Collisional stabilisation of trapped electron modes in tokamaks: pitch angle scattering vs energy scattering

P. Manas[1], Y. Camenen[1], S. Benkadda[1], W.A. Hornsby[2], A.G. Peeters[2]

[1] *Aix-Marseille Université, CNRS, PIIM UMR7345, 13397 Marseille, France*

[2] *Department of Physics, University of Bayreuth, 95440 Bayreuth, Germany*

In tokamaks, trapped electron modes (TEM) are known to be stabilised by collisions (see e.g. [1, 2]). Many turbulent transport studies have been carried out with a pitch angle scattering operator only as it is sufficient to obtain TEM stabilisation by collisional detrapping. In the following, the effect of energy scattering will be investigated.

The simulations are performed with the gyrokinetic flux tube code GKW using the δf approximation. The linearised landau collision operator can then be written as follows:

$$C^L(F,F) = C^T(\delta f, F_0) + C^F(F_0, \delta f) \tag{1}$$

with C^T the test-particle operator and C^F the field particle operator. The latter is hard to deal with [5, 4] because of the computationally intensive calculation of Rosenbluth potentials of the perturbed distribution function but is essential to conserve physical quantities such as momentum and energy. It is often simplified keeping the conservation properties of the collision operator. This is what is done in GKW where the loss of momentum and energy are artificially reintroduced.

$$\frac{\partial f}{\partial t} = C(f) + C_{mom} v_{\parallel} F_M \tag{2}$$

$$\frac{\partial f}{\partial t} = C(f) + C_{ene}\left(v^2 - \frac{3}{2}T\right) F_M \tag{3}$$

with C_{mom} and C_{ene} ensuring that:

$$C_{mom}\int \mathrm{d}v_{\parallel}\mathrm{d}\mu v_{\parallel}^2 F_M + \int \mathrm{d}v_{\parallel}\mathrm{d}\mu v_{\parallel} C(f) = 0 \tag{4}$$

$$C_{mom}\int \mathrm{d}v_{\parallel}\mathrm{d}\mu v^2\left(v^2 - \frac{3}{2}T\right) F_M + \int \mathrm{d}v_{\parallel}\mathrm{d}\mu v^2 C(f) = 0 \tag{5}$$

The test-particle operator is numerically implemented using a conservative second order 2D finite difference scheme.

$C^T(\delta f)$ can be decomposed as follows:

$$C^T(\delta f, F_0) = C_{\theta\theta} + C_{vv} + C_v \tag{6}$$

$C_{\theta\theta}$ being the pitch-angle scattering, C_{vv} the energy scattering and C_v the friction. $C_{\theta\theta}$ has been successfuly benchmarked with the gyrokinetic codes GS2 and GENE. As for C_{vv} and C_v comparisons with analytical thermalisation rate of two maxwellians showed good agreement.

The following simulations will make use of the full test particle collision operator to investigate the effect of energy scattering (energy scattering + friction) on micro-instabilities such as ITG and TEM.

First, to emphasize the importance of retaining the field particle operator and more precisely the part related to the conservation of energy, we look at the evolution of the growth rate of a purely temperature gradient driven TEM case ($R/L_{Ti} = 0$, $R/Ln = 0$, $R/L_{Te} = 9$, $k_\theta \rho_i = 1.5$) while varying the collisionality. All simulations were performed for concentric circular cross sections with $q = 2$, $\hat{s} = 1$, $\varepsilon = 0.166$ in a linear electrostatic case.

ν_{eff} is defined such that:

$$\nu_{eff} = 0.1 \frac{R_{ref} n_e Z_{eff}}{T_e^2} \tag{7}$$

with R_{ref} in meters, n_e in units of $10^{19} m^{-3}$ and T_e in units of keV.

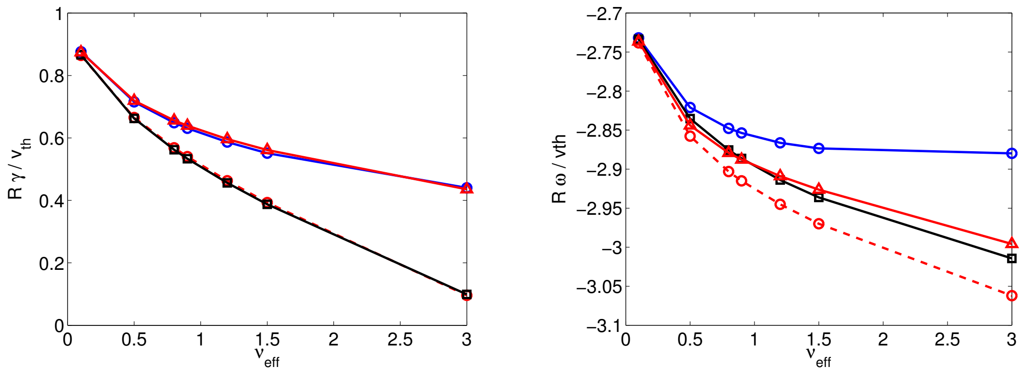

Figure 1: Scan in collisionality of a TEM at $k_\theta \rho_i = 1.5$ (left: growth rate, right: frequency). Blue circles: pitch-angle scattering Eq(8), red circles: pitch + energy scattering Eq(9), red triangles: pitch + energy scattering + conservation in energy Eq(10), black squares: pitch + energy lost when using energy scattering Eq(11).

