An analytic model to estimate the coupling efficiency of an ordinary polarized beam to extraordinary mode at the critical density layer

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An analytic model to estimate the transformation efficiency of a paraxial beam from O-mode to X-mode in the proximity of the critical density cutoff layer in a cold magnetized plasma is proposed. The evaluation of the power conversion efficiency of a generic distribution of field amplitude is obtained as the sum of the coupling efficiencies calculated for the components of the beam angular spectrum of plane waves, decomposed in Gauss-Hermite functions. The model allows to obtain a basic assessment of the conversion efficiency of a beam, more realistic than only considering a plane wave, including the local plasma parameters. As an example, the model is applied to the high density plasma of the Frascati Tokamak Upgrade (FTU) device, using the geometry of the new ECR H&CD launcher beam extended up to the proximity of the critical density layer.

Introduction

The coupling of an ordinary polarized wave to the extraordinary one in the electron cyclotron frequency range is exploited in several fusion devices where ECH is applied in conditions such that the electron density exceeds the critical one for the O-mode propagation. The plasma heating is obtained through the absorption of the electron Bernstein waves [1] excited with the O-X-B mode conversion scheme [2]. The power transmission function T_{OX} of the O-wave to the X-wave at the cutoff layer is typically evaluated by using the analytic formula [3]:

$$T_{OX}(N_y, N_z) = \exp\left\{ -\pi k_0 L_n \sqrt{\frac{Y}{2}} \left[2(1+Y)(N_{z,opt} - N_z)^2 + N_y^2 \right] \right\}, \quad (1)$$

where $L_n = n/(\partial n/\partial x)$ is the local density scale length, $k_0 = 2\pi/\lambda$ is the vacuum wavenumber, $Y = \omega_{ce}/\omega$ is the normalized electron cyclotron frequency and $N_{z,opt}^2 = Y/(Y+1)$. Here we consider a one-dimensional approximation (slab plasma) using a right-handed local Cartesian reference system, in which plasma density increases is along the x axis and the magnetic field is in the direction of z. Thus N_y and N_z are the components of the refractive index, in the plane at constant density $x = \text{const}$, respectively perpendicular and parallel to the magnetic field. A plane wave with $N_z = N_{z,opt}$ and $N_y = 0$ allows, in principle, 100% of power transmission, regardless of the parameters Y, k_0 and L_n in Equation (1). A more realistic evaluation should take into account

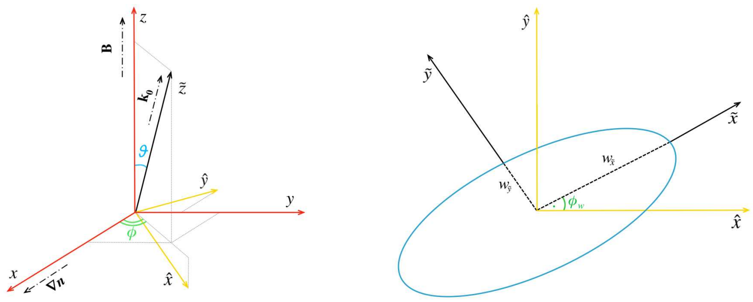

Figure 1: Reciprocal position between the slab plasma reference system x,y,z (red) and the beam system $\tilde{x},\tilde{y},\tilde{z}$ (black), used for the beam-modes analysis. The planes $\tilde{x}$-$\tilde{y}$ and x-y intersect at the $\hat{y}$ axis. The projection of B on the $\tilde{x}$-$\tilde{y}$ plane lies in the $\hat{x}$ direction.

the geometry of a beam-like field distribution in the proximity of the cutoff layer. When a finite wave dimension is considered the parameters Y, k_0 and L_n are always influential on the coupling efficiency, even though the wave propagates on the optimal direction. The model presented in this work is thought to estimate the O-X mode conversion efficiency of an ordinary polarized paraxial beam.

O-X transformation of a beam

Let us define a Cartesian coordinate system $(\tilde{x},\tilde{y},\tilde{z})$, locally defined at the transformation region, with $\tilde{z}$-axis in the direction of the beam propagation and the two orthogonal directions $\tilde{x},\tilde{y}$ conveniently defined in the plane perpendicular to it (Fig. 1). Under the assumption of paraxial approximation ($k_{\tilde{x}} \ll k$ and $k_{\tilde{y}} \ll k$), both a generic field distribution $\psi(\tilde{x},\tilde{y},\tilde{z}) = u(\tilde{x},\tilde{y},\tilde{z})\exp(-ik\tilde{z})$ and its angular spectrum of plane waves $A(k_{\tilde{x}},k_{\tilde{y}})$ (i.e. the 2D Fourier Transform of $\psi(\tilde{x},\tilde{y},0)$) can be written as a superposition of beam-modes, exploiting the properties [4] of the Gauss-Hermite functions $\psi_{\mathscr{H}_n}(x) = \exp\left(-\frac{x^2}{2}\right)\mathscr{H}_n(x)$:

