Investigating the anomalous Doppler effect for suprathermal electrons in tokamak plasmas using self-consistent kinetic simulations

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1. Introduction

Suprathermal electron populations can arise under a broad range of lower-density plasma conditions in tokamaks. The anomalous Doppler instability (ADI) is a key limiting mechanism for monotonically decreasing extended tails in the probability distribution of the component of electron velocity parallel to the magnetic field direction. It has an experimental pedigree extending over four decades in low density tokamak plasmas[1,2] with both Ohmic and lower hybrid current drive. The ADI remains topical, for example it is believed to explain the recently observed relaxation oscillations in directional soft X-ray signals emitted by the energetic electron population in the HT-7 tokamak[3]. These observations reflect the underlying physics of the ADI, which involves a shift of energy from parallel to perpendicular particle motion, accompanied by the excitation of waves at frequency and wavenumber $(\omega,\boldsymbol{k})$ satisfying the anomalous Doppler resonance condition $\omega - \boldsymbol{k}.\boldsymbol{v} = -\Omega$, where Ω is the cyclotron frequency and $\boldsymbol{v}$ is particle velocity. This is known to apply at both the classical single-particle level[4-6] and the collective level[7] of mathematical description. The X-ray signal oscillations are believed to arise where the energetic electron tail distribution cyclically relaxes through the isotropising ADI, and is then reconstituted by the current drive processes. Here we present, for the first time, a fully nonlinear kinetic treatment of the ADI for the electron population under tokamak plasma conditions. By means of a well diagnosed state-of-art particle-in-cell (PIC) code[8], we explore how the character of the velocity distribution and of the excited electromagnetic fields evolves in time and depends on plasma parameters. This is an essential step towards exploiting the potential of the ADI as a spontaneous *in situ* diagnostic of tokamak plasma conditions. It also contributes to the diagnostic and design capability for planned beam-plasma laboratory experiments[9] in this area.

2. Computational model

At the single particle level of description using linear analysis[4-6], the energy flows associated with a single electron undergoing ADI in a magnetised plasma are as follows: from the kinetic energy of motion in the direction parallel to the magnetic field direction, into the kinetic energy of motion perpendicular to $\boldsymbol{B}$ and into wave energy of the resonant excited electrostatic wave, in the ratio Ω:ω. The driving linear growth rate scales with the magnitude of the tail distribution, integrated over perpendicular velocity, at the anomalous Doppler resonant parallel velocity. For net linear growth, the corresponding Landau resonant velocity of the corresponding wave must lie beyond the bulk thermal population, otherwise linear Landau damping is usually sufficient to prevent the instability. These features carry over into linear analysis of collective energy flows calculated from ***J.E***[7]. Here we report computational studies of the collective kinetic evolution of electron populations undergoing ADI, together with the self-consistently excited wave fields. We carry out 1.5D PIC simulations where the full ion and electron dynamics of macroparticles, together with $\boldsymbol{E}(x,\boldsymbol{v},t)$ and $\boldsymbol{B}(x,\boldsymbol{v},t)$, evolve self-consistently under the Lorentz force law and relativistic Maxwell equations[8]. The initial distribution of parallel velocities of the suprathermal electron distribution is taken to be either flat or, using superposition of drifted Maxwellians, to be monotonically decreasing.

3. Simulation results

Figure 1 shows a time sequence of plots of the evolving electron distribution function *f*. The constant-*f* contours for the highest energy component of the electron population, initially extended in the direction of the parallel velocity axis, spread rapidly in the perpendicular velocity direction. The population evolves from pencil-like to pancake-like under the action of the ADI. Figure 1 also shows local flattening

