A Flux-Coordinate Independent Field-Aligned Approach to Plasma Turbulence Simulations

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Introduction

Plasma turbulence evolves from a considerable class of instabilities characterized by long parallel wavelengths, of the order of the system size, and short perpendicular wavelengths of the order of the ion gyro-radius. As a consequence, from the numerical viewpoint, one can conceive that much more efficient codes are made possible by a suitable choice of coordinates, that allow the smallest number of grid points in a certain direction. Indeed, field-aligned coordinates have been employed for already a couple of decades in tokamak turbulence simulations [1, 2, 3]. The gain in computational efficiency obtained by using optimal coordinates can be a couple of orders of magnitude for a turbulence simulation of a large device like ITER.

Field-aligned coordinates employed so far are derived from predefined flux coordinates. The scope of this work is to illustrate and validate a more general approach that rely only on fundamental coordinates independent of the flux surface variables [5].

Turbulent transport in tokamaks is studied via numerical simulations of a variety of model equations which retain the important physics one wants to study. A fairly general structure of model equations is given as follows:

$$\partial_t L \cdot S = E(S) + I \cdot S \tag{1}$$

where S is a structure of vectors representing the state of the system. $E(S)$ is a nonlinear operator that can be treated explicitly without much penalty. L and I represent linear operators such that the reduced problem obtained by setting $E(S) = 0$ could be treated implicitly. The splitting of the right hand side (r.h.s.) between E and I is by no means unique and depends on the physics to be studied. As a general rule, one aims at treating explicitly only the physics occurring at the timescale of interest for the specific problem. The main constraint with respect to a generic r.h.s. is that the implicit problem be linear. We further assume that L and I are time-independent.

Since we are interested in the treatment of the parallel dynamics, we now specialize Eq. (1) to the following normalized drift-wave model. This model will be used to illustrate our approach throughout the rest of this work:

$$\begin{cases} \partial_t n + [\phi, log(n_0)] + A\nabla_{\|} u = 0 \\ \\ \partial_t u + \frac{1}{\tau} A\nabla_{\|} n + A\nabla_{\|}\phi = 0 \\ \\ n = \phi - \rho_*^2 \nabla_\perp^2 \phi \end{cases} \tag{2}$$

Here n is the perturbed ion guiding center density, n_0 is the equilibrium density profile, u is the ion parallel velocity and ϕ is the electrostatic potential. We define two dimensionless parameters: $A = a/R \times 1/\rho_*$ where a is the tokamak minor radius, R is the tokamak major radius, $\rho_* = \rho_s/a$ is the reduced gyro-radius with ρ_s being the ion sound Larmor radius. Moreover τ is the ratio of electron temperature to ion temperature T_e/T_i. Note that time is normalized to the Bohm timescale $a^2/(\rho_s c_s)$, where c_s is the ion sound speed. The explicit expression of the parallel derivative operator $\nabla_{\|}$ depends on the magnetic field structure. In the case of a cylindrical geometry one can write $\nabla_{\|} = \partial_\phi + 1/q(r)\,\partial_\theta$ with (r,θ) the polar coordinates in the poloidal plane and $q(r)$ the safety factor.

The Flux-Coordinate Independent (FCI) Approach

The field-aligned coordinate approach presented in [4] still relies on flux coordinates, for instance (r,θ). A new method is presented in [5] that we refer to as FCI (Flux-Coordinate Independent) approach. One has to look for a change of coordinates from the original (x,y,z) to a new set (ξ^α, s) such that s can be treated as a slowly-varying coordinate and only the two ξ^α $(\alpha = 1,2)$ carry the information on the small scales. As discussed in [5], one divides the domain in a certain number of sectors centered around z_k, and extending to the boundary in the (x,y) directions, with k labeling a given sector. One then considers a set of transformations of the form:

$$\begin{cases} \xi^\alpha &= V^\alpha(\mathbf{x}) + C^\alpha(\mathbf{x})(z - z_k) \\ s &= z - z_k \end{cases} \tag{3}$$

where $V^\alpha(\mathbf{x})$ and $C^\alpha(\mathbf{x})$ are yet unknown functions.

