# Bifurcation effects on the plasma edge of tokamaks with ergodic limiter

C. Vieira Abud[1] and I. L. Caldas[1]

[1] *Universidade de São Paulo, São Paulo, Brazil*

**Abstract**

*In the present paper, we solve a twist symplectic map for the action of an ergodic magnetic limiter in a large aspect-ratio tokamak. In this model, we study the bifurcation scenarios that occur in the remnants regular islands that co-exist with chaotic magnetic surfaces. The onset of atypical local bifurcations created by secondary shearless tori are identified through numerical profiles of internal rotation number and we observe that their rupture can reduce the usual magnetic field line escape at the tokamak plasma edge.*

Appropriated modifications on the magnetic field have indicated an improvement to the plasma confined in tokamaks [1]. In this scense an adequate chaotization of field lines can reduces the plasma-wall interaction and the anomalous particle transport. Such chaotic field lines can be induced by external devices as the ergodic limiter [2] and the dynamic ergodic divertor [3].

As is well-known, magnetic field lines can be represented by a 1 + 1/2 degree of freedom Hamiltonian system. Thus, these field line Hamiltonian can be replaced by symplectic maps [4]. In general, symplectic maps are classified as twist, when the rotation numbers increase monotonically; otherwise they are nontwist. The twist condition is assumed in several mathematical theorems such as the Poincaré-Birkhoff, Aubry-Matter, and the Kolmogorov-Arnold-Moser (KAM) theorem. These theorems are the basis of the twist map scenario but their predictions are not verified for the nontwist maps, which present distinctive features in the phase space such as the presence of a nontwist invariant torus (shearless torus), and the separatrix reconnection process. Nevertheless, recent studies [5, 6] reported the onset of secondary shearless tori (twistless tori) that appear within islands of stability in the phase space of simple twist maps.

In the present paper, we solve a twist symplectic map introduced to describes chaotic field lines in tokamaks with ergodic limiter [7]. It is constituted by the convolution of two maps, $\hat{F}=F_1 \circ F_2$:

$$F_1 : \begin{cases} r_{n+1} = \frac{r_n}{1-a_1 sin\theta} \\ \theta_{n+1} = \theta_n + \frac{2\pi}{q_{eq}(r_{n+1})} + a_1 cos\theta_n \end{cases} \qquad F_2 : \begin{cases} r_{n+1} = r^*_{n+1} + \frac{mC\varepsilon b}{m-1}\left(\frac{r^*_{n+1}}{b}\right)^{m-1} sin(m\theta_{n+1}) \\ \theta^*_{n+1} = \theta_{n+1} - C\varepsilon\left(\frac{r_{n+1}}{b}\right)^{m-2} cos(m\theta_{n+1}) \end{cases}$$

where $F_1$ defines the equilibrium map with $a_1$ being the correction for toroidal effect. The function $q_{eq}$ is the monotonic equilibrium safety factor defined by:

$$q_{eq}(r) = \begin{cases} q_a \frac{r^2}{a^2} \left\{ 1 - \left(1 - \frac{r^2}{a^2}\right)^{\gamma+1} \right\}^{-1} & (0 \leq r \leq a) \\ q_a \frac{r^2}{a^2} & (a \leq r \leq b) \end{cases}$$

In the pertubation map, $F_2$, we have $C = 2mla^2/R_0 q_a b^2$ and $\varepsilon$ being the perturbation parameter related with the ratio between the limiter and plasma current, $\varepsilon = I_h/I_p$. In the following phase spaces, we will use the normalized coordinates $x = \theta/2\pi$ and $y = 1 - r/b$ and a set of parameters: $a_1 = -0.04$, $R_0 = 0.3m$, $b = 0.11m$, $a = 0.08m$ and $l = 0.08m$.

Recently, the presence of one shearless torus were analytically predicted for generic Hamiltonian system in the neighborhood of the tripling bifurcation of an elliptic fixed point [5]. Furthermore, numerical investigations suggest a specific profile containing two shearless tori near the quadrupling (1/4) bifurcation [6].

As we are interested in localized regular islands immersed in chaotic regions of the phase space of our model, we define the internal rotation number to measure the torsion of each torus with respect to its elliptic fixed point:

$$\omega_{in} = \lim_{n\to\infty} \frac{1}{2\pi n} \sum_{n=1}^{\infty} P_n(x,y)\hat{\theta}P_{n+1}(x,y), \tag{1}$$

where $P_n\hat{\theta}P_{n+1}$ means the angle $\theta$ between two consecutive points and $(x,y)$ are the coordinates of the two-dimensional map. By equation 1 a rational internal rotation number $(n/m)$ describes a periodic orbit while an irrational number represents a circular quasi-periodic orbit. For a chaotic orbit, equation 1 does not converge.