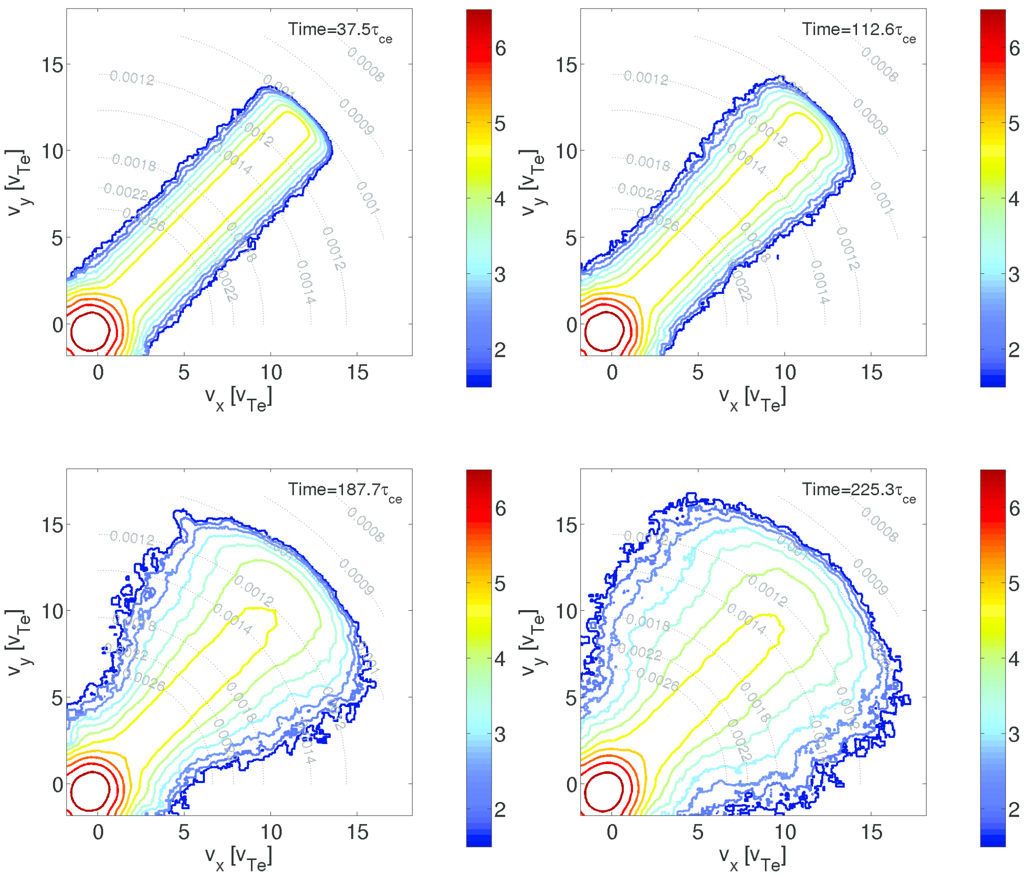

Fig.1. Time evolution of an electron population undergoing anomalous Doppler instability. Constant-*f* contour plots are shown at three times for an electron velocity distribution with initially 8% of the particles in a suprathermal tail. Parallel and perpendicular velocities are plotted in units of thermal velocity, and the colour density for the contours is in logarithmic scale. Panels (*a*) to (*c*) show snapshots from beginning to end of the linear phase of the ADI, at times t = (*a*) 37.5, (*b*) 112.6, (*c*) 187.7, and (*d*) 225.3, where the time units are electron gyroperiods. Magnetic field is oriented at 45°.

and extension of the bulk distribution at a few times the electron thermal velocity v_{Te}, which is a consequence of Landau damping there of tail-excited waves. The corresponding structure of the electrostatic field amplitude in (ω,k) space is shown in the sequence of Fourier transform plots in Fig.2, under conditions where the ratio of electron cyclotron frequency to electron plasma frequency is 1.31. Here each plot relates to a time window of duration 37.5 gyroperiods after the instant at which the corresponding snapshot is taken in Fig.1. Figure 2 shows that the excited field is dominated by electrostatic waves, with the peak frequency of the spectrum changing as the distribution evolves.