The different operators used for this case were:

$$C(f) = C_{\theta\theta} \tag{8}$$

$$C(f) = C_{\theta\theta} + C_{vv} + C_v \tag{9}$$

$$C(f) = C_{\theta\theta} + C_{vv} + C_v + C_{ene} \left(v^2 - \frac{3}{2} T \right) F_M \tag{10}$$

$$C(f) = C_{\theta\theta} - C_{ene} \left(v^2 - \frac{3}{2} T \right) F_M \tag{11}$$

As expected, pitch-angle scattering has a stabilising effect. For this specific case energy scattering applied with energy conservation (Eq 11) has no effect on the growth rates and a rather small effect on the mode frequency whereas applied without energy conservation (Eq 9), one obtains an artificial stabilisation. The loss of energy induced by the energy scattering alone, explains the further stabilisation of the mode compared to the pitch-angle scattering. The conservation of energy will always be applied when using the energy scattering operator in the subsequent simulations.

The growth rates and frequencies of three different basic instabilities have been studied: a pure ITG mode ($R/L_{Ti} = 9$, $R/Ln = 0$, $R/L_{Te} = 0$), a purely temperature gradient driven TEM ($R/L_{Ti} = 0$, $R/Ln = 0$, $R/L_{Te} = 9$) and a purely density gradient driven TEM ($R/L_{Ti} = 0$, $R/Ln = 9$, $R/L_{Te} = 0$).

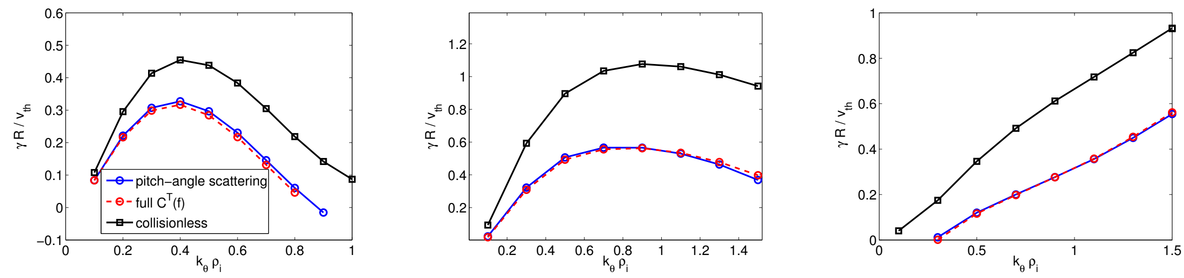

Figure 2: Linear spectrum of a pure ITG (a), a purely density gradient driven TEM (b) and a purely temperature gradient driven TEM (c) from collisionless case to a full collision operator with $\nu_{eff} = 1.5$.

As seen on Figure 2, the effect of energy scattering (with energy conservation) on these basic instabilities is negligible, even at rather high collisionality.

However for mixed ITG TEM cases with $R/Ln \neq 0$ and $R/L_{Te} > R/L_{Ti}$, the differences between pitch-angle and energy scattering increase with the collisionality. Furthermore these differences exhibits a maximum when varying R/L_n. The key parameter seems to be the mode frequency as the effect of energy scattering is greater when the frequency of the mode goes toward 0 with various R/L_{Ti} as can be seen on Fig(4b) . This dependency is currently under investigation.

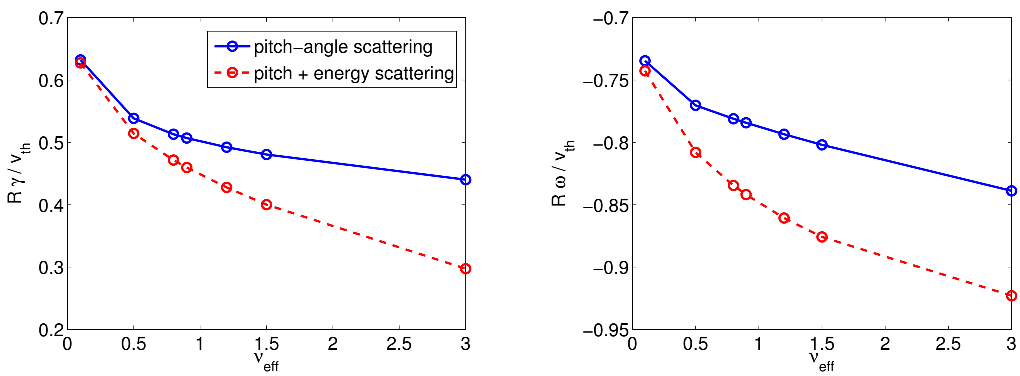

Figure 3: Scan in collisionality of a mixed ITG TEM instability ($R/L_{Ti} = 5, R/Ln = 3, R/L_{Te} = 12$) for $k_\theta \rho_i = 0.7$ (left: growth rate, right: frequency).