To illustrate the emergency of secondary shearless torus within islands of stability, we choose a value to $\varepsilon$ that presents a phase space whose the chaotic layer near the border $y = 0$ is composed by only one visible remaining island chain. See in figure 1 the period-5 island chain whose elliptic fixed point has bifurcated in a period-4.

The evolution of the internal rotation number profiles to the highlighted island in figure 1 are shown below of the same figure 1. For $\varepsilon = 0.185$ the internal rotation number profile is monotonic, i. e., each specific value of $\omega_{in}$ is related to only one circular invariant torus. Increasing the parameter to $\varepsilon = 0.188$ we observe the formation of two bumps. The presence of a minimum and a maximum point in the rotation profile indicates the existence of two tori without shear. For $\varepsilon = 0.189$, the maximum bump reaches the value $\omega_{in} = 1/4$ yielding four stable fixed points, so long as the minimum bump still exist and does not bifurcate for any lower order rational number and ends colliding with elliptic fixed point.

In spite of secondary shearless tori be a local phenomena and, consequently, do not interfere in global properties of the chaotic layer it is relevant, in the tokamak context, to study the transport process while these secondary shearless exist.

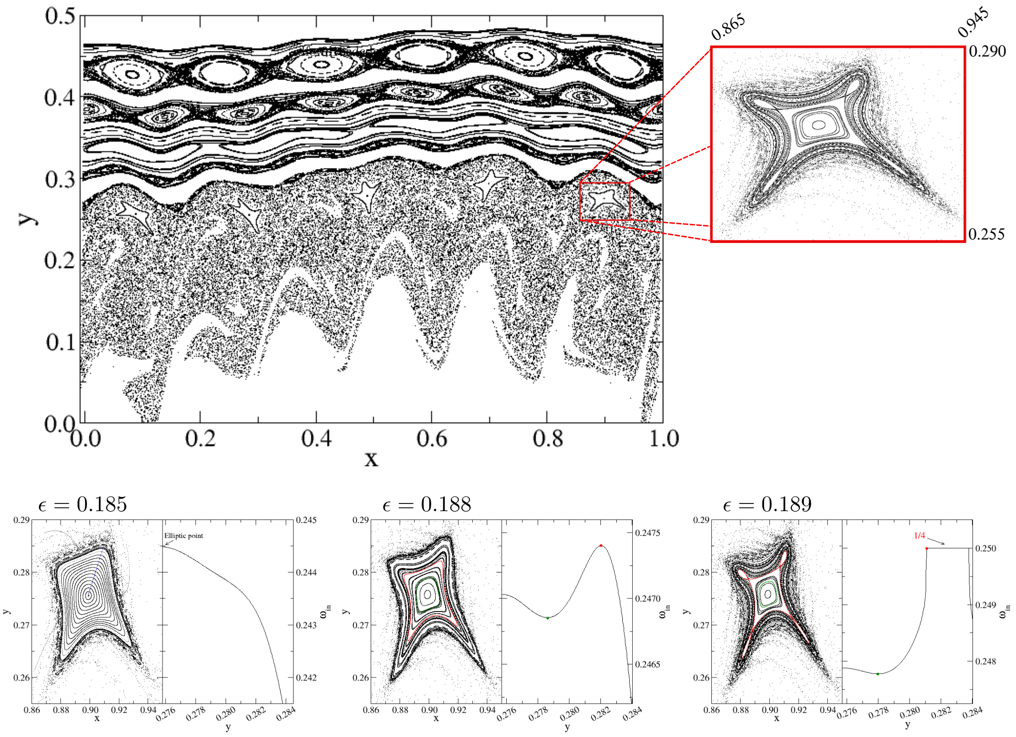


Figure 1: Phase space of the Ullmann map with $\varepsilon = 0.189$ and $m = 6$ (top). The quadrupling bifurcation created by secondary shearless torus (bottom).

