Suprathermal Electrons Flux and its Role in the Ambipolar Electric Field in Stellarators

Julio J. Martinell, Debjyoti Basu

Instituto de Ciencias Nucleares, UNAM, A. Postal 70-543, México D.F., Mexico

Introduction

In non-axisimmetric toroidal magnetic configurations such as stellarators the neoclassical transport is not intrinsically ambipolar as it is in axisymmetric tokamaks. The main contribution to the unequal transport of electrons and ions in low collisionality plasmas is due to trapped particles in the magnetic ripples. This gives rise to a radial electric field which can be measured over the whole plasma cross section using Heavy Ion Beam Probes (HIBP). Previous results for the calculated ambipolar electric field using neoclassical transport[1] have shown that this field is systematically smaller than the one measured by HIBP. A possible cause may be the suprathermal electrons that are created by the ECRH system used in most stellarators which are pumped out to the outer plasma. Here we explore this possibility by calculating the electric field produced by the suprathermal electrons. A kinetic description of the particle transport is followed in which a non-maxwellian velocity distribution is assumed for the electrons having a high velocity component. For the thermal particles, fluxes are computed from previous expressions due to Kovrizhnykh. Therefore only the high energy contribution is addressed to obtain a suprathermal flux. In this way we obtain an analytical expression for the electron fluxes which is used to obtain the ambipolar radial electric field. This electric field is compared with the one obtained from the thermal plasma and it will be shown that the population of suprathermal electrons should be less than 1%. The new flux is added to the thermal electron flux so that the relevant ambipolarity condition is

$$\Gamma_i = \Gamma_e + \Gamma_s. \tag{1}$$

Calculation of the electron flux. In order to obtain the macroscopic flux of suprathermal electrons it is necessary to start from kinetic theory. The drift kinetic equation is used together with a model collision operator for a magnetic geometry characteristic of a stellarator. Only suprathermal electrons are considered since it is assumed that thermal electrons and ions are described by the existing theories. Since high energy electrons have low collision frequencies it is possible to restrict the analysis to this regime. In the collisionality range contained between the bounce frequency in a ripple $\omega_b = \delta^{1/2} v_{the} N/R$ and the drift frequency in a superbanana

$\omega_{sb} = v_{the}^2 \delta/\omega_{ce} r^2$ (i.e. $\omega_{sb} < \nu_e/\delta < \omega_b$) the thermal particle fluxes are given by [2, 3]

$$\Gamma_j = -A(\alpha) n_j \left(\frac{T_j}{eBR}\right)^2 \frac{a_j}{\nu_j} \left[\frac{n_j'}{n_j} + b_j \frac{T_j'}{T_j} + e_j \frac{\Phi'}{T_j}\right] \tag{2}$$

where $A(\alpha)$ is a geometric coefficient that depends on the ripple parameter $\alpha = \varepsilon/qN\delta$, ν_j the collision frequency and $a_i = 27.42$, $a_e = 12.78$, $b_i = 3.37$, $b_e = 3.45$. Here the magnetic field is modeled by a single harmonic: $B \approx B_0[1 - \varepsilon\cos\theta - \delta\cos N\varphi]$. Using this same B-filed model and taking a momentum conserving collision operator $C(f)$ [4], the kinetic equation is solved for an equilibrium distribution function, $F_0(\mathbf{v})$, of the type expected for suprathermal electrons created by EC-heating. In particular. the energy perpendicular to the magnetic field is greatly increased thus creating an anisotropic distribution. The model $F_0(\mathbf{v})$ chosen is a ring Maxwellian in which the maximum in $v_\perp$ space is shifted forming a ring. It has the normalized form

$$F_0(\mathbf{r},\mathbf{v}) = n(\mathbf{r}) \left(\frac{m}{2\pi T_\parallel}\right)^{1/2} \frac{m K_\perp^p}{2\pi K_s^{1+p}\Gamma(1+p)} \exp(-K_\parallel/T_\parallel) \exp(-\frac{K_\perp}{K_s}) \tag{3}$$

where $K_\parallel$ and $K_\perp$ are the components of the kinetic energy, K_s represents the shift in $K_\perp$ and measures the energy of suprathermals and p is related to the perpendicular temperature. This function is shown in Fig.1. When solving the drift kinetic equation with heating term $H(f)$ for the gyroaveraged function,

$$\frac{\partial f}{\partial t} + (v_\parallel \hat{b} + \mathbf{v}_d) \cdot \nabla f + \frac{d\mathscr{E}}{dt}\frac{\partial f}{\partial \mathscr{E}} = C(f) + H(f) \tag{4}$$

