Influence of resonant magnetic perturbation on a rotating helical plasma

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Introduction

Physics of resonant magnetic perturbation (RMP), which is due to error fields and/or additional current coils, is of great interest to magnetic confinement fusion, because it might be applicable to a control method of magnetohydrodynamic (MHD) instability and turbulent transport. In tokamaks, it is well-known that screening effect of plasma flows plays a key role in suppression of RMP-driven magnetic islands, and excitation of magnetic islands is accompanied by slowing down of plasma flows and locking of mode rotation.

In helical systems, such as the Large Helical Device (LHD), sudden disappearance (self-healing) and appearance (penetration) of RMP-driven magnetic islands have been observed[1, 2]. These phenomena could be understood by the mode-locking theory, where the screening effect of neoclassical viscosity-driven plasma flows in helical systems plays an essential role[3, 4]. However, relationship between the mode-locking theory and the conventional theory of curvature-driven magnetic islands in helical systems has not been fully discussed. Recently, the mode-locking theory is revisited so as to take into account effect of averaged curvature in helical systems[5, 6]. In this study, the theoretical prediction is qualitatively compared with experimental observations in the LHD.

Simulation model

A model of RMP-driven magnetic islands in rotating helical plasmas is systematically derived in Ref.[5]. In the following, we outline the model.

Consider a helical plasma with an averaged minor radius a and a major radius R_0. Fluid equations are readily averaged in the toroidal direction. The toroidally-averaged equations are described in torus coordinates (r, θ, ϕ), where r is the averaged radial position, θ is the poloidal angle, and ϕ is the toroidal angle. Perturbation is dominated by a single mode with a poloidal mode number m and a toroidal mode number n, which is resonant at a rational surface $\iota = n/m$ located at $r = r_s$, where ι is the rotational transform normalized by 2π, and r_s is the averaged radial position of the rational surface. Near the rational surface, a boundary layer is formed, where an outer layer is described by ideal MHD and an inner layer is described by resistive MHD.

Without loss of generality, outer-layer poloidal magnetic flux perturbation is separated as $\psi_{\rm m}(r)\cos\Theta+\psi_{\rm c}(r)\cos(\Theta-\Delta\Theta)$, where $\Theta=m\theta-n\phi$ is the phase in rest frames of rotating magnetic islands and $\Delta\Theta$ is the phase difference between magnetic islands and vacuum magnetic islands by the RMP. Ideal MHD equations in currentless helical plasmas yield (misprints in Ref.[5] are corrected)

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi_{\rm m(c)}}{\partial r}\right)-\left(k_\theta^2-\frac{8\pi\kappa_{\rm h}k_\theta^2 p_0'}{B_0^2 k_\parallel^2}\right)\psi_{\rm m(c)}=0, \tag{1}$$

where $k_\parallel$ is the parallel wave number, $\kappa_{\rm h}(r)$ is the toroidally-averaged curvature, $k_\theta=m/r$, $p_0'(r)$ is the total pressure gradient, and B_0 is the toroidal magnetic field. Boundary conditions are: $\psi_{\rm m}(0)=\psi_{\rm c}(0)=0$, $\psi_{\rm m}(r_{\rm s}\pm w/2)=\psi_s$, $\psi_{\rm c}(r_{\rm s}\pm w/2)=0$, $\psi_{\rm m}(a)=0$, and $\psi_{\rm c}(a)=\psi_a$. Magnetic island width and vacuum island width are given by $w=4\sqrt{L_s\psi_{\rm s}/B_0}$ and $w_{\rm v}=4\sqrt{(r_{\rm s}/a)^m L_s\psi_{\rm s}/B_0}$, respectively, where L_s is the magnetic shear length at the rational surface. Tearing mode stability parameter is separated into two components:

$$\Delta'_{\rm mode}=\frac{1}{\psi_{\rm s}}\left[\frac{\partial\psi_{\rm m}}{\partial r}\right]_{r_{\rm s}-w/2}^{r_{\rm s}+w/2}, \tag{2}$$

$$\Delta'_{\rm coil}=\frac{1}{\psi_{\rm s}}\left[\frac{\partial\psi_{\rm c}}{\partial r}\right]_{r_{\rm s}-w/2}^{r_{\rm s}+w/2}, \tag{3}$$

where $x=r-r_{\rm s}$. In the limit of $\kappa_{\rm h}=0$, we obtain $\Delta'_{\rm mode}=\Delta'_0$ and $\Delta'_{\rm coil}=-\Delta'_0(w_{\rm v}^2/w^2)$, where $\Delta'_0=-2k_{\theta\rm s}/(1-r_s^{2m}/a^{2m})$ and $k_{\theta\rm s}=m/r_{\rm s}$. A finite value of $\kappa_{\rm h}$ modifies those quantities.