In order to investigate the transport properties on initial conditions, we computed the average escape time of orbits by the following numerical experiment. For a given value of $\varepsilon$, we tested a large number of initial points ($N_P = 2 \times 10^6$) placed on a regularly spaced grid of same size of the upper box of figure 1. Each initial condition was iterated until the corresponding orbit crossed the reference boundary $y = 0$. From figure 2 (escape time) we observe a strong dependence on $\varepsilon$ with several peaks. The higher peak, $\varepsilon = 0.18995$, corresponds to the lower order bifurcation ($\frac{1}{4}$) and it is also the limit to the existence of the secondary shearless torus. This atypical escape time around a specific value of $\varepsilon$ indicates some kind of trapping mechanism that hinders the transport. The fine details of the dependence of the escape time on the initial conditions is shown in figure 2 (right), for two different values of $\varepsilon$: one of them corresponding to short escape time (blue dotted line labeled as **a.** in figure 2) and the second to long escape time (red dashed line labeled as **b.** in figure 2). Each initial condition was iterated $3 \times 10^6$ times and the different escape times are indicated by a logarithmic color scale. It is clear that points inside the islands, evidently, do not escape and are identified with red color, but the orbits adjacent to them may spend long or short time to reach the reference boundary $y = 0$. Comparing figure 2 (a) and (b) we see that figure 2 (b) presents reasonable amount of initial conditions that spend long times ($\approx 1 \times 10^6$, orange color) encircling the island. This type of stickiness interferes in the average escape time and, consequently, in the global transport of the system.

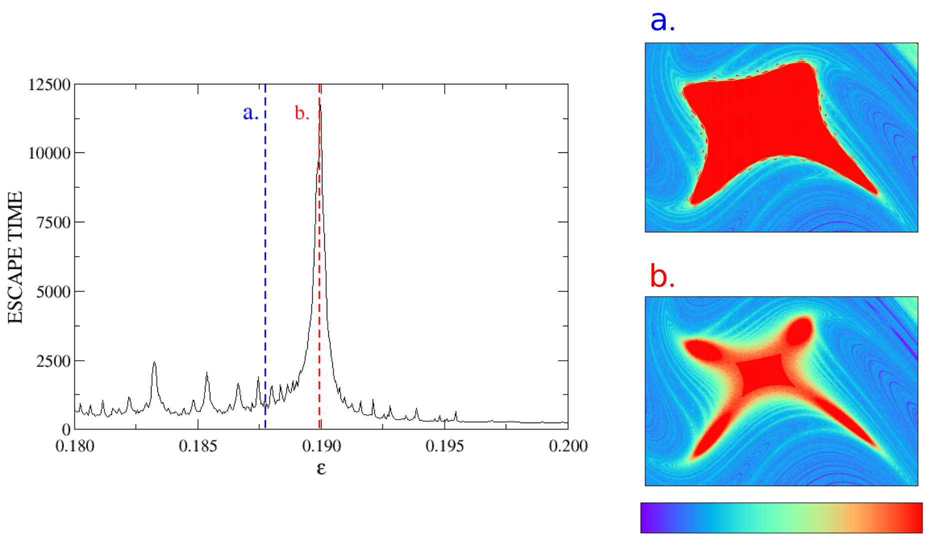


Figure 2: Escape time (left) and the details of label a. and b. in the phase space.

Concluding, we addressed the chaotic transport of magnetic field lines in tokamak plasma edge. In our work, the resonant perturbations that break the magnetic surfaces are those created by an external ergodic limiter. In this case, the perturbed field line mapping is approximately obtained, for large aspect ratio tokamaks. The obtained map allows us to analyze the influence of the relevant control parameters, related to the equilibrium and perturbing fields, on the transport of field lines. More specifically, we show the onset of secondary shearless curves in equilibria with monotonic safety factor profiles and how such bifurcation affects the plasma edge transport [8].

## References

[1] Boozer A. H. 2005 *Rev. Mod. Phys.* **76** 1071.

[2] Evans T. E. *et al* 2005 *Nucl. Mat.* **337-339** 691.

[3] Jakubowski M. W., Abdullaev S. S. and Finken K. H. 2004 *Nucl. Fusion* **44** S1.

[4] Morrison P. J. 2000 *Phys. Plasmas* **7** 2279.

[5] Dullin H. R., Meiss J. D. and Sterling D. 2000 *Nonlinearity* **13**, 203.

[6] Abud C. V. and Caldas I. L. 2012 *Chaos* **22** 033142.

[7] Ullmann K. and Caldas I. L. 2000 *Chaos, Solit. & Fract.* **11** 2129.

[8] Abud C. V. and Caldas I. L. *in preparation*.