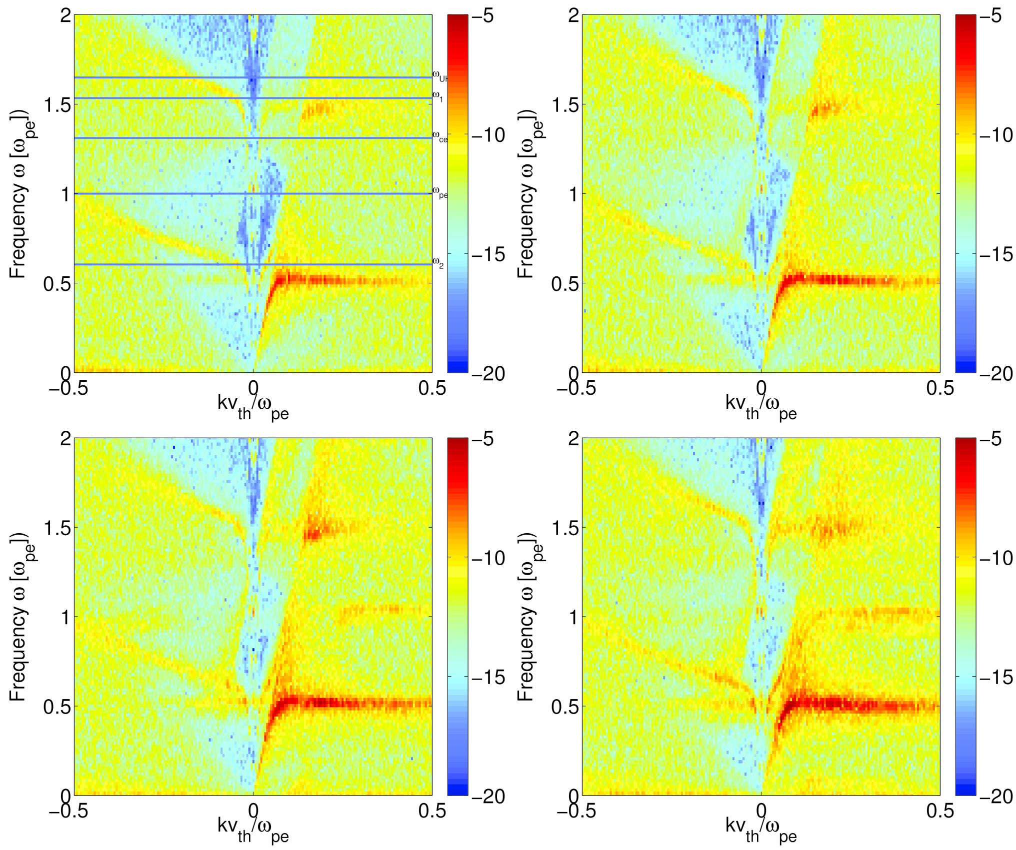

Fig.2 Time evolution of electric field amplitude in (ω,k) space during the linear phase of the ADI. This electric field is self-consistent with the electron velocity distribution evolving under ADI in Fig.1. Each panel is constructed by short-time Fourier transforming over a time interval lasting 37.5 gyroperiods and beginning at the instant of the corresponding snapshot in Fig.1. In the upper left panel, the horizontal lines show natural frequencies of the plasma: from top, upper hybrid frequency, higher frequency electrostatic normal mode, electron cyclotron frequency, electron plasma frequency, lower frequency electrostatic normal mode. The frequency axis is normalised in units of ω_{pe} and the wavenumber axis is normalised in units of v_{Te}/ω_{pe}. The same logarithmic colour scale is adopted in each plot to assist comparison.

4. Comparison of simulation results with analytical theory

The early time evolution of the field amplitude, computed from simulations using five different fractional concentrations ξ of suprathermal tail electrons in the range 2% to 10%, is shown in Fig.3. The initial gradients of these traces yield well defined linear growth rates for the different values of ξ, which have been compared to the analytical scaling derived by applying the formulae of Ref.[7] to each initial *f*. We find quantitative agreement to within error bars.

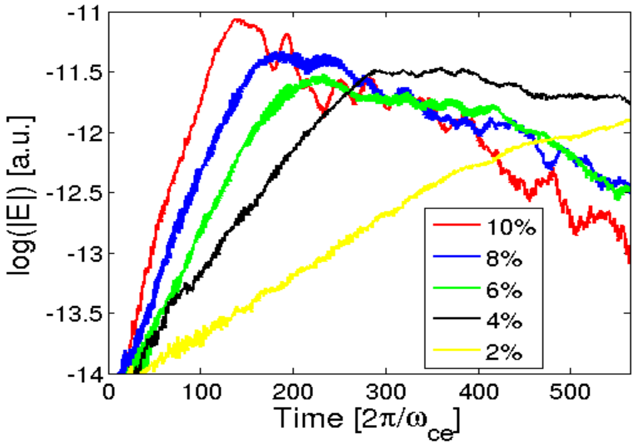

Fig.3 Time evolution of electric field amplitude for five different values of energetic particle concentration ξ in the range 2% to 10%. The linear growth rates inferred from the initial gradients agree with results from the analytical theory[7] for the specific model distributions considered.

5. Conclusions

The results presented here represent a significant new development in the theory of the anomalous Doppler instability for magnetised plasmas. Hitherto understanding has rested primarily on experimental observations[1-3], interpreted in terms of analytical theory[4-7] in the linear approximation. Here we have bridged experiment and analysis by presenting direct numerical simulations of the ADI. These embrace the fully kinetic time evolution of the electron population in velocity space together with the self-consistent electric and magnetic fields. Our simulations agree, where appropriate, with linear analytical theory. This approach creates opportunities to explore nonlinear evolution of the ADI, and strongly driven experimental scenarios where cyclic occurrence of the ADI is observed.

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