Tokamak MHD Equilibria with 3D Distortions

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The main goal of the hybrid scenario proposed in ITER is to prevent sawteeth oscillations by operation with a safety factor $q > 1$ throughout the plasma. The basic characteristics in such scenario correspond to conditions of weakly reversed core magnetic shear with minimum value of q_{min} above unity [1]. However, in tokamaks like MAST, the discharges evolve such that q_{min} approaches unity and a nonresonant $m = 1, n = 1$ Long-Lived Mode (LLM) develops [2]. These core internal structures can be simulated as static three-dimensional (3D) magnetohydrodynamic (MHD) equilibrium states [3] with the VMEC and ANIMEC stellarator solvers [4, 5]. Previously, fixed boundary ITER modelling with ANIMEC has demonstrated that bifurcated MHD equilibrium solutions exist; one branch is virtually axisymmetric and the second branch displays a helical core [6]. We extend in this work the application to free boundary solutions in ITER, though for simplicity we have retained so far up-down symmetric conditions. The vacuum magnetic fields from the currents in the ITER toroidal and poloidal field coils are calculated and provided as input for the equilibrium computations. Unlike the fixed boundary simulation, the internal helical displacement of the 3D branch solution can modify the shape of the last closed magnetic flux surface when the plasma-vacuum interface is allowed to move.

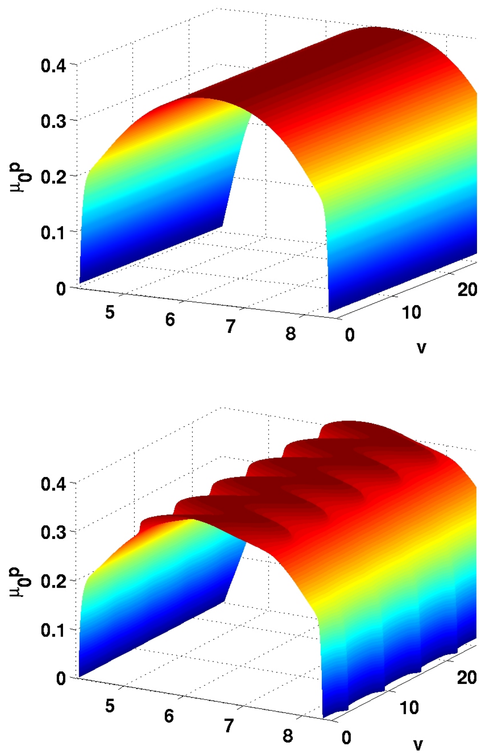

Figure 1: *The pressure distribution at the midplane ($4 < R < 8.5m$) in ITER as a function of the toroidal angle v for the axisymmetric equilibrium branch solution (top) and the helical branch solution (bottom).*

Bifurcated equilibrium solutions in ITER with free boundary in hybrid scenario conditions (weak reversed central shear) exist when q_{min} is close to unity at a radial position $\sqrt{s} \sim 0.427$. This illustrated in Fig. 1 where the amplitude of the pressure at the midplane over 5 toroidal transits is plotted for the axisymmetric branch solution and for the helical branch solution. A large core helical distortion with snake-like characteristics appears in the 3D equilibrium state. At the outer boundary around $R \sim 8.45m$, there is also an observable deformation dominated by a $m = 1, n = 1$ structure. In the axisymmetric solution, the outer boundary remains unperturbed.

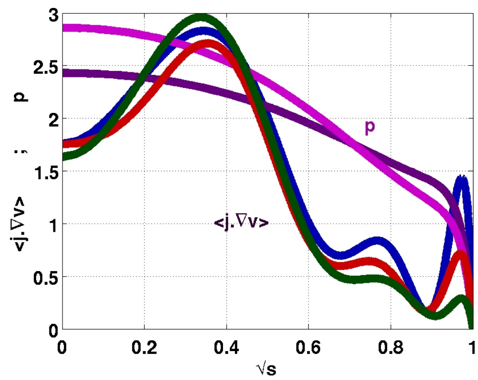

Figure 2: *The input pressure p and toroidal current $\langle \mathbf{j} \cdot \nabla v \rangle$ as a function of $\sqrt{s}$. The radial variable s is proportional to the enclosed toroidal magnetic flux.*

We investigate the sensitivity of the $m = 1, n = 1$ modulation at the outer boundary by independently varying the edge pressure gradient and the edge bootstrap current. We choose two pressure profile with large and small p' and the current profile with large, small and tiny edge bootstrap currents j_b. The value of the volume averaged $\langle \beta \rangle$ is fixed at 1.77% and the total toroidal current can vary from 11.7 to $14.9MA$. The profiles are displayed in Fig. 2. The internal helical core that is generated under hybrid scenario conditions when q_{min} approaches unity extends to the plasma boundary. However, the $m = 1, n = 1$ modulation at the outer edge depends on p' and j_b, as demonstrated in Fig. 3. For large p' and j_b, the peak to trough variation in ITER reaches $0.12m$. Maintaining j_b, but reducing the p' by half at the edge results in a reduction to about $0.09m$. The phase, however, is shifted by 180^o. This is may not be an effect of great significance because there are 18 toroidal field coils, hence