F_0 is assumed to be the zeroth order equilibrium solution, i.e. $C(F_0) + H(F_0) = 0$. To next order, one needs to integrate the equilibrium equation at constant energy ($f = F_0 + \tilde{f}$)

$$v_\parallel \hat{b} \cdot \nabla \tilde{f} + \mathbf{v}_d \cdot \nabla F_0 = C(\tilde{f}) \tag{5}$$

where the contribution of heating to this order is neglected. The important effect in $C(f)$ comes from pitch angle scattering described by the magnetic moment μ. Standard procedure [2, 3] is used to solve this equation for the low collisionality regime mentioned above, for ripple trapped electrons, leading to $\tilde{f} = (\sin\theta_0/eR\nu)(\mu - \mu_m)F_0'$, where θ_0 labels filed lines, $\mu_m = K/B_{max}$ at maximum turning point and $\nu = \nu_{ee}(K) + \nu_{ei}(K)$.

From $\tilde{f}$ the electron flux is computed integrating in angles, K and μ obtaining the result

$$\Gamma_s = -A(\alpha) n_s \left(\frac{T_\parallel}{eBR}\right)^2 \frac{c_1}{\nu_e} \left[\frac{n_s'}{n_s} - \frac{T_\parallel'}{2T_\parallel} - e\frac{\Phi'}{T_\parallel} + (\frac{c_2}{\hat{K}} - 2)\frac{\hat{K}'}{\hat{K}}\right] \tag{6}$$

where $A(\alpha)$ is the same geometrical coefficient of Eq. (2), $\hat{K} = K_s/T_\parallel$ represents the energy of suprathermal electrons relative to parallel thermal energy and the coefficients c_1 and c_2 are

$$c_1 = \frac{I_1^p}{\hat{K}^{1+p}\Gamma(1+p)},\ c_2 = \frac{I_2^p}{I_1^p} \text{ with } I_n^p = \int_0^\infty \frac{x^{3+p+n}e^{-x/\hat{K}}dx}{H(x)},\ \ \eta(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t}t^{1/2}dt \tag{7}$$

and $H(x) = \eta(x) + \eta'(x) - \eta(x)/2x + \eta(\hat{x}) + \eta'(\hat{x}) - \eta(\hat{x})/2\hat{x}$, $\hat{x} = m_i/m_e x$ comes from the energy dependence of collision frequency. They are computed numerically for a constant value of $\hat{K}(r)$ taken as its maximum. Some of their values are given in the Table. It is seen that the flux increases strongly as the energy of the supathermal electrons increases.

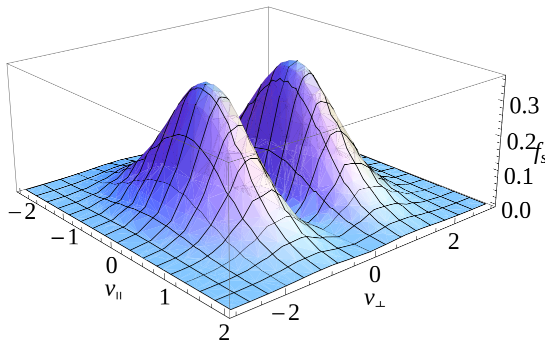

Figure 1: Ring Maxwellian distribution for suprathermal electrons.

c_1			
$\hat{K}\backslash p$	2	3	4
1	63.14	187.8	435.6
2	984.7	2941.5	6842.6
3	4942.7	14786	34430
c_2			
1	5.95	6.96	7.96
2	11.95	13.96	15.96
3	17.95	20.96	23.96

