Refraction of electromagnetic waves near electron cyclotron resonance surface in open magnetic trap

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1. Introduction

Because of the short length of EC waves, the geometrical optics approximation usually provides a sufficiently accurate description of their propagation in magnetic devices. By investigating geometrooptical rays, it is possible to qualitatively, and often quantitatively, estimate the possibility of plasma heating, as well as the heating efficiency and the sizes of the energy input region.

A qualitative pattern of geometrooptical rays can often be obtained by considering their behaviour near singular points---various resonances in the medium. For the cyclotron resonance, a such analysis was carried out in paper [1], where it was showed that the singular points can be either nodes or saddles, depending on the curvature of the surface $\omega = \omega_B(\mathbf{r})$ (where $\omega_B = eB/mc$ is the electron cyclotron frequency) and on the ratio between the radiation frequency and the plasma frequency $\omega_P^2 = 4\pi e^2 n_e / m$. These analytic results were obtained in an approximate model in which the magnetic field direction is constant. However, this model is inapplicable to confinement systems where the gradient of the magnetic field strength makes a small angle with the magnetic field itself (because in this case for such model $divB \neq 0$). Sooner it was demonstrated that variation in the magnetic field direction can qualitatively change the wave propagation pattern and can markedly affect the efficiency of electron cyclotron resonance plasma heating in a straight magnetic confinement system [2-4].

In this paper characteristic features of the propagation of electromagnetic electron cyclotron waves in the vicinity of the electron cyclotron resonance surface are investigated with allowance for variation in the magnetic field strength and a corresponding variation in the magnetic field direction. Also influences of plasma density inhomogeneity and spatial dispersion are discussed.

2. Ray trajectories near ECR surface in paraxial approximation

Magnetic field in axisymmetrical system outside the region of its sources is determined from Maxwell equations. Solution of these equations in paraxial approximation may be presented as:

$$\begin{cases} B_z = J_0\left(r\frac{\partial}{\partial z}\right)F(z) \\ B_r = -J_1\left(r\frac{\partial}{\partial z}\right)F(z) \end{cases}, \tag{1}$$

where $B_z(0,z)=F(z)$ is an arbitrary function, $J_{0,1}$ are the Bessel functions presented as power series. Near ECR-surface $F(z)=B_0\left(1-z/L_\parallel+z^2/2a\right)$ magnetic field takes the form:

$$B_z \approx B_0\left(1-\frac{z}{L_\parallel}-\frac{r^2}{4a}\right)\ B_r \approx B_0\frac{r}{2L_\parallel},\ B \approx B_0\left(1-\frac{z}{L_\parallel}-\frac{\xi r^2}{2L_\parallel^2}\right), \tag{2}$$

with $\xi=\frac{L_\parallel^2}{2a}-\frac{1}{4}$. In the expression for B we take into account first non-zero terms over both coordinates z and r. Correspondingly terms in power series for B_z and B_r resulting in the next order corrections into the value of B are omitted.

For modeling of plasma density distribution we can use such form:

$$n_e = n_0\left(1\mp\frac{r^2}{2L_\perp^2}\right). \tag{3}$$

In this approximation we neglect density variation along magnetic field. Plasma density distribution can be with maximum or minimum on axis dependently on sign in (3).

Near ECR region it is convenient to use ray Hamiltonian in form which do not include terms which tend to infinity at cyclotron frequency in spite of the fact, that some of dielectric tensor components possess such property. For example, such ray Hamiltonian for near longitudinal propagation of ECR waves may be written as:

$$H(r,z,N_r,N_z)=N_\perp^2+2\frac{\varepsilon_\parallel}{\varepsilon_-}\left(N_\parallel^2-\varepsilon_-\right)=0, \tag{4}$$

where $\varepsilon_\parallel=1-\upsilon;$, $\varepsilon_-=1-\omega_P^2/\omega(\omega-\omega_B)$ $\upsilon=\omega_P^2/\omega^2$, $N_\perp=N_r\cos\alpha-N_z\sin\alpha$, $N_\parallel=N_r\sin\alpha+N_z\cos\alpha$, $tg\alpha=B_r/B_z$.

In our approximation spatial dependences of coefficients in Hamiltonian take form $N_\perp\approx N_r-\alpha N_z$; $N_\parallel\approx N_z$, $\alpha\approx r/2L_\parallel$, $\frac{2\varepsilon_\parallel}{\varepsilon_-}=-\frac{2\varepsilon_\parallel^0\omega^2}{\omega_{P0}^2}\left(\frac{z}{L_\parallel}+\frac{\xi r^2}{2L_\parallel^2}\right)$; $\varepsilon_\parallel=\varepsilon_\parallel^0+\frac{r^2}{2L_\perp^2}$.