Results and Conclusions

We developed a new code that we called FENICIA: **F**lux indep**EN**dent f**I**eld-aligned **C**oord**I**nate **A**pproach, in which we implemented the FCI approach described in the previous section. A sketchy description of this new code is given in Appendix A of [5]. The actual implementation of the parallel derivative depends on the scheme of choice. In the following, we consider second order centered finite differences, since they are used to solve the drift-wave test model (2). Here we present numerical tests that were carried out with FENICIA to qualify the new method. These are of two types: a) a test that demonstrates the capability of the new method to simulate wave propagation accurately even when the toroidal mode number exceeds the Nyquist cut-off (half the number of toroidal points) at the given toroidal resolution. This is precisely the situation where the straightforward approach that computes the parallel derivative as a combination of the toroidal and poloidal derivatives would fail, and b) a test of convergence using a nonlinear ITG model. For the tests shown here the box size is $400 \times 400 \times 20$ and $m/n = 2$, with (m,n) ranging from $(4,2)$ to $(30,15)$. We start by showing results obtained from Eq. (2) with $\log(n_0) = 0$ (zero drift frequency). The initial velocity is such that there is a single wave propagating at frequency $\omega = A(1+1/\tau)^{1/2}(m/q(r)-n)$. The results are summarized in Fig. 1 where the error per unit time $E = (< (n_{exact} - n_i)^2 > / < (n_{exact})^2 >)^{1/2}$ is plotted as a function of the poloidal mode number for three cases: 1) the full model (2) with $L_n = 1/4$ and $A = 12.5$, 2) the same model with $1/L_n = 0$ (no density gradient), and 3) the model with $A = 0$. The latter case tests the effect of switching off the parallel dynamics so that the system reduces effectively to

$$\partial_t n + [n, log(n_0)] = 0 \tag{4}$$

The first thing to notice is that all the tests give an error per unit time much less than one. We

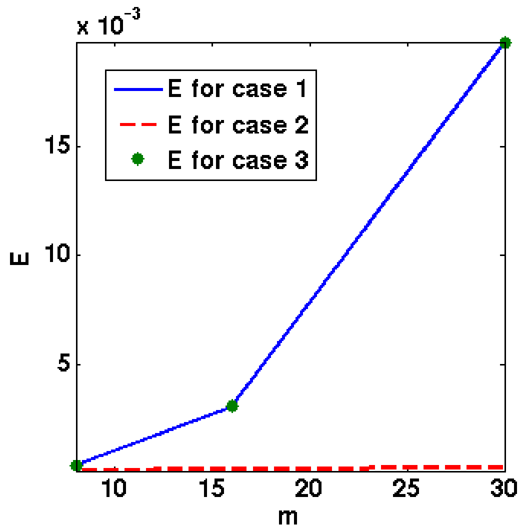

Figure 1: *The relative error between between the exact and the numerical solution as a function of the poloidal wavenumber m. For case* **1***: full model with finite drift frequency, we get the blue solid line; For case* **2***: full model with zero drift frequency, we get the red dashed line; For case* **3***: reduced model of Eq. (4) without sound wave terms, we get the green bullets*

remark that the relative difference between case **1** and case **3** is less than 10^{-3}, which explains why the data points for the two cases look almost superposed in Fig. 1. Since the model is normalized to the Bohm timescale $a^2/(\rho_s c_s)$, any physics effect occurring on a shorter time scale is treated accurately.

The second following test shows the convergence of the code using an ITG turbulence model. Convergence is achieved at $nz = 15$. Thus, with the new method, one needs only a few tens of toroidal points to get a good result, regardless of the toroidal mode number, provided that adequate resolution is available in the poloidal plane.

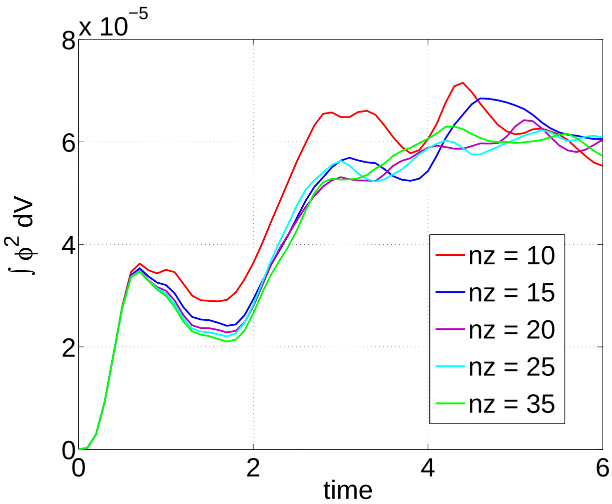

Figure 2: *convergence test using an ITG turbulence model*

In conclusion, a new approach to the problem of field-aligned coordinates for plasma turbulence simulations has been developed. We call it the FCI approach. It allows, in particular, a more natural treatment of the operations in the poloidal plane and deals without difficulty with X-point configurations and with O-points such as the magnetic axis, since it is constructed on coordinate systems with non-singular metric. Tests using the newly developed code FENICIA were carried out to show explicitly the capability to simulate drift-wave propagation with toroidal mode numbers exceeding the Nyquist cutoff and to show the convergence in a nonlinear regime using an ITG model.

References

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