An asymptotic matching of the outer-layer current and the inner-layer current gives time evolution equations of magnetic islands:

$$\frac{4\pi I_1}{\eta_\parallel c^2}\frac{\partial w}{\partial t}=\Delta'_{\rm mode}+\Delta'_{\rm coil}\cos\Delta\Theta+\sum_{\alpha={\rm i,e}}\frac{I_3 D_\alpha}{\sqrt{w^2+(I_3/I_2)^2 w_{\rm c,\alpha}^2}}, \tag{4}$$

$$\frac{\partial\Delta\Theta}{\partial t}=k_{\theta\rm s}(v_\theta+v_{*\rm e})|_{r=r_{\rm s}}, \tag{5}$$

where w is the magnetic island width, $I_1=0.827$, $I_2=6.6$, $I_3=6.35$, $\eta_\parallel$ is the parallel resistivity, c is the velocity of the light, $w_{\rm c,\alpha}=(\chi_{\parallel\alpha}/\chi_{\perp\alpha})^{1/8}\sqrt{8L_s/k_{\theta\rm s}}$, $\chi_{\parallel\alpha}$ is the parallel thermal diffusivity, and $\chi_{\perp\alpha}$ is the perpendicular thermal diffusivity. Curvature parameters of ion fluid and electron fluid are defined by

$$D_\alpha=\frac{8\pi\kappa_{\rm hs}p'_{\alpha0\rm s}L_s^2}{B_0^2}, \tag{6}$$

for $\alpha=$ i,e, where $\kappa_{\rm hs}=\kappa_{\rm h}(r_{\rm s})$, and $p'_{\alpha0}$ is the α-species *unperturbed* pressure gradient at the rational surface.

Poloidal flows in helical plasmas are mainly driven by neoclassical viscosity. Near magnetic islands, coupling of the RMP-induced magnetic field and current perturbation drives Lorentz force. A poloidal flow evolution equation is given by

$$\frac{\partial v_\theta}{\partial t} = \sigma \frac{k_{\theta s} v_{As}^2}{512 L_s^2} w^3 \Delta'_{\mathrm{coil}} \sin\Delta\Theta + \sum_{\alpha=\mathrm{i,e}} \nu_\alpha^{\mathrm{neo}} \left(V_\alpha^{\mathrm{neo}} - v_\theta\right) + \mu \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r v_\theta\right)\right], \tag{7}$$

where $\nu_\alpha^{\mathrm{neo}}$ is the α-species neoclassical damping rate, V_α^{neo} is the α-species neoclassical flow velocity, μ is the phenomenological, anomalous momentum diffusion coefficient, v_{A} is the Alfvén velocity at the rational surface, $\sigma = 1$ for $|r - r_{\mathrm{s}}| \leq w/2$, and $\sigma = 0$ for $|r - r_{\mathrm{s}}| > w/2$.

Criteria of self-healing and penetration without curvature effects

Using Eqs. (1)-(7), thresholds of self-healing and penetration in various parameter regimes are obtained[5]. In the following, we focus on a case, where ion temperature equals to electron temperature, the neoclassical viscosity is dominated by the ion viscosity, the anomalous viscosity is approximated by the Bohm's diffusion coefficient, the curvature effect is not important, and the island width is much smaller than $\lambda = \sqrt{\mu/\nu_i^{\mathrm{neo}}}$. Typical parameters in the LHD are: $a = 60$[cm], $R_0 = 360$[cm], $r_{\mathrm{s}}/a = 0.85$, $B_0 = 1 - 2.75[T]$, $\iota(r_{\mathrm{s}}) = 1$, the magnetic shear at the rational surface is -2.4, and $w_{\mathrm{v}} = 11.6$[cm].

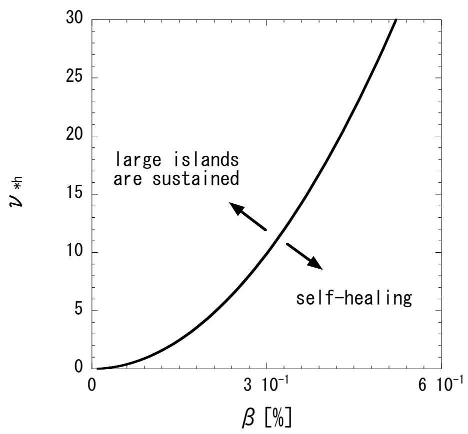

Figure 1: Self-healing threshold of RMP-driven magnetic islands.