Ambipolar electric field. The significance of the expression presented above for the su[rathermal electrons flux can be estimated by the way it modifies the radial electric field. This flux can be used in the extended ambipolarity condition Eq. (1) in order to find the resulting ambipolar electric field. Since the transport coefficients here do not depend on the electric field, as it occurs in more elaborate models that cover different collisionality regimes [5], it is easy to solve Eq. (1) together with Eqs. (2,6) for the radial electric field $E_r = -\Phi'$. This simple estimate gives

$$E_r = -\frac{T_e}{e}\frac{\frac{n'}{n}(1-2.1m_r t^{7/2}) + 3.4\frac{T_e'}{T_e} - 7.3\frac{T_i'}{T_i}m_r t^{7/2} + \frac{c_1}{12.8}f_s\frac{T_\parallel}{T_e}^2\left(\frac{n_s'}{n_s} - \frac{T_\parallel'}{2T_\parallel} + \frac{\hat{K}'}{\hat{K}}\left(\frac{c_2}{\hat{K}} - 2\right)\right)}{1 + 2.1m_r t^{5/2} + \frac{c_1}{12.8}f_s T_\parallel/T_e} \tag{8}$$

where $f_s(r) = n_s/n$, $t(r) = T_i/T_e$, $m_r = \sqrt{m_i/m_e}$. Since all plasma parameters vary with radius the expression provides a way to compute the radial electric field profile when the radial dependencies of the other quantities are provided. For this we use typical profiles for the TJ-II stellarator for different densities. In Fig. (2) it is shown the electric field profiles for four line averaged densities using polynomial profiles of n and T_j fitted to the experimental ones, ignoring suprathermal contribution ($f_s = 0$). Although they are not exactly like the profiles measured by HIBP, due to the restriction of low ν_e, they reproduce the main features like positive E_r at low n and completely negative E_r for high n.

In order to include suprathermal particles it is necessary to give profiles for n_s, $T_\parallel$ and $\hat{K}$. For these we assume on-axis heating so that the n_s and $\hat{K}$ profiles are centrally peaked and

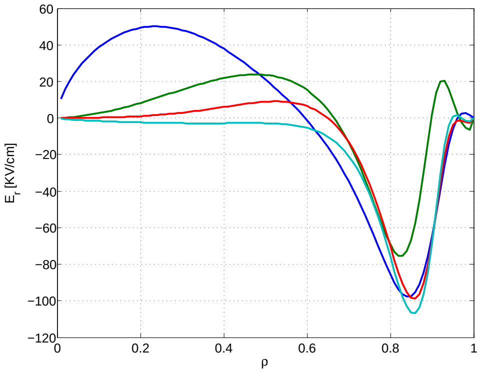

Figure 2: E_r profile with only thermal particles for 4 densities. High n is blue.

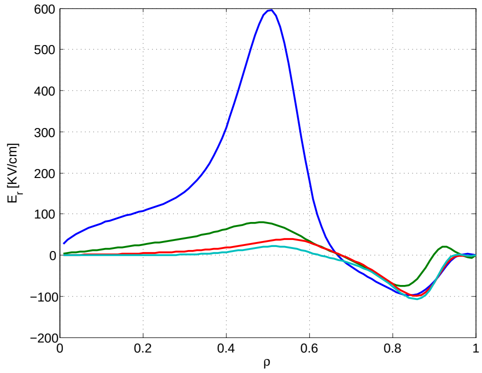

Figure 3: E_r profile with suprathermal electrons for same densities.

very narrow with a half mean width $\Delta \leq a/5$. For the temperature we assume it is similar to the thermal electrons $T_{\|} \approx T_e$. The suprathermal population is taken as 1% of the main density ($f_s \sim 0.01$). The corresponding electric field profiles are shown is Fig. 3 for the same densities. It can be appreciated that the magnitude of E_r is increased substantially, especially for the inner regions at low densities. For the high densities, the electric field can even become positive and it is no longer all negative. Of course this behavior cannot be reliable since at high densities the plasma is more collisional and our calculations for low collisionality are not applicable. As expected the effect is less important near the edge since the suprathermal population is negligible there.

In any case, the implications of the suprathermal flux are that the electric field should increase bringing E_r towards the electron root which would improve the agreement with the experiment. This is a natural result since an increased total electron flux gives rise to positive electric fields. More accurate computations are needed in order to compare with the experimental measurements which must include equations for Γ_s at higher collisionalities.This work is in process.

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