$$A(k_{\tilde{x}},k_{\tilde{y}}) = \sum_{n,m} C_{nm}\frac{(i)^{n+m}}{2\pi}\cdot\psi_{\mathscr{H}_n}\left(\frac{\sqrt{2}k_{\tilde{x}}}{w_{k_{\tilde{x}}}}\right)\cdot\psi_{\mathscr{H}_m}\left(\frac{\sqrt{2}k_{\tilde{y}}}{w_{k_{\tilde{y}}}}\right), \qquad (2)$$

where

$$C_{nm} = \iint_{-\infty}^{+\infty} u(\tilde{x},\tilde{y},0)\cdot\psi_{\mathscr{H}_m}\left(\frac{\sqrt{2}\tilde{x}}{w_{0,\tilde{x}}}\right)\cdot\psi_{\mathscr{H}_n}\left(\frac{\sqrt{2}\tilde{y}}{w_{0,\tilde{y}}}\right)\mathrm{d}\tilde{x}\,\mathrm{d}\tilde{y}.$$

$\mathscr{H}_n(x)$ is the *Hermite polynomial* of grade n, $w_{k_{\tilde{x}}} = 2/w_{0,\tilde{x}}$ and $w_{k_{\tilde{y}}} = 2/w_{0,\tilde{y}}$ are the beam width factors in the k-space, and $w_{0,\tilde{x}}, w_{0,\tilde{y}}$ are the beam waist on the $\tilde{x},\tilde{y}$ directions. Each component of the spectrum carries a power fraction proportional to $A(k_{\tilde{x}},k_{\tilde{y}})^2$ and the beam coupling efficiency $T_{OX,b}$ can then be computed applying (1) to each plane wave component properly weighted:

$$T_{OX,b}(\theta,\phi,\phi_w) = \iint_{-\infty}^{+\infty} T_{OX}\left(\frac{(\mathrm{R}_{\theta,\phi,\phi_w}\tilde{\mathbf{k}})_y}{k_0},\frac{(\mathrm{R}_{\theta,\phi,\phi_w}\tilde{\mathbf{k}})_z}{k_0}\right)\frac{A(k_{\tilde{x}},k_{\tilde{y}})^2}{\iint A(k'_{\tilde{x}},k'_{\tilde{y}})^2\mathrm{d}k'_{\tilde{x}}\mathrm{d}k'_{\tilde{y}}}\mathrm{d}k_{\tilde{x}}\mathrm{d}k_{\tilde{y}}, \qquad (3)$$

The projections (k_x, k_y, k_z) of each spectral component $\tilde{\mathbf{k}} = (k_{\tilde{x}}, k_{\tilde{y}}, \sqrt{k_0^2 - k_{\tilde{x}}^2 - k_{\tilde{y}}^2})$ in the plasma reference system (red axes in Fig. 1) are calculated through the rotation matrix $\mathrm{R}_{\theta,\phi,\phi_w}$. The k_x component becomes null at the critical density layer, as predicted by the wave dispersion relation. k_y and k_z, on the other hand, affect the mode coupling according to Eq. (1). The optimal aiming thus occurs when the beam axis far from the critical density layer is on the direction $(\sqrt{1 - N_{z,opt}^2}, 0, N_{z,opt})$ in the plasma reference system. A preservation of the spectrum along the beam path is assumed in this work, despite in the real case modifications may occur along the propagation path in the plasma. Under this assumption, for fixed plasma parameters, the coupling efficiency at the cutoff layer depends only on the beam waist $w_{0,\tilde{x}}, w_{0,\tilde{y}}$ and on the beam orientation, given by $\mathrm{R}_{\theta,\phi,\phi_w}$.