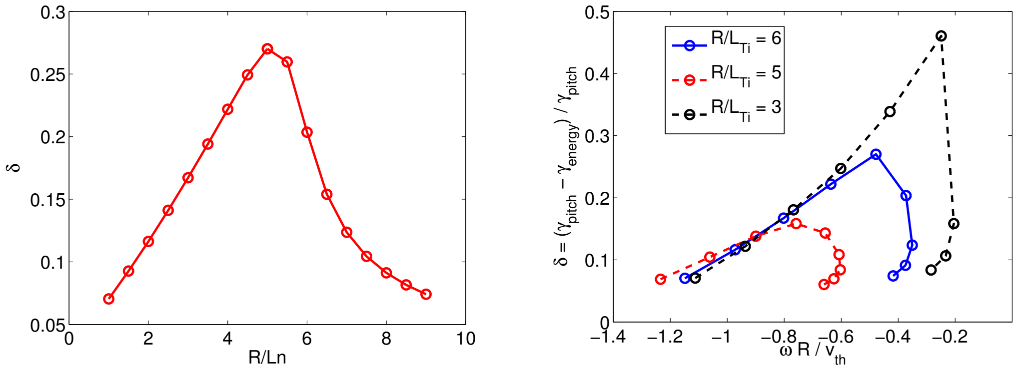

Figure 4: Mixed ITG TEM instability with $R/L_{Te} = 12$ for $k_\theta \rho_i = 0.7$ and $\nu_{eff} = 1.5$ (left: δ vs R/L_n, right: δ vs the normalised frequency when increasing R/L_n).

In some cases the energy scattering can even cause a transition from TEM to ITG as it is more effective on TEM (in a case with R/L_{Te} and $R/Ln \neq 0$).

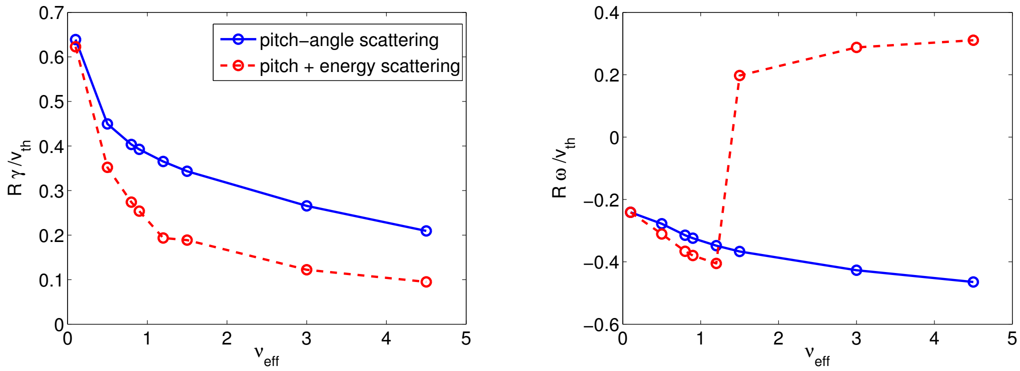

Figure 5: Scan in collisionality of a mixed ITG TEM instability ($R/L_{Ti} = 5, R/Ln = 5, R/L_{Te} = 10$) for $k_\theta \rho_i = 0.7$ (left: growth rate, right: frequency).

In conclusion, to study the effect of energy scattering on trapped electron modes, conservation of energy must be accounted for as this instability is very sensitive to the loss of energy introduced by the linearised Landau collision operator. Even though it is negligible in many basic cases, adding energy scattering to the well known pitch-angle scattering can have a strong effect for mixed ITG / TEM instabilities with intermediate values of R/L_n and $\nu_{eff} \sim 1$. In some cases it can even trigger a transition between TEM and ITG.

References

[1] M.A. Beer, Ph.D. thesis, Pinceton University, (1995)

[2] F.Y. Gang and P.H. Diamond, Phys. Fluids B **2**, 2976 (1990)

[3] A.G. Peeters et al., Computer Physics Communications **180**, 2650, (2009)

[4] E.A Belli, J. Candy, Plasma Phys. Control. Fusion, 54 (2012) 015015

[5] I.G. Abel, M.Barnes, S.C.Cowley, W.Dorland, A.A. Schekochihin, Phys. Plasmas 15, 122509 (2008)