Self-healing threshold of RMP amplitude is given by

$$\frac{B_{\mathrm{RMP}}}{B_0} = 1.06\, \varepsilon_{\mathrm{t}}^{1/4} \beta^{1/2} \nu_{*\mathrm{h}}^{-1/4} \rho_*^{3/4}, \tag{8}$$

where B_{RMP} is the RMP-induced magnetic field, $\varepsilon_{\mathrm{t}} = r_{\mathrm{s}}/R_0$, β is the ion (or electron) pressure normalized by magnetic pressure, $\nu_{*\mathrm{h}}$ is the collisionality normalized by $\varepsilon_{\mathrm{h}}^{3/2} r_{\mathrm{s}}/v_{\mathrm{ti}}$, $\varepsilon_{\mathrm{h}} B_0$ is the magnitude of rippled magnetic field, v_{ti} is the ion thermal velocity, and ρ_* is the ion Larmor radius normalized by r_{s}. Figure 1 shows a phase diagram of RMP-driven magnetic islands in $\beta - \nu_{*\mathrm{h}}$ space. A solid line is given by Eq. (8) as $\nu_{*\mathrm{h}} = 1.1 \times 10^6 \beta^2$, where in an evolution of the coefficient of this relation, we fixed ρ_* as $\rho_* = 2.0 \times 10^{-2}$ because change of ρ_* is small in the experiments. In Fig. 1, regions of sustainment and annihilation (self-healing) of large magnetic islands are shown. In the LHD, data of saturation states of RMP-driven magnetic islands are accumulated. In the experiments, a boundary between sustainment and annihilation is clear[1]. Our scaling qualitatively reproduces the experimental boundary.

Similarly, penetration threshold of RMP amplitude is given by

$$\frac{B_{\mathrm{RMP}}}{B_0} = 0.17\, \varepsilon_{\mathrm{t}}^{1/10} \varepsilon_{\mathrm{h}} \beta^{9/20} \nu_{*\mathrm{h}}^{-7/10} \rho_*^{3/4} s^{-1/5} \delta^{-1/5}, \tag{9}$$

where s is the magnetic shear at the rational surface, and δ is the ion skin depth normalized by r_{s}. This scaling is contrary to experimental observations shown in Ref.[7], where penetration threshold of the RMP amplitude is monotonically increasing function of the magnetic shear. This implies that there exists additional mechanism which enhances the penetration in low magnetic shear regime. The effect of the averaged curvature is one of candidates.

Criteria of self-healing and penetration with curvature effects

The criteria with the averaged curvature are also described in Ref.[5]. Figure 2 shows a phase diagram of RMP-driven magnetic islands in a space of $D = D_{\mathrm{e}} + D_{\mathrm{i}}$ and the RMP amplitude, where the thresholds are given by numerically solving Eqs. (1)-(7). In this analysis, D is amplified to examine the curvature effect. The self-healing threshold of the RMP amplitude becomes smaller in the presence of the unfavorable averaged curvature, i.e. RMP-driven islands tend to be sustained. The penetration threshold is not sensitive when the curvature is small, but becomes quite sensitive if the curvature-driven tearing mode becomes unstable, which enhances the penetration.

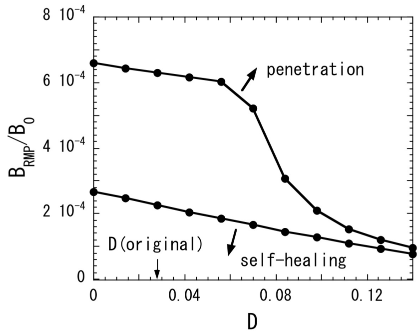

Figure 2: Stability diagram of RMP-driven magnetic islands.

Summary

The self-healing threshold of RMP-driven magnetic islands in $\beta - \nu_{*\mathrm{h}}$ space is qualitatively well reproduced by the mode-locking theory. While, the penetration mechanism of RMP is not fully understood, so far. In this study, we found that the unfavorable averaged curvature can modify the penetration threshold, however, further analysis is necessary to clarify this problem.

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