Application to the Frascati Tokamak Upgrade device

The application of the model to Frascati Tokamak Upgrade (FTU) device (major radius $R_0 = 0.935$ m, minor radius $a = 0.29$ m) is considered as an example. A typical high density plasma discharge with central density $n_{e,0} = 3.4 \cdot 10^{20}$ m^{-3}, and a beam launched from the upper mirror of the new ECRH antenna [5, 6] with $\omega = 2\pi \cdot 140$ GHz are considered for the calculations. The beam, launched from major radius $R = 1.258$ m and $Z = 0.148$ m above the equatorial plane, is Gaussian and stigmatic, with $w_{0,\tilde{x}} = w_{0,\tilde{y}} = 1.2$ cm. This leads to simpler calculations since only the first order beam-mode $m = n = 0$ in Eq. (2) plays a role in the description. The beam-tracing code GRAY [7] was used to find the launching angles allowing to reach the critical layer $X = 1$ at optimal incidence (Fig. 2). At the conversion layer the normalized magnetic field is

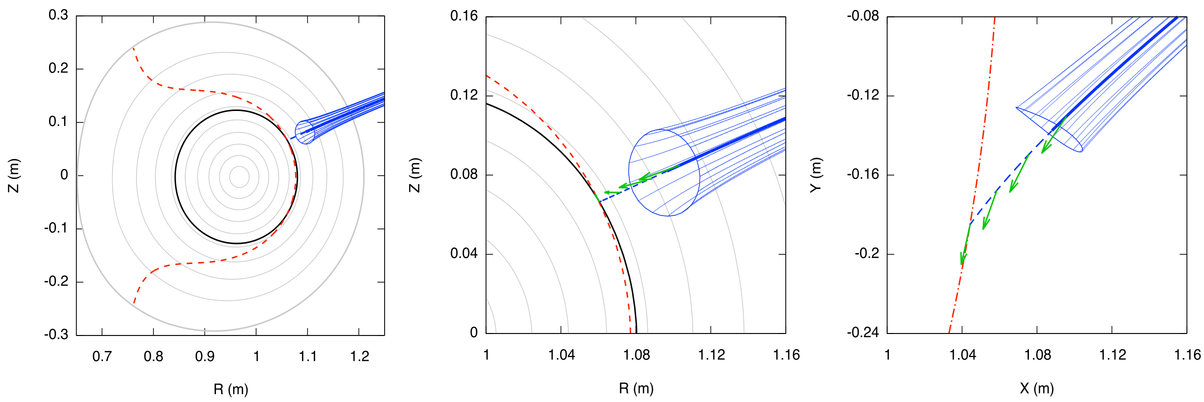

Figure 2: Poloidal view (left, centre) and top view (right) of the beam-tracing for optimal aim in FTU shot # 33717 at $t = 0.9$ s. O-mode cutoff $X = 1$ (black solid) and X-mode cutoff $X/(1+Y) = 1 - N_{\parallel}^2$ (red dashed) intersect at the conversion point (red dash-dot). Phase velocity direction (green arrows) and group velocity decouple approaching the cutoff.

$Y = 0.935$, and the density scale length is $L_n = 0.14$ m. Assuming that the beam shape evolution along its path is not affected by the plasma, we used Eq. (3) to evaluate the O-X conversion efficiency for optimal aiming (i.e. $\cos\theta = N_{z,opt}$ and $\phi = 0$), obtaining a power fraction $T_{OX,b}$ of O-mode transmitted to X-mode of about 30%. The use of the beam-tracing code also allowed us to asses the actual beam sizes $w_{\tilde{x}}, w_{\tilde{y}}$ and phase front curvatures $R_{C,\tilde{x}}, R_{C,\tilde{y}}$ as modified by the interaction with the plasma, at small distance ($\Delta x \simeq 3.5$ cm) from the conversion layer. The measured values of w and R_C correspond to those of an astigmatic beam with $w_{0,\tilde{x}} = 0.66$ cm and $w_{0,\tilde{y}} = 0.83$ cm. This beam waist is smaller than that of the launched beam in vacuum, implying a broader $A(k_{\tilde{x}}, k_{\tilde{y}})$ spectrum. As a consequence, a smaller efficiency $T_{OX,b} \simeq 15\%$ is found in this second case.

Conclusions

An analytic method has been proposed to estimate the O-X mode conversion efficiency of a generic paraxial beam field in a cold magnetized slab plasma, making use of the decomposition in Gauss-Hermite beam-modes. As an example, the model is applied to the high density plasma of the FTU tokamak. The conversion efficiency calculated assuming vacuum propagation of a circular beam unaffected by the plasma is around 30%. Considering, instead, the beam shape as computed by the GRAY code in proximity of the critical layer, the efficiency reduces near 15%. The evaluation technique here presented can be considered complementary to other models of O-X conversion available in literature, capable to take into account the plasma geometry in a more refined way [8]. Given the low efficiency expected in FTU an algorithm is being implemented in the automatic controller of the launcher [9], in order to keep the maximum coupling efficiency during the shot.

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