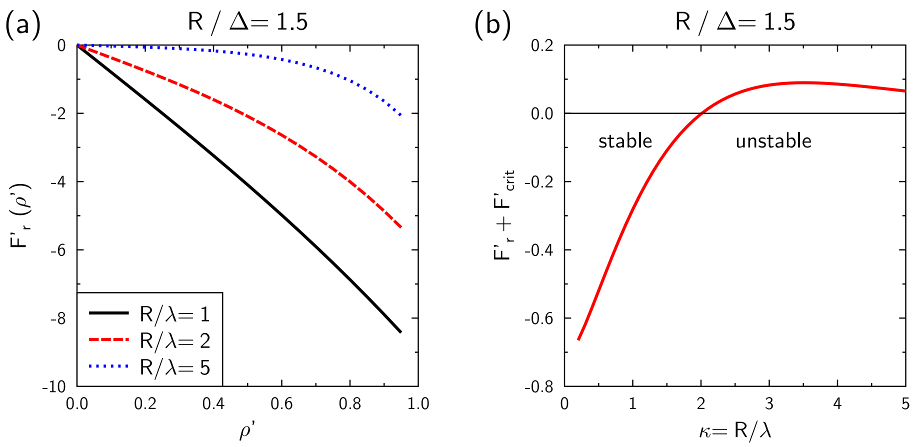

Figure 3: (a) The normalized restoring force $F_r'(\rho)$ for different values of the shielding factor κ. (b) The sum of the restoring force and the critical force for onset of the zig-zag instability at a normalied displacement $\rho' = 0.1$ as a function of the shielding factor κ.

The stability of the string against the zig-zag instablity can be analyzed using the model of an infinitely long string with nearest-neighbor interaction in a cylindrical harmonic potential trap. When we denote the transverse elongation of the n-th particle in the string by x_n and its position in the unperturbed string by $z_n = n\Delta$, the dynamics of the string is governed by the equation

$$m_d \ddot{x}_n = -m_d \omega_0^2 x_n + \frac{F_\Delta}{\Delta}(2x_n - x_{n-1} - x_{n+1}) . \quad (3)$$

Setting $x_n(z) = \hat{x}\exp[\mathrm{i}(kz - \omega t)]$ yields the dispersion relation $\omega^2 = \omega_0^2 - 4\omega_\Delta^2 \sin^2(k\Delta/2)$ with $\omega_\Delta^2 = F_\Delta/(m_\mathrm{d}\Delta)$.

Stable transverse modes are found for positive values of the r.h.s.. The maximum unstable mode is found for $k\Delta/2 = \pi/2$. The threshold for that mode is given by $\omega_\Delta^2 = \frac{1}{4}\omega_0^2$. From this condition, a critical (normalized) force can be defined

$$F'_\mathrm{crit} = 4\left(1 + \kappa\frac{\Delta}{R}\right)\exp\left(-\kappa\frac{\Delta}{R}\right)\rho'\frac{R}{\Delta} . \quad (4)$$

In Fig. 3(b), the stability analysis is discussed in terms of the net force $F'_\mathrm{r} + F'_\mathrm{crit}$ for a normalized displacement $\rho' = 0.1$ as a function of the shielding factor κ. It turns out that for $\kappa < 2$ the net force is negative, i.e., the restoring force overcomes the critical force and results in a stable string on the axis of the cylindrical cavity. In this way, stable string structures can be the result of the geometrical constraint by the cavity.

In summary, the role of ion drag forces, neutral flows and string stability in cylindrical cavities have been discussed in the context of string formation at low Mach numbers. We argue that the observed string pattern involves both the vortex flow and stabilization of the strings by suppression of the zig-zag instability.

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