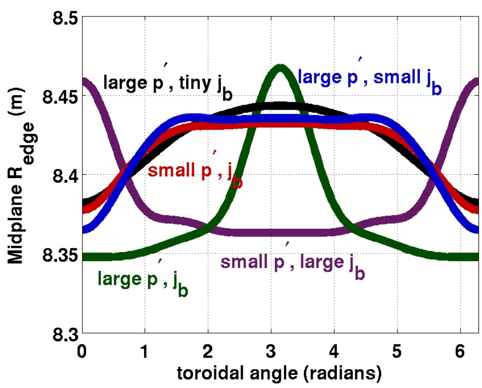

Figure 3: *The modulation of R at the midplane outer boundary in ITER as a function of the toroidal angle for five cases at $\langle \beta \rangle \simeq$ 1.77%: large p', large j_b (green), small p', large j_b (purple), large p', small j_b (blue), small p', small j_b (red), large p', tiny j_b (black).*

an 18-fold degeneracy in phase is possible. The outer boundary for the case of large edge p' and small j_b varies once around the torus by about $0.065m$. The variation of R at the edge for the cases with small p' - small j_b and large p' - tiny j_b are about $0.05m$ and $0.055m$, respectively.

The formulation of the MHD equilibrium problem in axisymmetric plasmas with finite isothermal toroidal rotation [7] can be resolved by the same energy minimisation technique as that applied to 3D static plasmas [8]. The first variation of the energy functional

$$\mu_0 W = \int\int\int d^3x \left[\frac{B^2}{2} + \frac{\mu_0 p(s,R)}{\Gamma - 1} \right] \tag{1}$$

yields the stationary MHD equilibrium state described in Ref. [7]. An approximate formulation of a stationary MHD equilibrium description in a 3D plasma with nested magnetic flux surfaces is proposed. For this purpose, we assume pure toroidal flow in cylindrical geometry. We write

$$V = RV^{\phi} \nabla Z \times \nabla R = R^2 V^{\phi} \nabla \phi \tag{2}$$

and impose that $V^{\phi} \equiv V \cdot \nabla \phi = \Omega(s)$. The MHD force balance relation then can be expressed as

$$\mu_0 \nabla F = -\mu_0 \nabla p + \frac{1}{2} \mu_0 \rho_M \Omega^2(s) \nabla R^2 + (\nabla \times B) \times B - \mu_0 \rho_M R^2 \Omega'(s)(V \cdot \nabla s) \nabla \phi = 0. \tag{3}$$

The complexity induced by the last term in Eqn. 3 conspires to severely limit any further analytical progress. Hence, another set of approximations is now considered based on experimental results in MAST. The toroidal plasma rotation flattens in the presence of a LLM in the core region of MAST discharges [2]. Resonant Magnetic Perturbations (RMP) greatly diminish (or even eliminate) the core toroidal flow to that at the pedestal level. Consequently, one of 2 approximations can be applied: a) $V \cdot \nabla s \simeq 0$ (the flow is basically toroidal and the flux surface geometry is only weakly 3D) or b) rigid flow $\Omega'(s) \simeq 0$ in domains with large 3D equilibrium structures like LLM or snakes. The solution of the parallel force balance relation under isothermal conditions $[T = T(s)]$ then yields

$$p(s,R) = P_0(s) exp \left[\frac{M_i}{4T(s)} \Omega(s)^2 R^2 \right]. \tag{4}$$

The MHD force balance becomes

$$\mu_0 \nabla F = -\mu_0 \left. \frac{\partial p}{\partial s} \right|_R + (\nabla \times B) \times B = 0 \tag{5}$$

and as a result, the energy functional in the axisymmetric limit described by Eqn. 1 applies to stationary 3D equilibria with isothermal toroidal rotation within the validity of the approximations invoked.

In conclusion, free boundary up-down symmetric ITER hybrid scenario bifurcated equilibrium states have been computed in the range of toroidal current $11.7MA \leq I_t \leq 14.9MA$ at $\langle\beta\rangle \simeq 1.77\%$ with weakly reversed core magnetic shear. The radial location of the q_{min} near unity is around $\sqrt{s} \sim 0.427$. The helical branch solution can also cause a deformation at the outer edge of the plasma boundary. The modulation depends mainly on the magnitude of the edge bootstrap current but also to a somewhat lesser extent on the size of the edge pressure gradient. An approximate formulation of the 3D MHD equilibrium problem with finite toroidal plasma rotation and imposed nested magnetic flux surfaces is proposed. This would be applicable to examine the impact of flows on Long-Lived Modes and on the screening of Resonant Magnetic Perturbations.

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