Equations for ray trajectories have following form

$$\begin{cases} \dfrac{dN_z}{dl} = 2\dfrac{\varepsilon_{\parallel}^0}{\upsilon_0}\dfrac{N_z^2}{L_{\parallel}} \\ \dfrac{dN_r}{dl} = 2\dfrac{\varepsilon_{\parallel}^0}{\upsilon_0}\dfrac{\xi r}{L_{\parallel}^2}N_z^2 + \dfrac{r}{L_{\perp}^2} + (N_r - \alpha N_z)\dfrac{N_z}{L_{\parallel}} \end{cases}; \quad \begin{cases} \dfrac{dr}{dl} = 2(N_r - \alpha N_z) \\ \dfrac{dz}{dl} = \dfrac{4\varepsilon_{\parallel}}{\varepsilon_-}N_z - 2(N_r - \alpha N_z)\alpha \end{cases}$$

First equation can be solved separately $N_z = -\dfrac{\upsilon_0}{2\varepsilon_{\parallel}^0}\dfrac{1}{\tau}$. Forth equation can be changed for conservation low for Hamiltonian. Finely we have one second order equation

$$\frac{d^2\rho}{d\tau^2} = \frac{2\rho L_{\parallel}^2}{L_{\perp}^2} + \frac{\upsilon_0}{\varepsilon_{\parallel}^0}\frac{\rho}{\tau^2}\left(\xi - \frac{1}{2}\right), \qquad (5)$$

where $r = L_{\parallel}\rho$. Solution of this equation can be obtained using Bessel functions in following form:

$$\rho = C_1\sqrt{\tau}J_{\frac{1}{2}\sqrt{1+4\frac{\upsilon_0}{\varepsilon_{\parallel}^0}\left(\xi-\frac{1}{2}\right)}}\left(-i\sqrt{2}\tau L_{\parallel}/L_{\perp}\right) + C_2\sqrt{\tau}J_{-\frac{1}{2}\sqrt{1+4\frac{\upsilon_0}{\varepsilon_{\parallel}^0}\left(\xi-\frac{1}{2}\right)}}\left(-i\sqrt{2}\tau L_{\parallel}/L_{\perp}\right), \quad (6)$$

this solution was obtained for plasma density profile with maximum on axis. For case of profile with minimum we mast change $-i/L_{\perp} \to /L_{\perp}$.

From this solution one can see that relatively far from ECR region ($|\tau| \sim 1$) ray trajectories depend on plasma density distribution, at that plasma density distribution with minimum on axis stabilize ray trajectories, and plasma density distribution with maximum on axis “extrude” rays to periphery. And only in closest region near ECR surface $\tau << L_{\perp}/L_{\parallel}$, ray trajectories depends on both magnetic field strength and direction. Such conclusions are in agreement with results of numerical modeling of wave propagation in axisymmetrical magnetic trap [5-7].

For $|\tau| \to 0$ we can use power series for Bessel function $J_{\nu}(x) \sim x^{\nu}$. And we can obtain solution for ray trajectories from paper [4], and criteria of attraction rays for resonance point on the axis from papers [2,3]

$$4\frac{\upsilon_0}{\varepsilon_{\parallel}^0}\left(\xi - \frac{1}{2}\right) < 0 \qquad (7)$$

When parameter ξ becomes greater than $1/2$ bifurcation takes place, and changing of type of resonance point is accompanied with appearance of new out-axis attractive points for undercritical plasma, and “turning” points for overcritical ones [3]. Positions of these points are determined from condition

$$\left(\cos\alpha\frac{\partial}{\partial r}-\sin\alpha\frac{\partial}{\partial z}\right)\omega_B=0\,. \qquad (8)$$

For overcritical plasmas these points separate trajectories which tend to axis of trap, and trajectories tends to periphery of trap.

3. Influence of spatial dispersion

Near ECR surfaces where $\omega\to\omega_B$ and $N_\parallel\to\infty$, effects of spatial dispersion can be considerable. In first order spatial dispersion can be taken in to account as following modification of Hamiltonian (without absorption)

$$H=N_\perp^2+\frac{2\varepsilon_\parallel}{\varepsilon_- - \dfrac{\omega_L^2\beta_T^2\omega N_\parallel^2}{(\omega-\omega_B)^3}}N_\parallel^2-2\varepsilon_\parallel=0 \qquad (9)$$

For this Hamiltonian critical point (resonance in terms of $N_\parallel^2\to\infty$) take place on surface

$$\omega_B=\omega+\left(\beta_L^2\omega_L^2\omega\right)^{1/3}\approx\omega\left(1+\left(\frac{p_e}{p_B}\right)^{1/3}\right), \qquad (10)$$

where p_e/p_B ratio of plasma pressure and magnetic pressure. Resonance surface shift to high field side. And this shift even for cold and not very dense plasma $T_e\approx 10эB\to\beta_T^2\approx 2\cdot 10^{-5}$, $\omega_L^2/\omega_B^2\approx 0.4$ noticeable $\left(\beta_L^2\omega_L^2/\omega_B^2\right)^{1/3}\approx 0.08$.

Consequently inhomogenity of magnetic field direction as well as plasma density distribution and spatial dispersion have a grate influences on ray trajectories in axisymmetrical magnetic traps. And these influences must be taken into account for estimation of efficiency of ECR heating of plasmas in open traps.

References

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